NON-UNITARY EXTENSIONS OF DIFFERENTIAL MATRIX MODULATION

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ABSTRACT

Differential matrix modulation is an attractive alternative for MIMO transmission since it does not require channel estimation at the receiver. Most proposals are based on unitary matrices. Assuming a receiver with relatively low complexity, we will show that the performance at higher bandwidth-efficiency can be improved by using non-unitary matrix constellations. We compare different non-unitary schemes and their soft-output detectors.

1. INTRODUCTION

Differential matrix modulation schemes which enable MIMO capacity gains and transmit antenna diversity without the need for MIMO channel estimation at the receiver have been proposed in [TJ00], [Hug00], [HS00]. In general, those schemes can be viewed as differential matrix modulation methods where the constellation consists of unitary matrices. The requirement of the constellation elements to be unitary matrices limits the achievable data rate and performance.

In order to increase the bandwidth-efficiency, we give up the restriction to unitary matrix constellations. Earlier proposals for non-unitary differential matrix modulation include the extension of unitary matrix modulation by an additional amplitude modulation [Xia02], [Bau03], [Bau04a]. Non-unitary differential matrix modulation based on orthogonal designs with QAM modulation was presented in [TC01], [CZSL03], [HNCT03b], [HNCT03a]. For those schemes, we propose novel soft-output detectors. Furthermore, we discuss the possibility of increasing the bandwidth-efficiency by a kind of differential spatial multiplexing scheme. Finally, we will compare the performance of those differential matrix modulation schemes.

2. CHANNEL MODEL

We consider a flat fading multiple-input multiple-output (MIMO) channel with n_T transmit and n_R receive antennas. The channel coefficients $h_k^{(ij)}$ are collected in the matrix $\mathbf{H}_k$ where $h_k^{(ij)}$ is the channel coefficient from transmit antenna i to receive antenna j at time k. At the receiver, we observe

$$\mathbf{Y}_k = \mathbf{H}_k\mathbf{X}_k + \mathbf{N}_k, \tag{1}$$

where $\mathbf{X}_k$ and $\mathbf{Y}_k$ contain the transmitted and received symbols, respectively, and $\mathbf{N}_k$ contains the noise samples which are assumed to be independent and Gaussian with variance $\sigma^2 = \frac{N_0}{2}$ per real dimension.

3. DIFFERENTIAL UNITARY MATRIX MODULATION

Differential unitary matrix modulation was introduced simultaneously in [HS00] and [Hug00]. The info bits are mapped on an $L \times L$ info matrix $\mathbf{C}_k$. The $n_T \times L$ transmit matrix $\mathbf{X}_k$ is determined by $\mathbf{C}_k$ and the previously transmitted matrix $\mathbf{X}_{k-1}$ according to the differential encoding rule

$$\mathbf{X}_k = \mathbf{X}_{k-1}\mathbf{C}_k. \tag{2}$$

In order to allow simple non-coherent detection, $\mathbf{C}_k$ must be unitary, i.e.

$$\mathbf{C}_k\mathbf{C}_k^H = \mathbf{I}_L, \tag{3}$$

where $\mathbf{C}^H$ denotes the conjugate transpose of $\mathbf{C}$ and $\mathbf{I}_L$ is the $L \times L$ identity matrix. A unitary reference matrix $\mathbf{X}_0$ has to be transmitted first. All transmit matrices $\mathbf{X}_k$ are unitary. A suitable construction of unitary matrices is using orthogonal designs [TJC99] with PSK symbols $c_{k,l}$. E.g. for $n_T = 2$ transmit antennas, we have the Alamouti scheme where $K = 2$ symbols are mapped to the matrix

$$\mathbf{C}_k = \begin{bmatrix} c_{k,1} & c_{k,2} \\ -c_{k,2}^* & c_{k,1}^* \end{bmatrix}. \tag{4}$$

Since unitary space-time modulation is based on PSK symbols, the performance will be poor for more bandwidth-efficient transmission, i.e. when the unitary matrix is based on M-PSK symbols for $M > 8$.

4. EXTENSIONS OF DIFFERENTIAL UNITARY MATRIX MODULATION

For more bandwidth-efficient differential matrix modulation, we propose to give up the restriction to unitary transmit matrices. We consider three approaches, where the differential encoding can be written as

$$\mathbf{X}_k = \frac{1}{A_k}\mathbf{X}_{k-1}\mathbf{C}_k. \tag{5}$$

The three proposals differ in the design of the info matrix $\mathbf{C}_k$ and in the meaning of the factor $1/A_k$: First, we describe differential amplitude and unitary matrix modulation in Section 4.1, where $\mathbf{C}_k$ is still an unitary matrix and $1/A_k$ is an additional amplitude or matrix norm modulation, i.e. $1/A_k$ carries information. In Section 4.2, we introduce two schemes, where the matrix $\mathbf{C}_k$ is not unitary any more and $1/A_k$ is just a transmit energy normalization factor which does not carry information.

4.1 Differential Amplitude and Unitary Matrix Modulation

Extensions of unitary matrix modulation by an additional amplitude or matrix norm modulation for higher bandwidth-efficiency with improved performance have been proposed in [Xia02], [Bau03], [Bau04a], [Bau04b]. The general approach according to [Bau04a], [Bau04b] which can be applied as an extension of any differential unitary matrix modulation scheme is depicted in Figure 1. Here, the input bits of the differential matrix modulator are grouped into two sets. The first $\log_2 M_1$ bits are mapped on a unitary matrix $\mathbf{C}_k$ as it is done in unitary differential matrix modulation. The last $\log_2 M_2$ bits and the previously

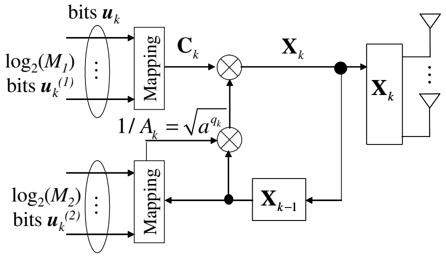

Figure 1: Differential Amplitude and Unitary Matrix Modulation. Transmitter.

transmitted matrix $\mathbf{X}_{k-1}$ determine the amplitude modulation factor a^{q_k}, where a is a real constant and $q_k \in \{-M_2+1, -M_2+2, ..., -1, 0, 1, ..., M_2-1\}$. The transmit matrices are not unitary any more but satisfy

$$\mathbf{X}_k \mathbf{X}_k^H = a^{z_k} \mathbf{I}_{n_T}, \quad (6)$$

where a^{z_k} can take the discrete real values $a^{z_k} \in \{1, a, a^2, ..., a^{M_2-1}\}$. Depending on the $\log_2 M_2$ last input bits, the amplitude a^{z_k} as defined in (6) is cyclic increased compared to the previously transmitted matrix by a factor of $1, a, a^2, ...,$ or a^{M_2-1}.

The amplitude difference exponent is given by

$$q_k = z_{k-1} - M_2 \lfloor (z_{k-1} + d_k)/M_2 \rfloor, \quad (7)$$

where $\lfloor . \rfloor$ is the floor function. The input bits $\mathbf{u}_k^{(2)}$ are mapped on an integer $d_k \in \{0, 1, ..., M_2-1\}$ and z_{k-1} denotes the amplitude exponent of the previously transmitted matrix $\mathbf{X}_{k-1}$ which is determined by

$$z_k = z_{k-1} + q_k \quad (8)$$

with the arbitrary choice $z_0 = q_0 = 0$.

A disadvantage of this scheme is that the amplitude modulation is performed per matrix rather than per symbol which means a limited improvement in bandwidth-efficiency especially for larger matrices.

4.2 Non-Unitary Orthogonal and Non-Orthogonal Differential Matrix Modulation

Another idea is to increases the bandwidth-efficiency by using non-unitary differential space-time modulation. In contrast to the scheme described in the previous section, we do not perform separate unitary matrix modulation and amplitude modulation. Instead, a differential matrix modulation with non-unitary matrices is performed. The transmitter is depicted in Figure 2. The input bits for the transmit matrix

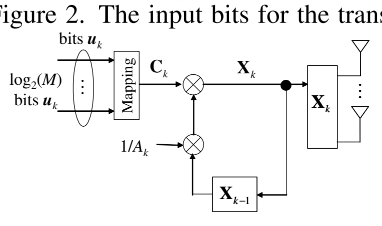

Figure 2: Non-unitary differential matrix modulation. Transmitter

$\mathbf{X}_k$ are mapped on a matrix $\mathbf{C}_k$ with complex entries $c_{k,ij}$. One possibility is, to choose the mapping rule according to an orthogonal design. In contrast to unitary differential matrix modulation where the entries $c_{k,ij}$ are restricted to PSK symbols, we allow $c_{k,ij}$ to be taken from a QAM constellation. We refer to this approach as *non-unitary orthogonal matrix modulation*.

Another approach is to fill the $L \times L$ matrix $\mathbf{C}_k$ with independent symbols of a PSK or QAM constellation. This is similar to spatial multiplexing in coherent MIMO transmission. If the cardinality of the modulation scheme is M, we can transmit $\log_2 M \cdot L \cdot L$ bits per matrix. However, care has to be taken that the differential encoding does not cause a zero sequence. In order to avoid this, we propose to rotate the constellation of each entry of the matrix by multiplication with a different phase factor. We call this approach *non-orthogonal differential matrix modulation*.

In unitary differential matrix modulation, the differential encoding (2) does not change the transmit energy since $\mathbf{C}_k$ and $\mathbf{X}_{k-1}$ are unitary. However, in case of non-unitary matrices $\mathbf{C}_k$, we need to normalize the transmit matrix by muliplication with the normalization factor $1/A_k$, i.e.

$$\mathbf{X}_k = \frac{1}{A_k} \mathbf{X}_{k-1} \mathbf{C}_k, \quad (9)$$

in order to meet the power constraint

$$\mathscr{E}_{k,l} \left\{ \sum_{n=1}^{n_T} |x_{k,l}^{(n)}|^2 \right\} = E_s, \quad (10)$$

where $\mathscr{E}_{k,l}$ denotes expectation with respect to k and l and E_s is the total average transmit energy per time slot, i.e. for a hypersymbol $\left[x_{k,l}^{(1)}, \cdots, x_{k,l}^{(n_T)} \right]$ which consists of the symbols which are transmitted simultaneously from different antennas. Normalization avoids blowing up the transmit power as well as a vanishing signal.

The power constraint (10) is met if the normalization factor is determined by the energy of the previously transmitted matrix, i.e.

$$A_k = \sqrt{\frac{1}{L} \text{trace } \left\{ \mathbf{X}_{k-1} \mathbf{X}_{k-1}^H \right\}}. \quad (11)$$

If $\mathbf{C}_k$ is an orthogonal design, we have

$$\mathbf{C}_k \mathbf{C}_k^H = a_k \mathbf{I}_L, \quad (12)$$

where $\mathbf{I}_L$ is the $L \times L$ identity matrix. Consequently, the amplitude of the matrix $\mathbf{C}_k$ can easily be defined as a_k. Then, condition (11) is equivalent to choosing the normalization factor

$$A_k = \sqrt{a_{k-1}}. \quad (13)$$

4.3 Receiver

We consider a simple non-coherent receiver which takes into account two successively received matrices

$$\mathbf{Y}_{k-1} = \mathbf{H}_{k-1} \mathbf{X}_{k-1} + \mathbf{N}_{k-1} \quad (14)$$

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{N}_k = \frac{1}{A_k} \mathbf{H}_k \mathbf{X}_{k-1} \mathbf{C}_k + \mathbf{N}_k, \quad (15)$$

where the channel is assumed to be constant during transmission of two matrices, i.e.

$$\mathbf{H}_{k-1} \approx \mathbf{H}_k. \quad (16)$$

Plugging (14) into (15) yields

$$\begin{aligned} \mathbf{Y}_k &= \frac{1}{A_k} \mathbf{Y}_{k-1} \mathbf{C}_k - \frac{1}{A_k} \mathbf{N}_{k-1} \mathbf{C}_k + \mathbf{N}_k \\ &= \frac{1}{A_k} \mathbf{Y}_{k-1} \mathbf{C}_k + \tilde{\mathbf{N}}_k. \end{aligned} \quad (17)$$

This describes the transmission of the info matrix $\mathbf{C}_k$ over an equivalent channel with L transmit and n_R receive antennas, channel coefficients $\tilde{\mathbf{H}} = \frac{1}{A_k}\mathbf{Y}_{k-1}$ and additive Gaussian noise with variance

$$\tilde{\sigma}^2 = \sigma^2 \left(1 + \frac{1}{A_k^2}\mathrm{trace}\left\{\mathbf{C}_k\mathbf{C}_k^H\right\}\right) \tag{18}$$

per real dimension at each virtual receive antenna, where σ^2 is the noise variance per real dimension at each physical receive antenna.

For detection, we need an estimate on the effective channel tap, i.e. we need an estimate on $1/A_k$. For soft-output detection, we even need an estimate on the effective noise variance according to (18). As long as $\mathbf{C}_k$ is chosen as orthogonal design, the parameter

$$v_k = \frac{\mathrm{trace}\left\{\mathbf{Y}_k\mathbf{Y}_k^H\right\}}{\mathrm{trace}\left\{\mathbf{Y}_{k-1}\mathbf{Y}_{k-1}^H\right\}} \approx \frac{1}{LA_k^2}\mathrm{trace}\left\{\mathbf{C}_k\mathbf{C}_k^H\right\} \tag{19}$$

can be used as an approximation of the last term in (18). In case of noiseless transmission, v_k is exactly $\frac{1}{LA_k^2}\mathrm{trace}\left\{\mathbf{C}_k\mathbf{C}_k^H\right\}$. For differential amplitude and unitary matrix modulation, $\mathbf{C}_k$ is unitary and hence v_k can also be used as approximation for the effective channel coefficient in (17). Moreover, v_k is a direct estimate on the data part which is transmitted via amplitude modulation. For more details we refer to [Bau04a].

In case of non-unitary orthogonal matrix modulation, v_k can be used for estimating the effective noise variance (18). However, we also need to know the effective fading coefficient which requires an estimate on $1/A_k$. We propose three methods for estimation of $1/A_k$:

1. A first solution is to ignore the normalization factor $1/A_k$ at the receiver by assuming $1/A_k = 1$. This works surprisingly well particularly in time varying channels, where the channel is not exactly constant during transmission of two successive matrices.
2. Another possibility is to estimate $1/A_k$ from the hard decisions up to matrix k. Since the first transmitted matrix $\mathbf{X}_0$ is a known reference matrix, estimates $\hat{A}_k$ on all normalization factors A_k can be obtained based on the hard decisions $\hat{\mathbf{C}}_t$, $t < k$ taking into account the differential encoding rule (9). More precisely: Knowing $\mathbf{X}_{k-1}$ yields $\hat{A}_k$ for detection of $\mathbf{C}_k$. From the hard decision $\hat{\mathbf{C}}_k$, we obtain an estimate for $\mathbf{X}_k$ which yields $\hat{A}_{k+1}$ etc.. The problem with this approach is that it imposes error propagation.
3. Error propagation can be avoided on the expense of neglecting noise if $1/A_k$ is estimated using (19) and (12), i.e.

$$v_k = \frac{\mathrm{trace}\left\{\mathbf{Y}_k^H\mathbf{Y}_k\right\}}{\mathrm{trace}\left\{\mathbf{Y}_{k-1}^H\mathbf{Y}_{k-1}\right\}} \approx \frac{a_k}{A_k^2}, \tag{20}$$

where a_k is the amplitude of the orthogonal design as defined in (12). From (13), we know that

$$A_{k+1}^2 = a_k. \tag{21}$$

Since the first transmitted matrix $\mathbf{X}_0$ is a known reference matrix, A_1 is known at the receiver. Typically, the reference matrix will be the identity matrix and, hence, A_1=1. For $k > 1$, an estimate $\hat{A}_k$ is obtained using (20) and (21) from

$$\hat{A}_{k+1}^2 = v_k\hat{A}_k^2. \tag{22}$$

Method 3 turns out to be advantageous in quasistatic fading channels, whereas method 2 performs better in time varying channels, where the condition (16) is violated. In case of non-orthogonal unitary designs, the approximation (19) doesnot hold true and, hence, only method 1 is appropriate.

Using those estimates on $1/A_k$ and the noise variance $\tilde{\sigma}^2$, the computation of soft-output decisions in the form of a max-log approximation of log-likelihood ratios

$$L(\hat{u}_{k,t}) \approx \max_{\substack{\mathbf{C}_k \\ u_{k,t}=+1}} \log p(\mathbf{C}_k|\mathbf{Y}_{k-1}\mathbf{Y}_k) - \max_{\substack{\mathbf{C}_k \\ u_{k,t}=-1}} \log p(\mathbf{C}_k|\mathbf{Y}_{k-1}\mathbf{Y}_k) \tag{23}$$

is straight forward (see [Bau04a] for details). In order to avoid estimation of the noise variance σ^2, we propose to simply multiply all log-likelihood ratios by σ^2 in order to make the right hand side of (23) independent of σ^2. If the noise variance is constant over a frame, which is a reasonable assumption, all log-likelihood ratios are scaled by the constant factor σ^2. This has no effect on the hard output of an outer Viterbi or Max-Log-type APP decoder. However, the log-likelihood ratios of the outer decoder will also be scaled by the same factor.

5. SIMULATION RESULTS

We compare the performance of the schemes mentioned above for $n_T = 2$ transmit and $n_R = 2$ receive antennas in a spatially uncorrelated quasistatic channel, i.e. the channel is constant during transmission of a coded block and changes independently from one block to the next. For forward error control (FEC) coding, we use a rate $R = 1/2$ convolutional code with constraint length 5 and generators [23,35]. We consider transmission of 8 bits per matrix. This can be achieved with unitary differential matrix modulation based on 16-PSK, with non-unitary orthogonal matrix modulation based on 16-QAM or with non-orthogonal differential matrix modulation based on rotated QPSK. Alternatively, we can use the differential amplitude and unitary matrix modulation scheme described in Section 4.1 based on 8-PSK modulation plus 4-ary amplitude modulation ($M_1 = 8$, $M_2 = 4$).

BER results for uncoded transmission are depicted in Figure 3.

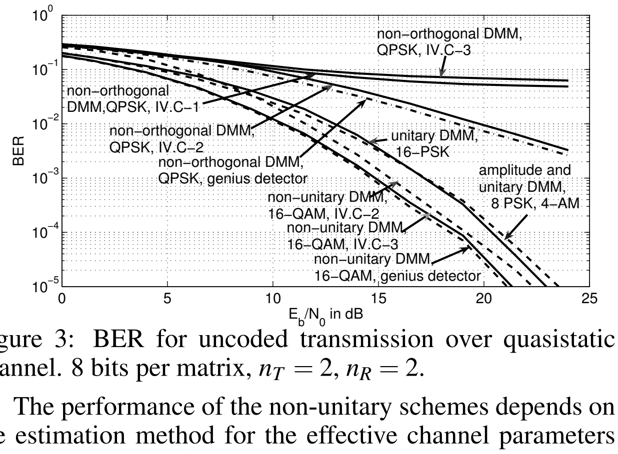

Figure 3: BER for uncoded transmission over quasistatic channel. 8 bits per matrix, $n_T = 2$, $n_R = 2$.

The performance of the non-unitary schemes depends on the estimation method for the effective channel parameters as discussed in Section 4.3. For comparison, we include the performance of a genius detector which perfectly knows the otherwise estimated effective channel parameters.

For non-unitary orthogonal matrix modulation (non-unitary DMM), estimation method 3 of Section 4.3 (IV.C-3) virtually achieves the performance of the genius detector,

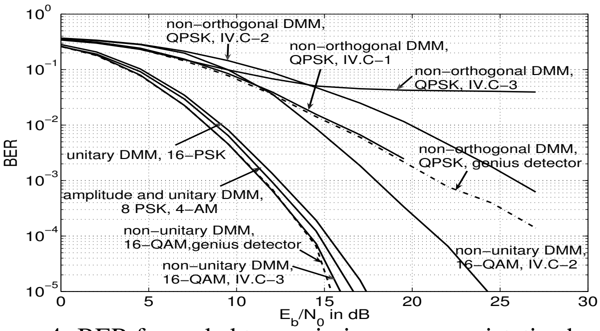

Figure 4: BER for coded transmission over quasistatic channel. 8 bits per matrix, $n_T = 2$, $n_R = 2$, convolutional code $[23,35]$ with $R = 1/2$.

whereas method 2 performs 0.6 dB worse. Method IV.C-1 results in an error floor (not shown). It is noticeable that with the optimum detector according to IV.C-3, non-unitary DMM performs 2.4 dB better than differential amplitude and unitary matrix modulation which shows similar performance as differential unitary matrix modulation.

For non-orthogonal differential matrix modulation, method IV.C-2 shows the best performance. Here, method IV.C-3 fails since (19) is not true. However, even with a genius detector, non-orthogonal DMM shows poorer performance than the other schemes. This is expected since as a kind of spatial multiplexing method, it does not provide diversity in uncoded transmission.

The respective BER results for FEC coded transmission are depicted in Figure 4. Here, the performance advantage of non-unitary orthogonal differential matrix modulation over differential amplitude and unitary matrix modulation reduces to $0.5dB$ which indicates that due to inaccurate estimation of the effective channel parameters, the quality of the soft-output log-likelihood ratios of the non-unitary scheme is worse than for differential amplitude and unitary matrix modulation resulting in a performance degradation of the FEC decoder. In the FEC coded scheme, differential amplitude and unitary matrix modulation outperforms unitary matrix modulation by 0.4 dB even though both schemes showed similar uncoded average BER. This can be explained by the fact that the bits which are transmitted in the unitary part of differential amplitude and unitary matrix modulation have a lower BER than those which determine the amplitude modulation. In FEC decoding, the larger number of bits transmitted with lower BER helps to correct errors in the amplitude modulation bits.

Even with a genius detector, non-orthogonal differential matrix modulation shows significantly poorer performance than the orthogonal schemes. This is due to the fact that the effective channel matrix $\mathbf{Y}_{k-1}$ might be ill conditioned due to differential encoding and, thus, the effective channel does not support spatial multiplexing but rather pure transmit diversity which is exploited by the orthogonal schemes. Interestingly, with FEC coding, method IV.C-1 shows the best performance and clearly outperforms IV.C-2. This demonstrates that not only the uncoded BER but also the quality of the log-likelihood ratios is decisive for the performance of an outer FEC decoder.

6. CONCLUSIONS

We have presented several non-unitary differential matrix modulation schemes with various novel versions of non-coherent soft-output detectors. In quasistatic fading channels, non-unitary orthogonal matrix modulation shows the best performance and outperforms standard unitary matrix modulation by 2.4 dB in uncoded transmission and 0.9 dB in FEC coded transmission at a rate of 8 bits per matrix. Non-orthogonal matrix modulation appears to be less suitable for non-coherent detection since the differential encoding causes the effective channel to be ill conditioned.

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