Modeling of a rat muscle using fractional multimodels

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***Abstract* - In this paper, the response of the rat muscle *peroneus digiti quarti*, is studied. Modeling of the isometric contraction due to Motor Unit (MU) stimulations is presented. The modeling is realized through multimodels and fractional differentiation. The multimodels structure allows distinguishing asymmetric contraction and relaxation mechanisms. Some recent works [1],[7] highlight a fractal structure of the muscle, which consolidate the approach based on the use of a fractal model to characterize its dynamic behavior. A non integer model, due to its infinite dimension nature, is particularly adapted to model complex systems with few parameters and to obtain a real time exploitable model. A comparison between Cole-Cole and Davidson-Cole models is presented, for FF, and FR MU stimulations. A study about dynamic behavior variations is realized, to obtain a mean fibre type response model for both MU types.**

I. Introduction

THE importance of the striated muscles contraction in animal organism physiology is considerable. Indeed, these muscles are involved during partial or global organism moves (locomotion). The knowledge of the effective contribution of muscular contractions to locomotor activity, makes possible to associate cinematic changes to physiological (tiredness) or pathological origin modifications of the muscle fibre properties (myopathies).

The striated muscle structure is widely described in biology. Three fibre types [10] make up this muscle: Fast (IIB), Medium (IIA) and Slow (I) fibres. These fibre types present different characteristics, like contraction, relaxation delays and feeding types.

This study deals with *peroneus digiti quarti* muscle modeling. New experiments allow to stimulate each MU separately. MUs are classified as FR, FF, and S types, which are respectively connected to IIA, IIB, and I fibre types. The muscle is stimulated through three FF MUs and four FR MUs.The most important problem when modeling muscle comes from its irreproducible nature. Applying a 10 Hz pulses pattern stimulation, amplitude variations and dynamic beavior variations appear.

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The experimental protocol is detailed in Section 2. Section 3 presents identification system with fractional differentiation. In Section 4, Multimodels and sub-models are detailed. Then, Section 5 proposes rat muscle multimodels for IIA and IIB muscle fibre.

II. Methods

Experiments were carried out on a rat. The animal was anesthetized with an initial intraperitoneal dose of pentobarbital sodium (45 mg/kg), supplemented, as necessary by additional intravenous doses to maintain full anesthesia. The nerve to the *peroneus digiti quarti* was freed, and the distal tendon of the muscle was attached to a force transducer (Kulite BG300) fixed on the shaft of a servo-controlled puller (LDS 201). All other muscles of the himb limb were denervated. The region containing the muscle was formed into a pool filled with paraffin oil.

A laminectomy was performed between L_4 and S_2 to expose the lumbosacral cord, and the skin flaps were elevated to form a pool that was filled with paraffin oil. The dorsal and ventral roots were cut near their entry into the spinal cord. Ventral roots were slit under oil into filaments and were raised onto a silver electrode that was used as the anode. A similar electrode, placed on the body mass near the root entry, served as cathode. Impulses in motor axons were detected by electrodes placed on the muscle nerve, which was elevated into oil. Potentials were amplified by Grass AC amplifiers and displayed on a Gould digital oscilloscope. A ventral root filament was shown to contain a single motor axon innervating the *peroneus digiti quarti* muscle when its stimulation evoked a unique potential in the muscle nerve.

The muscle length-twitch force curve was determined during stimulation of the muscle nerve. The muscle length was then set to the optimal length for the muscle twitch force. Isometric contraction forces developed by single MUs were measured for three stimulation frequencies (10, 20, 40 Hz). The MU type of each MU was determined according to the protocol described in [7]. Briefly, the type of the MU is determined using the amplitude of the force oscillations and the mean level of the force developed by the MU during stimulation at 20 Hz and stimulation at 40 Hz. The force signal and the muscle length signal were digitized at 2,000 Hz and stored using a CED 1401 interface coupled to a PC computer running the Spike2 software.

III. System identification using fractional model

A. Fractional differentiation definition

The fractional derivative of the function $f(t)$ at ν order is defined and also simulated at every sample period as [2],

$$D^{\nu} f(t) \approx \frac{1}{h^{\nu}} \sum_{k=0}^{\infty} (-1)^k \binom{\nu}{k} f(t - k\,h), \tag{1}$$

with $t = Kh, K \in \mathbb{N}^+$ and h is the sampling period.

Assuming that $f(t) = 0 \,\forall\, t < 0$, the D^{ν} Laplace transform is [3], [5], [6], [9]:

$$\mathrm{L}\left(D^{\nu} f(t)\right) = s^{\nu} \cdot \mathrm{L}\left(f(t)\right), \tag{2}$$

where ν can be real or imaginary number order.

Linear model described with the fractional differential equation,

$$\sum_{l=1}^{L} a_l \frac{d^{n_l}}{dt^{n_l}} \ddot{y}(t) = \sum_{q=1}^{Q} b_q \frac{d^{m_q}}{dt^{m_q}} u(t), \tag{3}$$

where

$$n_1, ..., n_L, m_1, .., m_Q \in \mathbb{R}, \quad \mathrm{L+Q} \tag{4}$$

can be modeled as the following fractional transfer function, providing that the system is relaxed at $t=0$,

$$\ddot{y}(s) = \frac{b_1 s^{m_1} + b_2 s^{m_2} + ... + b_q s^{m_Q}}{a_1 s^{n_1} + a_2 s^{n_2} + ... + a_L s^{n_L}} u(s) \ . \tag{5}$$

The output model can then be simulated:

$$\ddot{y}(Kh) = \frac{1}{\sum_{l=1}^{L} \frac{a_l}{h^{n_l}} \binom{n_l}{0}} \left(\begin{array}{c} \sum_{k=0}^{K} \sum_{q=1}^{Q} (-1)^k \frac{b_q}{h^{n_l}} \binom{n_q}{k} u((K-k)h) \\ - \sum_{k=1}^{K} \sum_{l=1}^{L} (-1)^k \frac{a_l}{h^{n_l}} \binom{n_l}{k} \ddot{y}((K-k)h) \end{array} \right). \tag{6}$$

Generally system identification of fractional model (linear or non linear) can be carried by minimizing the quadratic output error criterion J,

$$J = \sum_{k=1}^{N} \left(\varepsilon(kh)\right)^2 , \tag{7}$$

where $\varepsilon(kh) = y^*(kh) - \ddot{y}(kh)$ (8)

is the output error.

A non linear optimization algorithm can then be used for example, Newton, Marquard, SimplexeÉ The non linear Simplex algorithm is chosen for the Output Error Model (figure 1) to minimize J by evaluating coefficients $a_1,..., a_L, b_1,...,b_L$ and orders $n_1,..., n_L, m_1,...,m_L$.

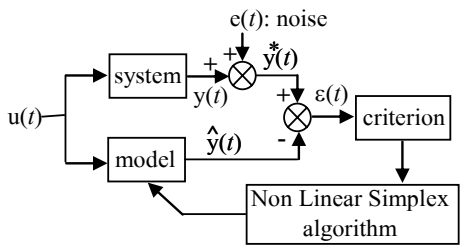

Fig. 1. Output Error Model

IV. Rat muscle modeling by fractional multimodels

A. Multimodels structure

The multimodels [4] necessity is highlighted by asymmetric mechanisms during muscle activation. The first one is an active phenomenon (contraction) and the second one, a passive phenomenon (relaxation). Thus multimodels is chosen to process separately contraction and relaxation phases [11].

The multimodels is made up of two branches, each one corresponding to a sub-model. On Fig. 2, the top-branch is dedicated to contraction model and the bottom-branch is dedicated to the relaxation model. $P(t)$ is a weight-function, which is set to 1 during $T_{junction}$ when a pulse is detected, (9).

The delay function is added on the bottom-branch to shift the M_2 response, corresponding to the relaxation. The weight-function allows to use M_1 and M_2 addition, even through both are active separately.

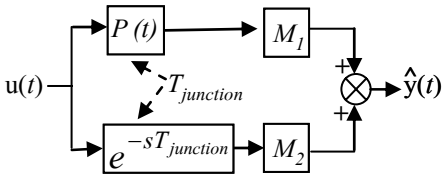

Fig. 2. Multimodels

$$P(t) \begin{cases} 0, \forall t > T_{junction} \\ 1, \forall t \le T_{junction} \end{cases} \tag{9}$$

Thus next stage of modelling consists in estimating M_1 and M_2 parameters. $T_{junction}$ is fixed as the switch time of the muscle response.

B. Linear sub-models

The two linear sub-models, M_1 and M_2, can be fractional Cole-Cole model or fractional Davidson-Cole model. When orders are integers, both models led to the rational transfer function.

1) Fractional Cole-Cole

Fractional Cole-Cole model is an extension of Cole-Cole model to fractional orders and is defined as

$$F_{Cole-Cole}(s) = \frac{A}{s^{\nu} + \lambda}, \tag{10}$$

where A, ν, λ are respectively real gain, positive real order and positive real pole to estimate. Identification can be based on models (11) using sum or/and product of (10), where orders and poles are let free.

$$M_{CC}(s) = \sum_{i=1}^{N} \prod_{j=1}^{M} F_{Cole-Cole}\left(s, A_{ij}, \nu_{ij}, \lambda_{ij}\right) \tag{11}$$

When orders ν are fixed to integer values, the fractional Cole-Cole model is rational transfer function

$$M_{RTF}(s) = \frac{b_1 s^{m_1} + b_2 s^{m_2} + ... + b_q s^{m_Q}}{a_1 s^{n_1} + a_2 s^{n_2} + ... + a_L s^{n_L}} \ , \tag{12}$$

where the orders, $n_1,...,n_L,m_1,...,m_Q$, are integer, and $a_1,...,a_L,b_1,...,b_Q$ are coefficients

.

2) Fractional Davidson-Cole

Fractional Davidson-Cole model is an extension of Davidson-Cole model to fractional orders and is defined as

$$F_{Davidson-Cole}(s)=\frac{A}{(s+\lambda)^{\nu}}, \quad (13)$$

where A,ν,λ are respectively real gain, positive real order and positive real pole to estimate. Identification can be based on models (14) using sum or/and product of (13), where orders and poles are let free.

$$M_{DC}(s)=\sum_{i=1}^{N}\prod_{j=1}^{M}F_{Davidson-Cole}(s,A_{ij},\nu_{ij},\lambda_{ij}). \quad (14)$$

V. IIB AND IIA RAT FIBRES IDENTIFICATION

A. Identification results for IIB and IIA fibres and modelling result

Identifications are realized with two particular responses in order to compare M_{CC}, M_{RTF} and M_{DC} efficiencies. The goal is to find the finest model with few parameters.

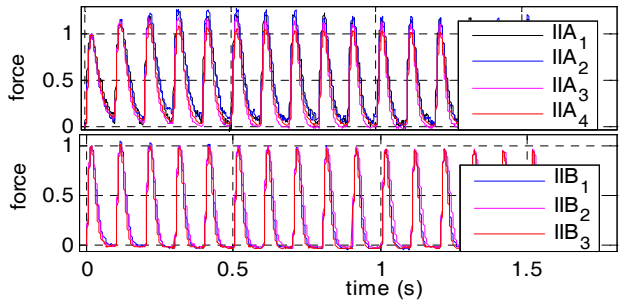

Fig. 3. Muscles response, (top) with every FR MU, stimulation at 10 Hz, (bottom) with every FF MU, stimulation at 10 Hz;

The last response of one FR MUs (respectively FF), which have the maximum amplitude are chosen to be modeled (Fig. 3). Muscle responses are normalized, since amplitude variations do not represent a major interest at this stage of the study. Contraction and relaxation using sub-models M_{CC}, M_{RTF} and M_{DC} models, are presented. $1/s$ terms come from identifications through Heavyside function as solicitation. Orders and Coefficients optimization
is realized. Only the structure size is fixed. Sub-models size below presented are chosen to obtain similar quadratic error, whatever sub-models types.

1) Fractional Cole-Cole

$$\left\{\begin{array}{l} M_1^{IIA}(s)=\frac{1}{s}\cdot\frac{1}{s^{1.38}+218}\cdot\frac{10573.9}{s^{1.4}+60.82} \\ M_2^{IIA}(s)=\frac{1}{s}\cdot\frac{1}{s+3.3606}\cdot\frac{172.8}{s^{1.22}+27.48} \end{array}\right\} \quad (15)$$

$$\left\{\begin{array}{l} M_1^{IIB}(s)=\frac{1}{s}\cdot\frac{1}{s^{1.54}+3816.39}\cdot\frac{1.56\cdot10^{7}}{s^{1.35}+5598.6} \\ M_2^{IIB}(s)=\frac{1}{s}\cdot\frac{1}{s^{1.27}+692.34}\cdot\frac{1}{s+102.88}\cdot\frac{1.015\cdot10^{5}}{s^{0.55}+141.5} \end{array}\right\} \quad (16)$$

2) Rational Transfer Function

$$\left\{\begin{array}{l} M_1^{IIB}(s)=\frac{1}{3.7\cdot10^{-8}s^4+9.1\cdot10^{-6}s^3+9.3\cdot10^{-3}s^2+0.6s-0.031} \\ M_2^{IIB}(s)=\frac{1}{s}\frac{1}{6.096\cdot10^{-5}s^2+0.01235s+1} \end{array}\right\} \quad (17)$$

$$\left\{\begin{array}{l} M_1^{IIA}(s)=\frac{1}{2.92\cdot10^{-5}s^4+1.77\cdot10^{-3}s^3+0.072s^2+0.99s-0.493} \\ M_2^{IIA}(s)=\frac{1}{s}\cdot\frac{1}{0.012s^2+0.19s+1} \end{array}\right\} \quad (18)$$

3) Fractional Davidson-Cole

$$\left\{M_1^{IIB}(s)=\frac{1.787\cdot10^{10}}{(s+502.86)^{3.787}};M_2^{IIB}(s)=\frac{5.376\cdot10^{5}}{(s+212.28)^{2.89}}\right\} \quad (19)$$

$$\left\{M_1^{IIA}(s)=\frac{1}{s}\frac{1.2152\cdot10^{6}}{(s+46.5)^{3.64}};M_2^{IIA}(s)=\frac{1}{s}\frac{355.3}{(s+11.77)^{2.38}}\right\} \quad (20)$$

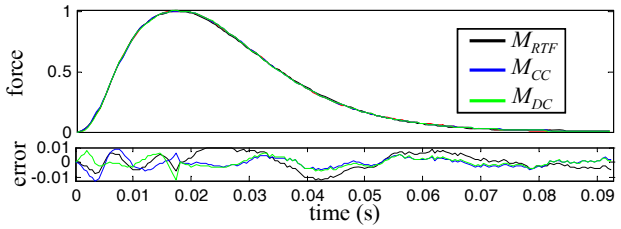

Fig. 4. Experimental muscle response for a particular FR MU stimulation, rational transfer function, Cole-Cole, and Davidson-Cole models in the multimodels structure. The output error allows to compare response models.

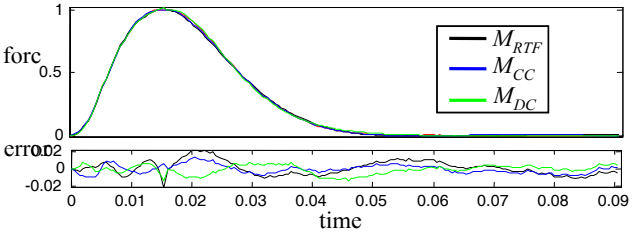

Fig. 5. Experimental muscle response for a particular FF MU stimulation, rational transfer function, Cole-Cole, and Davidson-Cole models in the multimodels structure. The output error allows to compare models efficiency.

Fig. 4 and Fig. 5 present the muscle response in the time domain and M_{CC}, M_{RTF}, and M_{DC} models response for IIA and IIB fibres.

The fractional Davidson-Cole needs only 6 parameters (resp. 6), compared to 8 (resp. 8) for rational transfer function, and 10 (resp. 12) for the fractional Cole-Cole for IIA (resp. IIB). The quadratic error in the third cases are quite similar, closed to 5.10^{-5}, but is minimum for the fractional Davidson-Cole case. Thus the Davidson-Cole structure is adopted to model response of fibres.

Muscle response variations appear between different MUs with the same characteristics. Furthermore, variations occur

also for every MU.

B. Mean model computing with IIB and IIA fibres

Identifications are realized for every MUs for every pulse of the 10 Hz pulses pattern. In this part, mean sub-models parameters $\{\nu_1, \lambda_{1,Tj_{unction\ 1}}\}\{\nu_2, \lambda_{2,Tj_{unction\ 2}}\}$ are determined to take the place of every muscle response.

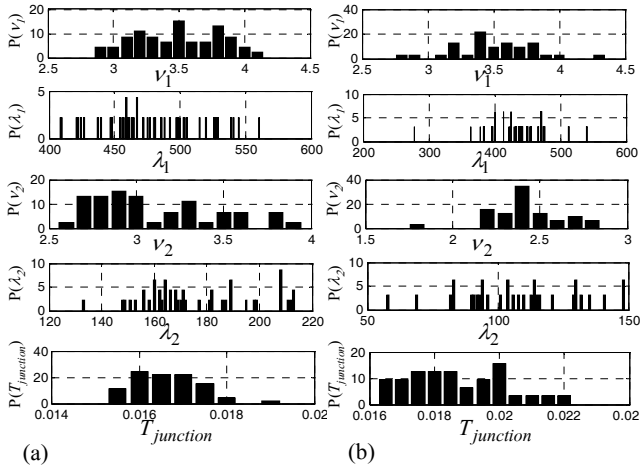

Fig. 6. $T_{junction}$, order (ν), and pole (λ) probabilities, for contraction and relaxation, (a) for IIA and (b) for IIB fibres

Fig. 6 present M_{DC} parameters ($T_{junction}$, orders and poles) variations around a mean value. The mean response model (20), (21), in each case, is based on mean parameters (Fig. 7 and Fig. 8). Sub-models presented in this paper report globally every isolated pulse responses. Indeed, multimodel has to be improved to describe smooth behavior-amplitude variations, which appear in the course of time, when 10 Hz stimulation is applied.

$$\left\{ M_1^{IIA}(s) = \frac{A_3}{(s+432.9)^{3.5}} ; M_2^{IIA}(s) = \frac{A_4}{(s+107.59)^{2.4}} \right\} \quad (21)$$

$$\left\{ M_1^{IIB}(s) = \frac{A_1}{(s+476.92)^{3.48}} ; M_2^{IIB}(s) = \frac{A_2}{(s+176.57)^{3.11}} \right\} \quad (22)$$

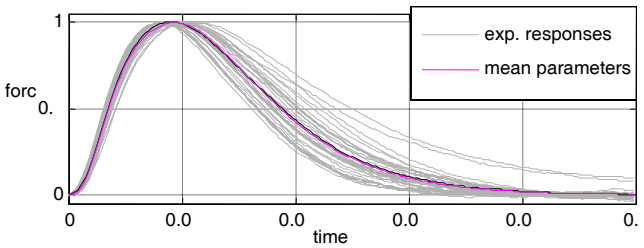

Fig. 7. Every IIA experimental responses and the mean M_{DC} model

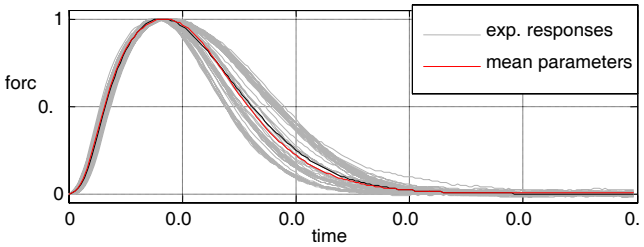

Fig. 8. Every IIB experimental responses and the mean M_{DC} model

VI. Conclusion and Prospects

A comparison between three specific models, fractional Cole-Cole, rational transfer function, and fractional Davidson-Cole is realized. Fractional Davidson-Cole presents the advantage of a minimum parameters number for a minimum output error. Two sub-models are proposed for IIA and IIB MUs stimulations. These models are obtained through fractional Davidson-Cole structure with mean parameters, determined from every experiment responses at 10 Hz. Each mean sub-model depends on 6 parameters but can not reproduce every activation as well as a particular one. In fact, at the beginning of the activation, there is a transitional stage responsible of large behavior variations. The next study will consist to understand transitional mechanisms and to model them. By this way, a completed model based on the results presented in this study will allow a good modeling of every MU in the course of stimulations. A specific muscle will be also studied to obtain the last I fibre response, which is still hard to reach with the *peroneus digiti quarti*. The muscle model will be also completed to take initial muscle length into account.

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