An Equivalent FIR Filter for an IIR Filter with Double Frequency Initialization

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Abstract— **IIR filters designed for steady state operation suffer from the transient effect caused by processing a finite number of samples. Their performance can be improved by using the initialization technique. This paper presents an equivalent FIR filter for an IIR filter operating in the transient and using double frequency initialization. This equivalent filter simplifies the transient analysis of IIR filters and provides a direct formula for calculating the transient frequency response and usable bandwidth.**

I. Introduction

Steady state digital IIR filters such as Chebychev and Elliptic filters suffer from the transient effect when they process only a limited number of samples. In this case, their frequency responses will be a function of the number of processed samples. Such responses are called the transient frequency responses and they are different from the steady state responses. The degradation is severe in applications such as radar and sonar where the IIR filter is used to reject the clutter signal. One way of improving the transient frequency responses of IIR filters is by initializing their internal memories with values other than zeros. This requires a special processor which is called the initialization processor to calculate the steady state values of the filter for a given input signal. The input signal may be approximated by a step function and this type is called step initialization [1]. Alternatively, the input signal may be approximated by a single tone sine wave and this type is called single frequency initialization [2]. The initialization will force the filter to reach its steady state response at a particular frequency, however, it will degrade the response slightly at other frequencies. This will result in improving the rejection characteristic of the filter provided that the filter has one z-plane zero at that initialized frequency.

Recently a new initialization processor was introduced which can work at two frequencies [3]. This was achieved by breaking the second order real filter into a multiplication of two first order complex sections and initializing the internal memories of each section with its steady state values. This resulted in forcing the filter to reach its steady state frequency response at these two frequencies irrespective of the number of processed samples. The new processor operates in real time and use only the first received input sample of the quadrature and in-phase channels.

Other types of initialization exist for improving the transient performance of IIR filters, however they are suitable for batch processing. The projection initialization is used to remove the transient component from the output of the filtered signals [5]. Exponential initialization [4] suppress the transient of an IIR filter by utilizing a complex exponential with frequency equal to the estimated mean frequency. Regression filters can be used to remove the clutter in color flow imaging [7]. A regression filter calculates the best least square fit of the signal to a set of the curve forms modeling the clutter signal, and subtracts this clutter approximation from the original signal.

This paper derives an equivalent FIR filter for an IIR filter operating in the transient with double frequency initialization. The equivalent filter simplifies the transient analysis of IIR filters with initialization by providing a direct formula for the transient frequency responses and usable bandwidth. This filter will be used to analyze real and complex second order IIR filters.

II. Initialization of Real Filters

The transfer function of a real second order IIR notch filter is given by the following equation:

$$H(z)=\frac{1+a_1 z^{-1}+a_2 z^{-2}}{1-b_1 z^{-1}-b_2 z^{-2}}, \tag{1}$$

where a_1, a_2 are the feed forward and b_1, b_2 are the feedback coefficients of the filter.

The z-transfer function of this filter can be rewritten as a product of two complex sections:

$$H(z)=\frac{1-(a_r+ja_i)z^{-1}}{1-(b_r+jb_i)z^{-1}}\times\frac{1-(a_r-ja_i)z^{-1}}{1-(b_r-jb_i)z^{-1}}, \tag{2}$$

where $a_r=\cos(2\pi f_1 T)$, $a_i=\sin(2\pi f_1 T)$, T is the sampling period ($1/f_s$), and f_1 is the notch frequency. The constants b_r and b_i are the real and imaginary parts of the complex conjugate pair of poles.

The initialization processor, shown in **Fig. 1**, calculates the steady state values of the internal memories for each section from the first received input sample and feeds them to the associated section. Each first order section will have zero transient frequency response at that particular frequency. The output of each section is fed to a multiplier so that the filter will have two zeros in the transient frequency response irrespective of the number of processed samples [3].

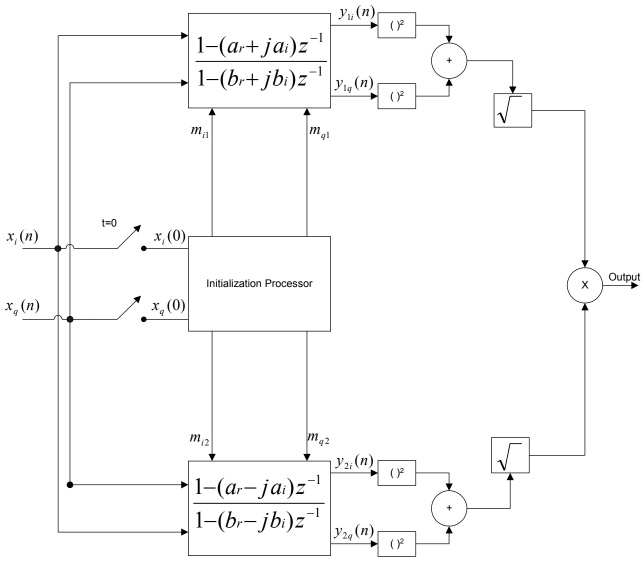

Fig. 1. Two complex first order filters with their initialization processor

The steady state values of the internal memories of the complex sections are given by:

$$m_{i_1} = \mathrm{Re}\left\{ \frac{[x_i(0)+jx_q(0)]z_1^{-1}}{1-(b_r+jb_i)z_1^{-1}} \right\} \tag{3}$$

$$m_{q_1} = \mathrm{Im}\left\{ \frac{[x_i(0)+jx_q(0)]z_1^{-1}}{1-(b_r+jb_i)z_1^{-1}} \right\} \tag{4}$$

$$m_{i_2} = \mathrm{Re}\left\{ \frac{[x_i(0)+jx_q(0)]z_1}{1-(b_r-jb_i)z_1} \right\} \tag{5}$$

$$m_{q_2} = \mathrm{Im}\left\{ \frac{[x_i(0)+jx_q(0)]z_1}{1-(b_r-jb_i)z_1} \right\} \tag{6}$$

III. EQUIVALENT FILTER

The impulse response of the equivalent FIR filter of an IIR filter processing only N samples is the same as the first N samples of the original filter. Thus, the impulse response of the first complex IIR section shown in **Fig. 1**, when processing N samples can be described by the following equation:

$$h_1(n) = \{h_1(0), h_1(1), \cdots\cdots, h_1(N-1)\} \tag{7}$$

It should be noted that the values of the impulse response will be complex. The initialization will force the frequency response of the filter to reach its steady state value (i.e. zero) at f_1. Consequently, the impulse response of its equivalent FIR filter can be described by this equation [6].

$$\hat{h}_1(n) = \left\{h_1(0), h_1(1), \cdots, h_1(N-2), \sum_{i=0}^{N-2} -(h_1(i)e^{-j2\pi i f_1 T}) \right\} \tag{8}$$

The impulse response of the equivalent FIR filter for the second section will be:

$$\hat{h}_2(n) = \left\{h_2(0), h_2(1), \cdots, h_2(N-2), \sum_{i=0}^{N-2} -(h_2(i)e^{-j2\pi i f_2 T}) \right\} \tag{9}$$

As can be seen from equation (8) and (9) the first *N-1* samples will be same as the samples of the original impulse response but the last sample will be different. The last sample is responsible for forcing the frequency response of the equivalent to have zero at the initialized frequency. From **Fig. 1** the outputs of the two initialized complex sections are multiplied together to force the total frequency response to have zero output at both frequencies (f_1 and f_2), therefore the impulse response of the equivalent FIR filter will be the convolution of the two impulse responses.

$$h_{eq} = \hat{h}_1 \otimes \hat{h}_2 \tag{10}$$

Since the first (*N-1*) samples in the equivalent filter are the same as the original filter, equation (7) can be rewritten as follows:

$$h_{eq}(n) = \begin{cases} h(n) & 0 < n < N-1 \\ \sum_{k=0}^{N-1} \hat{h}_1(k)\hat{h}_2(n-k) & N-1 \le n < 2N-2 \end{cases} \tag{11}$$

where $h(n)$ is the impulse response of the original second order filter described in equation (1).

It should be noted that although the second order filter is processing N samples, its equivalent FIR filter will have $2N-1$ coefficients.

The equivalent filter will allow us to calculate the transient response of the second order filter with initialization. The square magnitude of the frequency response of the equivalent filter can be written as

$$|H(f)|^2 = d_0 + 2\sum_{i=1}^{2N-2} d_i \cos(2\pi i f T), \tag{12}$$

where $d_0 = \sum_{j=0}^{2N-2} h_{eq}^2(j)$ and $d_i = \sum_{i=0}^{2N-2-i} h_{eq}(i)h_{eq}(i+j)$

The response is first normalized for a unity white noise average gain, i.e.

$$\sum_{j=0}^{2N-2} h_{eq}^2(j) = 1 \quad so\ that\ d_0 = 1 \tag{13}$$

Using equation (12) is simpler than simulating the in-phase and quadrature channels of the circuit in **Fig. 1** to calculate the transient frequency response of the filter. In addition, the useable bandwidth which is defined as the bandwidth over which the filter has a response more than 0 dB can be calculated by finding the frequencies that satisfy the following equation:

$$\sum_{i=1}^{2N-2} d_i \cos(2\pi i f T) = 0 \qquad (14)$$

A. *Example I*

Consider a second order Chebychev filter with the following transfer function:

$$H(z) = \frac{(1-(0.996 - j0.0888)\,z^{-1})\,(1-(0.996 + j0.0888)\,z^{-1})}{(1-(0.8111 - j0.1727)\,z^{-1})\,(1-(0.8111 + j0.1727)\,z^{-1})}$$

Table 1 shows the impulse responses of this filter when processing only 7 samples. It is clear from this table that the first 6 samples of the impulse responses of the two complex sections with and without initialization are the same. It should be noted here that the impulse response h_1 is the complex conjugate of h_2 because the second order filter is real.

The only difference between the cases with and without initialization is the 7^{th} sample. However, the impulse response of the equivalent filter will have the same impulse response as the original real filter in the first 6 samples. The samples between the 7^{th} and the 13^{th} will be different.

Table 1 The impulse response of the Chebychev real filter and its equivalent filter when processing 7 samples.

n	$h(n)$	$h_1(n)$	$\hat{h}_1(n)$	$h_1(n)$	$\hat{h}_2(n)$	$h_{eq}(n)$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	-0.3698	-0.1849 + 0.0839i	-0.1849 + 0.0839i	-0.1849 - 0.0839i	-0.1849 - 0.0839i	-0.3698
2	-0.2877	-0.1645 + 0.0361i	-0.1645 + 0.0361i	-0.1645 - 0.0361i	-0.1645 - 0.0361i	-0.2877
3	-0.2124	-0.1396 + 0.0009i	-0.1396 + 0.0009i	-0.1396 - 0.0009i	-0.1396 - 0.0009i	-0.2124
4	-0.1467	-0.1134 – 0.0234i	-0.1134 - 0.0234i	-0.1134 + 0.0234i	-0.1134 + 0.0234i	-0.1467
5	-0.0919	-0.0879 – 0.0386i	-0.0879 - 0.0386i	-0.0879 + 0.0386i	-0.0879 + 0.0386i	-0.0919
6	-0.0482	**-0.0647 – 0.0465i**	**-0.1647 - 0.3561i**	**-0.0647 + 0.0465i**	**-0.1647 + 0.3561i**	**-0.2482**
7	0.0150					**0.0589**
8	0.0088					**0.0663**
9	0.0246					**0.0671**
10	0.0339					**0.0632**
11	0.0380					**0.0564**
12	0.0384					**0.1539**

Fig. 2 shows the frequency responses of the Chebychev filter with initialization. It is clear that the rejection characteristic of this filter is improved in comparison with the transient frequency response without initialization. However the pass band characteristics were slightly degraded.

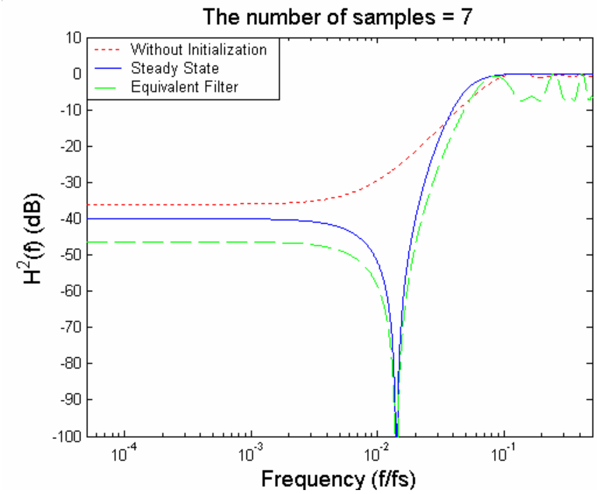

Fig. 2. The square magnitude of the frequency responses of the real second order Chebychev filter and its equivalent filter.

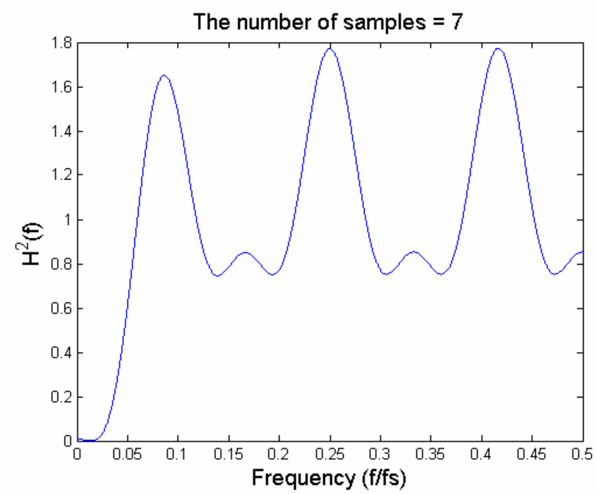

Fig. 3. The pass-band characteristics of the equivalent filter of the Chebychev filter.

Fig. 3 demonstrates the degradation in the pass-band characteristic which resulted from the initialization process. From this figure the useable bandwidth for the Chebychev filter processing 7 samples with initialization is 0.4015 f_S.

B. *Example II*

As a second example, consider a second order Elliptic filter with the following transfer function:

$$H(z) = \frac{(1-(0.9504 - j0.3111)\,z^{-1})\,(1-(0.9504 + j0.3111)\,z^{-1})}{(1-(0.4612 - j0.4957)\,z^{-1})\,(1-(0.4612 + j0.4957)\,z^{-1})}$$

Fig. 4 shows the frequency responses of this Elliptic filter with initialization. It is clear that rejection characteristic of this filter is improved in comparison with the transient frequency response without initialization. Note that the degradation in the pass band characteristics were less than those for the Chebychev filter.

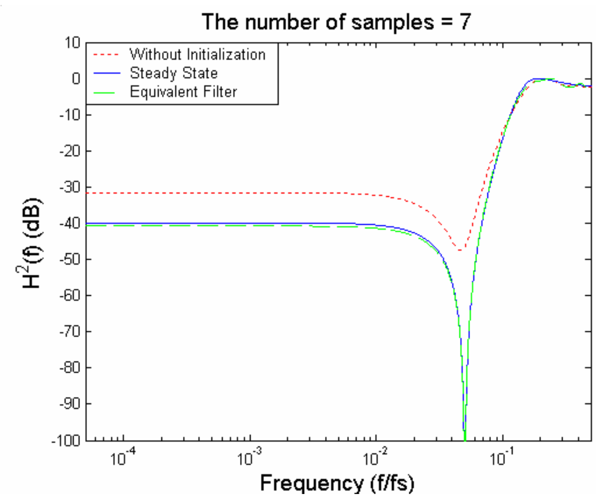

Fig. 4. The square magnitude of the frequency responses of the real second order Elliptic filter and its equivalent filter.

IV. Complex filters

The initialization processor described in section III was used to initialize a second order real filter at two frequencies (f_1 and f_S-f_1). The same techniques can be extended to complex second order filters with zeros that are not complex conjugate pairs.

A. Example

Consider a complex second order filter with transfer function:

$$H(z)=\frac{(1-(0.9511+ j0.3090)\,z^{-1})\,(1-(0.8090\ +\ j0.5878)\,z^{-1})}{(1-(0.8111 - j0.1727)\,z^{-1})\,(1-(0.8111+j0.1727)\,z^{-1})}$$

Table 2 The impulse response of the complex filter and its equivalent filter when processing 7 samples.

n	$h(n)$	$h_1(n)$	$\hat{h}_1(n)$	$h_2(n)$	$\hat{h}_2(n)$	$h_{eq}(n)$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	-0.1379 - 0.8968i	0.0021 - 0.4151i	0.0021 – 0.4151i	-0.1400 – 0.4817i	-0.1400 - 0.4817i	-0.1379 - 0.8968i
2	-0.3236 - 0.6458i	0.0734 - 0.3363i	0.0734 – 0.3363i	-0.1967 – 0.3666i	-0.1967 - 0.3666i	-0.3236 - 0.6458i
3	-0.4301 - 0.4308i	0.1176 - 0.2601i	0.1176 – 0.2601i	-0.2229 – 0.2633i	-0.2229 - 0.2633i	-0.4301 - 0.4308i
4	-0.4752 - 0.2548i	0.1403 - 0.1907i	0.1403 – 0.1907i	-0.2262 – 0.1751i	-0.2262 - 0.1751i	-0.4752 - 0.2548i
5	-0.4750 - 0.1170i	0.1467 - 0.1304i	0.1467 – 0.1304i	-0.2137 – 0.1030i	-0.2137 - 0.1030i	-0.4750 - 0.1170i
6	-0.4438 - 0.0146i	**0.1415 - 0.0804i**	**0.0417 – 0.3900i**	**-0.1911 – 0.0466i**	**-0.2910 + 0.2629i**	**-0.6436 - 0.0146i**
7	-0.3933 + 0.0567i					**-0.3658 + 0.2358i**
8	-0.3328 + 0.1021i					**-0.2682 + 0.2311i**
9	-0.2694 + 0.1266i					**-0.1835 + 0.2127i**
10	-0.2081 + 0.1352i					**-0.1132 + 0.1861i**
11	-0.1523 + 0.1322i					**-0.0575 + 0.1556i**
12	-0.1040 + 0.1215i					**0.0904 + 0.1244i**

Table 2 shows the impulse responses of a second order complex filter processing 7 samples. The impulse response of the equivalent filter will have the same impulse response as the original filter in the first 6 samples. The samples between the 7th and the 13th will be different.

Fig. 5 shows the frequency responses of the filter described in **Table 2**. It is clear that the rejection characteristics of this filter are improved in comparison with the transient frequency response without initialization.

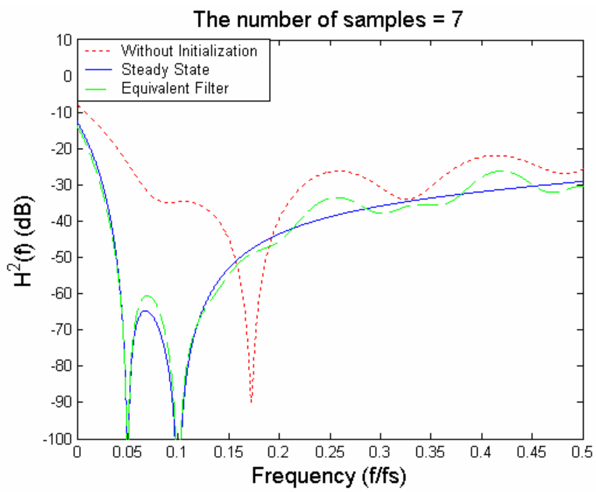

Fig. 5. The square magnitude of the frequency responses of a complex second order filter and its equivalent filter.

V. Conclusion

An equivalent FIR filter for a second order IIR filter operating in the transient with double frequency initialization was derived. The equivalent FIR filter is applicable to both real and complex IIR filters. This equivalent filter simplified the transient analysis of IIR filters with initialization by providing a direct formula for the transient frequency responses and usable bandwidth. The equivalent filter will pave the way for optimizing the transient characteristic of IIR filters.

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