MULTICOMPONENT SIGNAL: LOCAL ANALYSIS AND ESTIMATION

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ABSTRACT

In previous published works [8, 3], we have studied the estimation of nonstationary monocomponent signals on short time-windows. Both of the instantaneous amplitude and frequency (IA/ IF) were modeled by polynomial functions. The maximization of the likelihood function was achieved by using a stochastic optimization technique: the Simulated Annealing (SA). The proposed algorithm was superior to the existing methods in terms of estimation accuracy and robustness in the presence of low Signal to Noise Ratio (SNR). Motivated by its efficiency and optimality in the monocomponent case, this paper is an extension for multicomponent signals. The synthesis algorithm iteratively reconstructs the signal, one component per iteration. During each iteration, the IA and IF of each component are synthesized by using Maximum Likelihood (ML) estimators and the SA technique. Monte Carlo simulations are presented and compared to the appropriate Cramer-Rao Bounds (CRB). This proves the efficiency and the performance of the algorithm. Moreover it underscores the superiority on previous methods to estimate the crossing frequency trajectories which is a great challenge related to the low sample number.

1. INTRODUCTION

In this work, we consider multicomponent signals with time-varying amplitude and frequency defined as:

$$s[n] = {}^{K}_{i=1} A_i[n]\, exp\,(j \quad {}_i[n])\,. \tag{1}$$

We assume the presence of K components. $\quad {}_i[n]$ is the phase of the i^{th} component. Its amplitude $A_i[n]$ is assumed to be positive. We note $F_i[n]$ its instantaneous frequency. Such signals arise in many real life applications such as mechanics, radar, sonar and wireless communications.

In [1, 2, 7, 8, 9], polynomial phase and amplitude models were assumed. In [2] ML estimators were derived. The optimization problem was solved using the Higher Ambiguity Function (HAF) which is a suboptimal technique. Then it was followed by an iterative minimization algorithm: BFGS quasi-Newton technique. It is well-known that this does not ensure global optimality. Not only this method needs high SNR, but its efficiency is limited due to the cross terms in presence of multiple components. Thus, it is adapted to linear Frequency Modulation and fails to estimate higher nonlinear modulation.

As we are interested in modeling any kind of nonstationarity, we propose to locally track modulation changes. The analysis proposed in this paper extends some previously published works [4, 5, 6], where only monocomponent signals were processed. We considered the signal on short contiguous segments. Then, on each one, we approximated both the IA and IF by second order polynomials. The model parameter estimation was carried out using a ML principle, optimized via the SA technique. The algorithm presented in [4, 5, 6] was robust in the presence of low SNR and more efficient than the HAF.

Now, we extend the study to multicomponent signals on local segments. We give the IA and IF polynomial models for each component in Section 2. The ML procedure and the SA concept, useful for the estimation process, are briefly described too. In Section 3, the signal reconstruction is accomplished by an iterative algorithm, we perform a component-by-component estimation. At each iteration, the IA and IF estimates of one component are provided. The algorithm ending is controlled by a whiteness test on the residual signal. In Section 4, the CRBs are established for parameter models. In section 5, some numerical examples illustrate the performance in estimating frequency crossing-trajectories. Section 6 concludes on the efficiency of the proposed algorithm in presence of low SNR, quadratic Amplitude and Frequency Modulation (AM/FM).

2. PROBLEM FORMULATION

Let us consider $y[n]$ a discrete time process consisting in the sum of deterministic multicomponent signals with AM and FM modulation embedded in an additive white Gaussian noise $e[n]$ with zero mean and unknown variance.

$$y[n] = s[n] + e[n], \qquad for\, \frac{-N}{2} \leq n \leq \frac{N}{2}, \tag{2}$$

where $N+1$ is the sample number, assumed to be odd for simplicity. $s[n]$ is given by (1). We propose to locally follow highly modulations. We consider the IA and IF on short time segments whose lengths are about three time periods.

2.1 Parametric model

According to Weierstrass theorem and thanks to the shortness of the segment, we assumed in [6], second order polynomial functions are sufficient to approximate the IA and IF on $[\frac{-N}{2}, \frac{N}{2}]$. More specifically, let us consider $g_0[n], g_1[n]$ and $g_2[n]$ a second order polynomial base, defined on $[\frac{-N}{2}, \frac{N}{2}]$, with order equal to $0, 1$ and 2 respectively. The parametric description of the IA, the IF and the continuous phase of the

i^{th} component are given by the following:

$$\begin{aligned} A_i[n] &= \quad {}^{2}_{k=0}\, a_{i,k}\, g_k[n] \\ F_i[n] &= \quad {}^{2}_{k=0}\, f_{i,k}\, g_k[n] \\ {}_i[n] &= \quad {}_{i,0} + 2 \left({}^{n}_{k=\frac{-N}{2}} F_i[k] - {}^{0}_{k=\frac{-N}{2}} F_i[k] \right). \end{aligned} \tag{3}$$

The initial phase ${}_{i,0}$ is referenced to the center window in order to minimize estimation errors [3]. All model parameters are real valued. Actually, we have to estimate $= \{ {}_1, ..., {}_K \}$ where ${}_i = \{a_{i,0}, a_{i,1}, a_{i,2}, {}_{i,0}, f_{i,0}, f_{i,1}, f_{i,2}\}$ is a set of seven parameters of the i^{th} component.

In [4, 5, 6], we employ a discrete orthonormal polynomial base. It allows uncoupled estimation of amplitude parameters for a monocomponent signal. This is no longer true for parameters that belong to distinct components. Nevertheless, a good estimation accuracy is still obtained by applying this base.

2.2 Maximum Likelihood estimator

Since the noise is assumed to be a white Gaussian process, the ML procedure is equivalent to the least squares (LS). So, we have to minimized the following equation

$$\hat{} = \underbrace{argmin}_{\in \ {}^{7K}} \ {}^{\frac{N}{2}}_{n=\frac{-N}{2}} |y[n] - \hat{s}[n]|^2 \tag{4}$$

Where $y[n]$ is the noisy observations. $\hat{s}[n]$ is the signal model, computed by substituting (3) into (1) for a given . Due to the nonlinearity of equation (4), this cannot be solved analytically. In [4, 5, 6], the SA technique was used because of its significant efficiency, when a desired global extremum is hidden in many local extrema [10]. Its implementation was relatively simple and has provided accuracy in estimating parameters for monocomponent signals using ML estimators. Hence, it had advantages on suboptimal techniques. For more details, see [6]. Aiming to use it in further sections, the main steps, involved in the SA technique, are summarized as follows. For simplicity, we here note by the set of parameters to estimate.

Given the initialization of , we run I iterations of three first steps. I is a fixed iteration number, which is asymptotically determinate in order to accelerate the convergence.

1. We generate new candidates ${}_C$ from a Gaussian probability law, centered on and with variance . is an agitation value which avoids converging to local minimum.
2. If ${}_C$ minimizes the LS, then we set $= {}_C$, otherwise value is not modified.
3. Then, generate u from a uniform law on $[0,1]$, if $u \leq \frac{2}{3}$, then $= 0.97 *$. This step linearly reduces the agitation value in a random way in order to increase the convergence rate.
4. Since the I^{th} iteration is achieved, we compare the mean square errors (MSE) of the parameter estimates with an asymptotic MSE threshold. We restart the estimation if the evaluated MSE is not the lowest.

3. ITERATIVE RECONSTRUCTION SIGNAL

Instead of simultaneously considering all the component parameters, which induces a high computational cost, we develop an iterative algorithm. We process the signal component-by-component. During each iteration, the estimation of $A_i[n]$ and $F_i[n]$ for the i^{th} component are carried out by using the SA technique and equation (4). Thus, we avoid to estimate 7 K parameters at the same time. The algorithm main steps are as follows.

1. Set $i = 1$,
2. Initialize the parameter values of the i^{th} component from the Fast Fourier Transform (FFT) of the noisy signal $y[n]$. ${}_i = \{a_{FFT}, 0, 0, {}_{FFT}, f_{FFT}, 0, 0\}$,
3. Apply the SA algorithm in order to estimate ${}_i$.
4. Once the frequency and the amplitude of the i^{th} component are evaluated using (3), we reconstruct the component $s_i[n] = A_i[n].e^{j \ {}_i[n]}$. We remove it from the noisy signal to generate a new noisy signal $y[n]$.
5. Check if the remained signal $y[n]$ is a white process. In this case, the component estimation is finished. If the answer is negative, set $i = i+1$ and restart step 2 in order to estimate the next component.

Since the estimation algorithm is iterative, the success of estimating one component depends on all the previous component estimates. So, we are optimal for a monocomponent case only. Nevertheless, as we show in Section 5, the accuracy on the estimation is sufficiently high. Moreover this synthesis algorithm provides an estimation of the component number K. Here, we note that the resolution on the Time-Frequency plan is critical due to the low sample number. So estimating K from ridges in the spectrogram or in the MCE-TFD representation (minimum cross entropy time frequency distribution) [1] is difficult. Furthermore the estimation of K is conditional to the SNR level.

4. CRAMER RAO BOUNDS

In [2], the Fisher Information Matrix (FIM) was given for amplitude and phase parameters for multicomponent signals. So, we derive it for amplitude and frequency parameters. The FIM for is then given by

$$FIM(\ ) = \frac{2}{{}^2} Re \left\{ \begin{bmatrix} \mathbf{A}_i^H \mathbf{A}_j & \mathbf{A}_i^H \ {}_j \\ {}_i^H \mathbf{A}_j & {}_i^H \ {}_j \end{bmatrix}_{\substack{1 \leq i \leq K \\ 1 \leq j \leq K}} \right\} \tag{5}$$

where $\mathbf{A}_i = [g_0(\underline{n}) \bullet e^{j \ {}_i(\underline{n})}, g_1(\underline{n}) \bullet e^{j \ {}_i(\underline{n})}, g_2(\underline{n}) \bullet e^{j \ {}_i(\underline{n})}]$, and ${}_i = j[\ {}_{-1}(\underline{n}) \bullet s_i(\underline{n}),\ {}_0(\underline{n}) \bullet s_i(\underline{n}),\ {}_1(\underline{n}) \bullet s_i(\underline{n}),\ {}_2(\underline{n}) \bullet s_i(\underline{n})]$.

${}_i(\underline{n})$ and $s_i(\underline{n})$ are vectors of the phase and signal values of the i^{th} component at each time n. $s_i(\underline{n})$ is equal to $A_i(\underline{n}) \bullet e^{j \ {}_i(\underline{n})}$. We note $\underline{n} = [\frac{-N}{2}, \frac{-N}{2}+1, ..., \frac{N}{2}]$, ${}_{-1}[n] = 1$ and ${}_i[n] = 2 \ ({}^{n}_{k=\frac{-N}{2}} g_i[k] - {}^{0}_{k=\frac{-N}{2}} g_i[k])$ for $i = 0,1,2$ and $n \in [\frac{-N}{2}, \frac{N}{2}]$. $(\bullet)$ denotes the multiplication element by element of the vector entries.

The CRB for is the inverse of the FIM matrix given by (5). For a monocomponent signal, $\mathbf{A}_i^H \ {}_j$ is purely imaginary. So amplitude and frequency parameters are decoupled. An orthonormal base makes $\mathbf{A}_i^H \mathbf{A}_i$ a diagonal matrix and amplitude parameters become uncoupled. It is not yet the case in presence of multiple components. Moreover, we note from (5) that the FIM for the frequency and the amplitude parameters are functions of the signal components $s_i[n]$, the phases $_i[n]$ and the base functions $g_i[n]$. The FIM depends on the frequency and the amplitude parameters only through the phase and the amplitude waveform. We also note that the FIM is a badly conditioned matrix, when crossing frequency trajectories occur and tends to a singular matrix when the difference in component IFs approaches zero.

5. EXAMPLES

In this section, we give some numerical examples demonstrating the synthesis algorithm. We also evaluate the CRB that was given in Section 4. All the considered signals are of 33 samples. The sampling frequency is 1 Hz. The SNR is defined as the ratio of the energy of a constant amplitude signal, whose energy equals that of the time-varying signal, to noise variance. That is a global definition on all the non-stationary signal. So the get value is not locally correct. Two-component of quadratic AM/FM signals, embedded in Gaussian noise, are used. The experimental plots are based on 50 independent noise realizations. The IA and IF are depicted in Figure 1. Two cases are discussed.

- Case I: The Frequency trajectories are well separated Fig.1(a). The bottom left figure shows the IA of this case.
- Case II: The Frequency trajectories are crossing one the other Fig.1(b). The corresponding IA are shown in the bottom right figure.

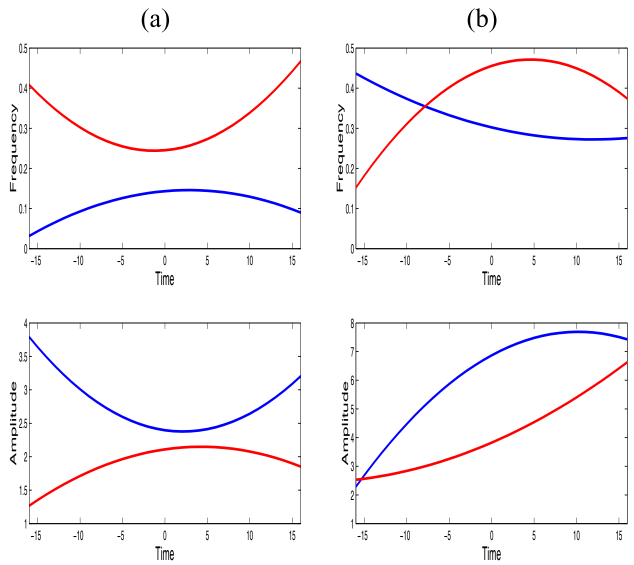

Figure 1: IF and IA of the two-component signal: Left figures illustrate the case I. Right figures illustrate the case II.

It is shown in Fig.2(a) and (b) the reconstruction of the frequency, the amplitude and the signal of case I, using the iterative algorithm. Estimated curves are plotted versus the original ones for SNR equal to 20 dB and 10 dB. Fig.3 displays the IF and IA estimates versus the original ones in case II for SNR equal to 20 dB and 10 dB too.

The estimated curves of the IF are close to the original ones. In the opposite, the IA estimation is less accurate. This effect is due, as we say before, to the estimation dependence

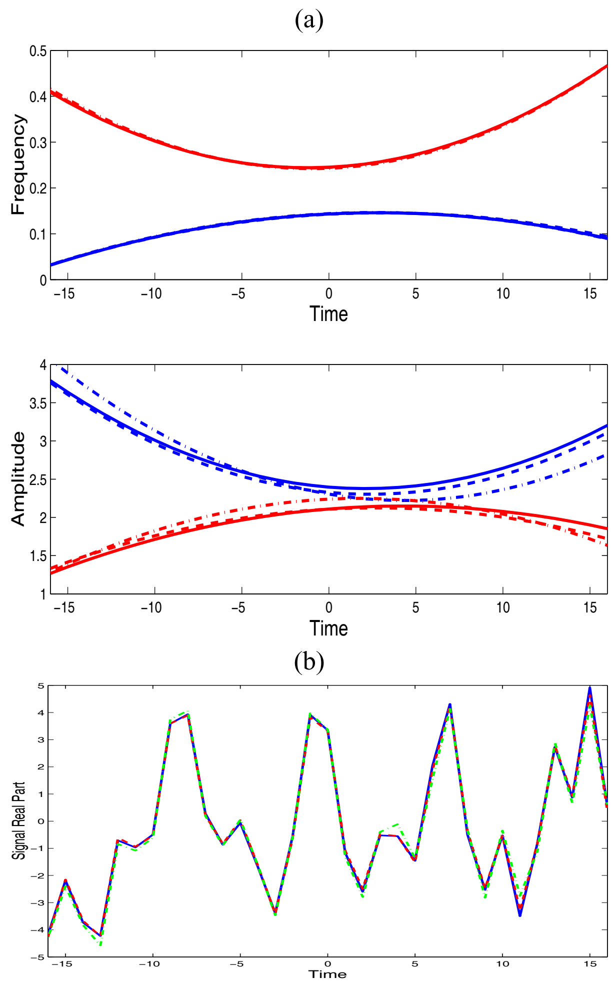

Figure 2: Case I: (a) AM/FM Estimation and (b) signal reconstruction: (dashed line) and (dashed-dotted line) for SNR equal to 20 dB and 10 dB respectively, versus the original curves (solid line).

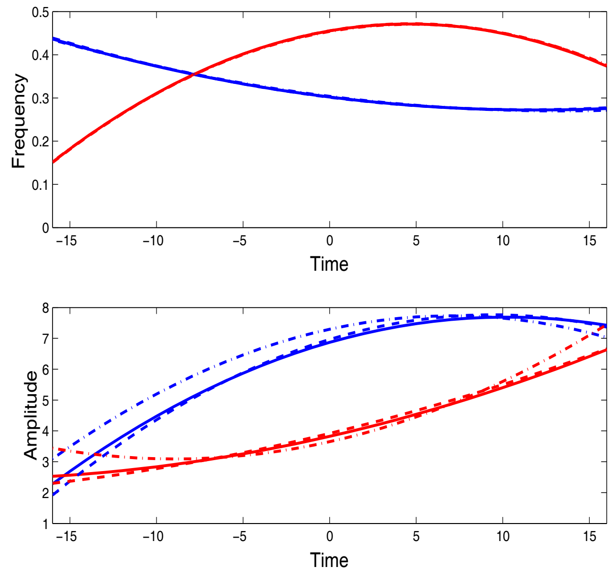

Figure 3: Case II: AM/FM Estimation: (dashed line) and (dashed-dotted line) for SNR equal to 20 dB and 10 dB respectively versus the original AM/FM modulation(solid line).

on the ability to estimate the individual IA and IF of the two signal components. We also have to take into account the low number samples, the nonlinear FM which means cubic phase, and especially the nonlinear AM. However, the proposed algorithm is able to estimate crossing or close frequency trajectories which was a challenge.

In the following, we consider the case I for a statistical parameter study. The solid line denotes the CRB. In Fig.4, the performance estimation of frequency parameter $f_{i,0}$ and amplitude parameter $a_{i,0}$ are reported. Fig.4 shows that the MSE on the variance of parameter estimation is close to the CRB. Similar results are obtained for the other parameters. This highlights the performances of the proposed method in noisy environnement.

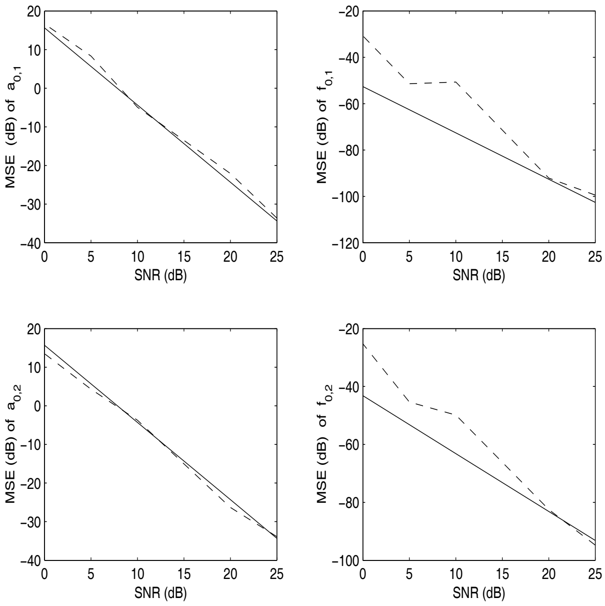

Figure 4: Estimation performance of amplitude and frequency parameters. The MSE of parameter estimation (dashed line) versus the CRB (solid line).

6. CONCLUSION

In this paper, the estimation of nonstationary multicomponent signals is adressed. The frequency and the amplitude are both nonlinear time-varying functions. Based on a previous published technique, whose efficiency was proved for monocomponent signals, we present an iterative algorithm to estimate multicomponent signals. Each component is reconstructed using a Maximum Likelihood procedure solved by a Simulated Annealing technique. This technique is a compromise between optimality and computation complexity. Monte Carlo simulations are compared to the appropriate CRB. It is shown that the estimation is closed to the CRB, even if crossing frequency trajectories occur. After studying signals in contiguous short segments, we aim now to merge all processed segments in order to reconstruct the entire modulations. This will provides a robust way to estimate any class of nonstationary signals.

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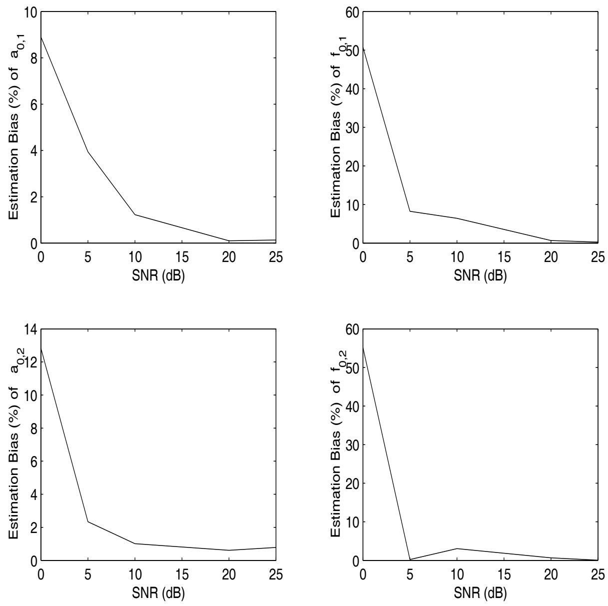

Figure 5: Estimation performance of amplitude and frequency parameters. The Bias of parameter estimation.

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