# A New Quadtree-Based Symmetry Transform (QST) With Application to Arab-andalusian images Indexing 

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#### Abstract

This paper describes a new method for the Arab-andalusian images features extraction for indexing. Our original approach is based on the extraction of symmetry information using a quadtree-based symmetry Transform (QST). Our method is inspired by the way that the artits (Maalam) draw the shapes of the patterns. We build a quadtree on the image center then we give a measure of symmetry, a feature vector is constructed and a similarity measure is given to compare two images. We have evaluated the algorithm on our Arab-andalusian database and found that the algorithm is able to discriminate small variations of symmetry. The obtained results significantly support the capabilities of our proposed approach to classify and recognize arab-andalusian images in terms of symmetry information.


## I. Introduction

Islamic Decorative Art consists of three main elements: calligraphy in various forms of Arabic script, arabesques (floral or plant-like designs) and geometrical designs called zillij in Moroccan Arabic.
The geometric decoration is the most vital of the Andalusian/Moroccan tradition and form a part of the human cultural heritage. Nowadays the zillij is widely used in decoration industry, particularly in commercial applications such as apparel design and interior design [8], they have been also widely used as a fundamental resource to archive decorative effects. In computer graphics, such images may be used for texture mapping.
In a real context, like textile and decoration production, there are a huge collections of zillij images that we cannot access or make use unless they are organized so as to allow efficient browsing searching, and retrieval of those images.
A work on Arabo-andalusian images indexing and classification focuses on the classification of the patterns by their symmetric groups [2], [15], [11]. They consider a pattern as a repetition of some motifs. In [14], the authors describe the structure of a pattern design by the use of some descriptors based on the theory of symmetry groups. This approach is not appropriate because the symmetry group theories do not relate to the way of thinking of the artists involved, and it has completely ignored the attributes of the unit pattern and has focused exclusively on arrangement formats [4].
A work accomplished by Benslimane et al. [6], [1] consists of the extraction of the spine of an image manually and the

[^0]characterization of the extracted spine by the Fourier Shape Descriptor (FSD). The major inconvenience of such approach is that it is supervised.
In this paper, we propose a new method for characterizing zillij images based on a symmetry measure derived from the quadtree representation of the image. This method is called Quadtree-based Symmetry Transform (QST) which gives us both global and a local measure of symmetry.
The quadtree data structure [12] has been used in several areas in digital image processing and computer graphics.
Ahmed et all. in [3] use the quadtree decomposition for image indexing based on the relationship between the extracted nodes. Another approach called relational linear quadtree which provides information at multiple resolutions to describe spatial features together with descriptive information in desired structures is presented in [16]. But none of those approaches attempt to give a scheme for image indexing based on the extracted symmetry information from the quadtree representation of images.
In computer vision, measuring the amount of symmetry is an important task.
Atallah [5] detected the axis of reflectional symmetry by first determining the centroid position and then using a string pattern matching technique, which considered all possible lines passing through the centroid. However, this approach can only be applied to algebraically defined patterns. Reisfeld [9], [7] describes an operator that measures symmetries using the image gradient. The algorithm to detect points of symmetry is developed in two phases: 1) compute the gradient of the input image; 2) compute the symmetry value for each point of the gradient image. This method is computer time expensive. Zabrodesky et all. [17] introduced a symmetry measure to quantify the deviation of shapes and objects from perfect symmetry. Those approaches are important contributions for computational geometry, but they are not appropriate for images indexing based on the symmetry information.
The rest of this paper is organized as follows: In section II, we present the Arab-andalusian images and figure out some of its characteristics. The formulation of our Quadtreebased Symmetry Transform and the definition of a symmetry measure that is able to capture both local and global symmetry is presented in Section III. Section IV presents some experimental results on the Arab-andalusian images indexing using this measure of symmetry. Finally, in section V some conclusions are drawn, and future works are discussed.

## II. ARAB-ANDALUSIAN IMAGES

The most striking characteristic of Arab-andalusian images (Figure 1) is the prominence of star and rosette shapes. Such shapes with five, six, eight, ten, twelve and sixteen rays are the ones that occur most frequently [8], [2], but patterns containing other number, particularly in multiples of eight up to ninety six, can be found [8]. Every star is surrounded by a ring of regular hexagons. The pattern can be regarded as being composed of these surrounded stars, or rosettes. An essential characteristic is the symmetry, in


Fig. 1. Example of Arab-andalusian images
fact, symmetry plays a part in most Arab-andalusian images. There may be a single line of reflective symmetry, usually from the top to the bottom, or there may be three or four lines of symmetry. Straight (translation) and turning (rotational) movements are also used. Sometimes reflective symmetry and the two kinds of movement are found in the same design. In many Arab-andalusian images, different elements seem to dominate, depending on how you look at the design. Colours can enrich the motif by breaking down some symmetry while respecting others.
As a whole, the technique of designing the figures are highly kept secret among the artists, but, according to the recent research, we found out that there is a certain traditional way through which some artists in Morocco could shape the figure preliminary first, they draw a fundamental region on a folded paper, then, they unfold the paper to get the ultimate shape of the decoration. The fundamental region is a piece of the tiling you can shift around to tile the entire plan.
So, an Arab-andalusian image will necessarily contain information that could be copied from another part of the image. In this paper we aim at using the quadtree image decomposition to factoring up all the symmetries of an Arabandalusian image. So to reduce it to a minimal set of non redundant information.

## III. OUR APPROACH

## A. Quadtree

A quadtree is a tree in which each node has four descendants. The root of the tree of Figure 2 corresponds to the entire image array. Each child of a node represents a quadrant of the region corresponding to that node. The children form left to right, correspond to the NW (Northwest), NE, SW and SE quadrants leaf nodes correspond to those quadrants
for which no further subdivision is necessary (See Figure 2).


Fig. 2. (a) Image partition. (b) The corresponding quadtree

## B. Quadtree Symmetry Transform (QST)

Generally a symmetry group $g$ of a $2 D$ image $I$ is a distance preserving mapping (translation, rotation, reflection, or composition of these) that maps every pixel in the image to a pixel of the same Gray value or colour.
We refer to the symmetries defined below, although the definitions and methods presented in this work can be extended to capture any other symmetry type.
A planar image is said to be reflection-symmetric if it is invariant to reflection with respect to one or more straight lines, denoted as reflection-symmetric axes. We define three groups of reflection symmetry namely $g_{1}, g_{2}$ and $g_{3}$ based on the quadtree decomposition of the image $I$ (see Figure 3). For the sake of simplicity, let us take the case of a one


Fig. 3. The three basic Symmetry operations
level quadtree $Q$ represented by the four blocks $N W, N E$, $S W$ and $S E$. The $g_{1}$ group models the horizontal translation symmetries (Figure 3(a)).
If $N W$ is symmetric with $N E$ and $S W$ is symmetric with $S E$ we say that $Q$ is $g_{1}-$ symmetric, if only $N W$ is symmetric with $N E$ or $S W$ is symmetric with $S E$ we say that $Q$ is $g_{1}$ - halfsymmetric.
In the same manner, the $g_{2}$ models the vertical translation symmetries and the $g_{3}$ models the diagonal translation symmetries. Let $O$ be the central point of the image decomposition, and let $Q$ be a quadtree centred on $O$, we say that two blocks $b_{1}$ and $b_{2}$ of $Q$ are symmetric and we denote by $\left(b_{1} / / b_{2}\right)$ if:

1) The two blocks $b_{1}$ and $b_{2}$ have the same decomposition level (same size),
2) the position of $b_{1}$ is symmetric with the position of $b_{2}$ according to a horizontal, vertical or diagonal translation vector passing through the origin $O$,
3 ) and $b_{1}$ is similar to $b_{2}$, (or some rotated $b_{2}$ ).

The similarity problem can be treated as a rigid comparison problem between the original block $b_{1}$ and the reflected block $b_{2}$. Therefore, one can use different similarity measures known in literature.
In this paper we use the block matching method defined in [10].


Fig. 4. Example of two symmetric blocks


Fig. 5. QST example: (a) an image grid, (b) his quadtree representation, (c) the Symmetric image according to the vertical axis and (d) the symmetric image corresponding quadtree.

To determine the presence of a certain kind of symmetry in an image represented by his quadtree $Q_{i}$ we must determine for every block $b$ the symmetric blocks according to the $g_{1}$, $g_{2}$ and $g_{3}$ groups. The symmetry will be measured in terms of the number of blocks symmetric and similar to $b$. Figure 5(d) presents the symmetric block according to $g_{2}$ of the block represented in Figure 5(b).
An algorithm for deriving the symmetric quadtree $Q T s$ of a quadtree $Q T$ according to the group $g_{2}$ is given as below:

1) We give the following denotations: $N W=1, N E=$ $2, S W=3$ and $S E=4$ and we mark the position and the number of the block $b$ (the bold squares in figure 6).
2) for every block $b_{i}$ replace his number according to the transformation below (Figure 7). The position of the marked block remain unchanged,
3) reorganize the tree by ordering the numbers, the position of the marked block must change
to obtain the symmetric quadtree $Q T s$ of $Q T$ according to $g_{1}$ or $g_{3}$ we should replace the transformation in the second step of this algorithm by the corresponding one.
This symmetry representation and measurement is called the QST (Quadtree Symmetry Transform)


Fig. 6. The transform according to vertical axis


Fig. 7. Illustration of our algorithm steps

## C. Symmetry Measure

We seek to answer the following: How much symmetry is there in an image with respect to its quadtree representation? Let $I$ be the image and let $Q$ be the quadtree corresponding to $I$. Suppose that $Q$ is of depth $d$. We define the symmetry measure as follows: For every block $b$ we determine the similarity between $b$ and the three symmetric blocks according to $g_{i} i=1,2,3$ and we represent the measure as a matrix called the Symmetry Matrix $(S M)$. Every element $e$ of $S M$ has the following form:

$$
\begin{equation*}
e=\left(2 * S_{v}\right)+\left(3 * S_{h}\right)+\left(4 * S_{d}\right) \tag{1}
\end{equation*}
$$

where $S_{v}, S_{h}$ and $S_{d}$ are flags to indicate wether $b$ is similar to the symmetric block $b_{s}$ according to the vertical axis (respectively the horizontal and diagonal axis). We remind that the similarity measure can be any of those known in literature.
This representation is global but can be restricted to be local by evaluating the local matrix of some nodes. There are several advantages of this representation and measure of symmetry:

- The first advantage of this symmetry measurement model is that it can give a measure of symmetry for every quadrant. This property allows to measure the local symmetry, and it is useful when we are interested in a local feature. Indeed, all images cannot be assumed to have a single global symmetry, but contain a collection of symmetric and almost symmetric patterns.
- This representation has the multi-resolution property, i.e. that the measure can be evaluated at different resolutions according to $d$. we note that the decomposition process plays a very important role on the measured symmetry, for example an image can exhibit symmetry at higher level but not at a lower one.
- This symmetry measure is flexible, one can give the amount of symmetric blocks according to a given axis, and for example images containing the human body are vertically symmetric but not horizontal. This information is useful for distinguishing between images.
We note that the depth $d$ of the quadtree decomposition can affect the efficiency of the image retrieval process


## IV. EXPERIMENTS AND RESULTS

We have conducted performance experiments on a small image database of 80 zillij images. This database can be divided into two categories of decors, the first category contain rosette and star shapes, the second category contain
some incomplete shapes that often break the property of symmetry.
The retireval performance is measured using recall and precision, as is standard in all CBIR systems [13]. Recall measures the ability of retrieving all relevant or similar items in the database. It is defined as the ratio between the number $r$ of relevant images retrieved and the total relevant images $R$ in the database. Precision measures the retrieval accuracy and is defined as the ratio between the number of relevant images and the total number of images $N$.

$$
\begin{gather*}
\text { precision }=\frac{r}{N}  \tag{2}\\
\text { recall }=\frac{r}{R} \tag{3}
\end{gather*}
$$



Fig. 8. The precision/recall curve
An exhaustive evaluation was done for 10 queries using example image. The query images including complete and incomplete rosette shapes. The images relevant to the query were identified by scanning the database.
The graph in figure 8 shows the retrieval performance of our method. For a recall of 0.5 , precision is about $78 \%$
Results of the image research are presented in Figure 9, where the target images are ranked according to the similarity measures obtained with the query image. The query image is the left-most image in the first row.
It's important to note that the similarity is measured in terms of symmetry and not in terms of color.


Fig. 9. Retrieval results

## V. Conclusion

In this paper we described a new method for Araboandalusian images indexing and retrieval. The proposed method is based on the Quadtree Symmetry Transform, which gives a robust measure of symmetry of an image.
A flexible measure of symmetry is given, this measure can evaluate the amount of symmetry into an image or a region of image. Results show good performance using our method for indexing the Arab-andalusian images data base in terms of symmetry information
Future work includes investigating whether our QST may be used to reconstruct noised decors.

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