

MITIGATING UNCORRELATED PERIODIC DISTURBANCE IN NARROWBAND ACTIVE NOISE CONTROL SYSTEMS

Muhammad Tahir AKHTAR¹ and Wataru MITSUHASHI²

¹The Education and Research Center for Frontier Science and Engineering,

²Department of Information and Communication Engineering,

University for Electro-Communications, 1-5-1 Chofugaoka, Chofu, Tokyo 182-8585, JAPAN

Email¹: akhtar@ieee.org, akhtar@ice.uec.ac.jp

ABSTRACT

This paper concerns the problem of uncorrelated periodic disturbance appearing at the error microphone in single-channel feedforward active noise control (ANC) systems. This disturbance, being uncorrelated with the primary noise, cannot be controlled by standard filtered-x least mean square (FxLMS) algorithm, and increases the residual noise. Furthermore, it significantly degrades the steady-state performance of ANC systems. Here we propose a new method which can simultaneously control both the correlated and uncorrelated noise signals. The proposed method comprises three adaptive filters; 1) the FxLMS algorithm-based ANC filter to cancel the primary noise, 2) a separate FxLMS algorithm-based ANC filter to cancel the uncorrelated disturbance, and 3) the LMS algorithm based supporting adaptive filter to generate appropriate signals for the two ANC filters. Computer simulations are carried out which demonstrate that the proposed method can effectively mitigate the correlated and uncorrelated primary disturbances, and gives significantly improved noise reduction performance. This improved performance is achieved at an increased computational complexity.

1. INTRODUCTION

Active noise control (ANC) is based on the principle of destructive interference between acoustic waves. Essentially, the primary noise is cancelled around the location of the error microphone by generating and combining an antiphase cancelling noise [1]. As shown in Fig. 1, a single-channel feedforward ANC system comprises one reference sensor to pick up the reference noise $x(n)$ correlated with the primary disturbance $d(n)$, one cancelling loudspeaker to propagate the cancelling signal $y(n)$ generated by ANC filter $W(z)$, and one error microphone to pick up the residual noise $e(n)$. The most famous adaptation algorithm for ANC systems is the filtered-x least mean square (FxLMS) algorithm [2], which is a modified version of the LMS algorithm [3]. Here the reference signal $x(n)$ is filtered through a model of the so-called secondary path $S(z)$, following the adaptive filter, and hence the name filtered-x algorithm. The FxLMS algorithm is a popular ANC algorithm due to its robust performance, low computational complexity and ease of implementation [2].

The FxLMS algorithm is widely used in ANC systems. However the performance the FxLMS algorithm in steady state will be degraded due to the presence of the uncorrelated disturbance at the error microphone, shown as $v(n)$ in Fig. 1. This situation arises in many real-world applications. For example, in electronic mufflers for automobiles

[4], the undesired disturbance such as the noises generated by other passing-by automobiles will affect the stability and performance of the ANC systems. In industrial installations, neighboring machinery near to the location of error microphone may generate uncorrelated disturbance.

Up to the best knowledge of Authors, little research has been done to cope with the uncorrelated disturbance problem. In [5], an adaptive algorithm consisting of two interconnected adaptive notch filters is proposed to reduce the disturbance problem. However, this algorithm is effective only for narrowband, single-frequency ANC systems. In [6], this algorithm has been generalized to multifrequency narrowband feedforward ANC systems using a single high-order adaptive filter, and a cascaded active noise control system is proposed. The main idea is to cascade an LMS-based supporting adaptive filter, that generates a disturbance free error signal for the adaption of the FxLMS-based noise control filter. This supporting adaptive filter is excited by the reference signal $x(n)$ and the error signal $e(n)$ is used as a desired response. The output of the supporting filter is used as a an error signal for adaptation of the FxLMS algorithm. This method improves the convergence of the FxLMS algorithm, however, cannot mitigate the effect of the uncorrelated disturbance $v(n)$ from the residual noise $e(n)$.

In this contribution we propose a new method which can simultaneously control both the correlated and uncorrelated noise signals. The proposed method comprises three adaptive filters; two ANC filters to cancel correlated and uncorrelated disturbances, $d(n)$ and $v(n)$, respectively, and the LMS algorithm based supporting adaptive filter. The supporting filter generates desired error signal for adaptation of ANC filter for $d(n)$, and an appropriate reference signal for excitation of ANC filter for $v(n)$. The outputs of the two ANC filters are summed before being propagated through the cancelling loudspeaker. Computer simulations demonstrate that the proposed method can effectively mitigate the correlated and uncorrelated primary disturbances, and gives an improved noise reduction performance as compared with the cascading method of [6].

The remainder of the paper is organized as follows: Section II reviews the FxLMS algorithm, and its performance in the presence of uncorrelated disturbance. Section III presents the cascading algorithm of [6], and details the proposed method. Section IV presents computer simulations to validate the performance of the proposed algorithm, and Section V gives concluding remarks.

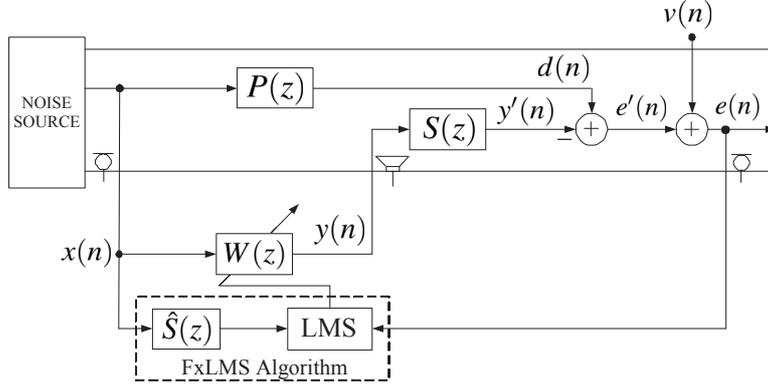


Figure 1: Block diagram of FxLMS algorithm based single-channel feedforward ANC systems.

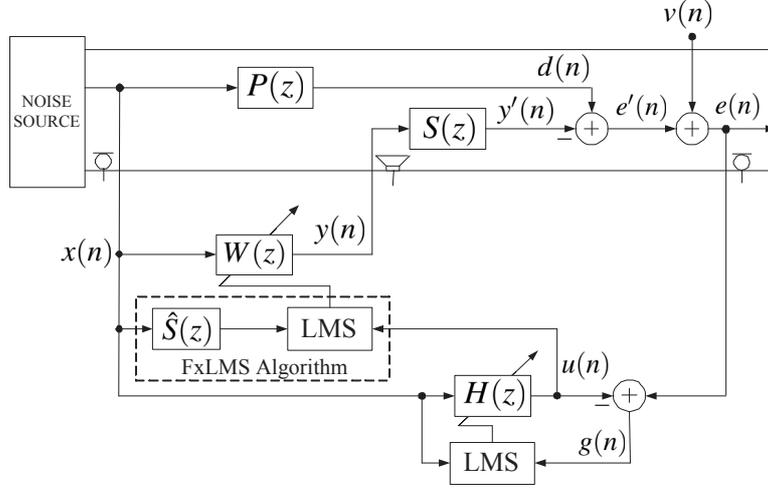


Figure 2: Block diagram of the cascading ANC system for improving adaptation of FxLMS algorithm in the presence of uncorrelated disturbance $v(n)$ [6].

2. FXLMS ALGORITHM AND EFFECT OF UNCORRELATED DISTURBANCE

The block diagram of FxLMS algorithm based single-channel feedforward ANC system for duct applications is shown in Fig. 1. Here components inside the duct are in the acoustic domain, and the rest of the components are in the electrical domain. Assuming that $W(z)$ is an FIR filter of tap-weight length L , the secondary signal $y(n)$ is expressed as

$$y(n) = \mathbf{w}^T(n)\mathbf{x}(n) \quad (1)$$

where $\mathbf{w}(n) = [w_0(n), w_1(n), \dots, w_{L-1}(n)]^T$ is the tap-weight vector, and $\mathbf{x}(n) = [x(n), x(n-1), \dots, x(n-L+1)]^T$ is an L sample reference signal vector. The residual error signal $e'(n)$ is given as

$$e'(n) = d(n) - y'(n) \quad (2)$$

where $d(n) = p(n) * x(n)$ is the primary disturbance signal, $y'(n) = s(n) * y(n)$ is the secondary cancelling signal, $*$ denotes linear convolution and $p(n)$ and $s(n)$ are impulse responses of the primary path $P(z)$ and secondary path $S(z)$, respectively. This is the desired error signal required for the

adaptation of FxLMS algorithm for $W(z)$. As stated earlier, in many application the error microphone may pick a noise component uncorrelated with the reference signal $x(n)$. Denoting this uncorrelated noise as $v(n)$, the error signal picked-up by the error microphone is thus given as

$$e(n) = e'(n) + v(n) = d(n) - y'(n) + v(n). \quad (3)$$

Minimizing the mean square error cost function; $J(n) = E\{e^2(n)\} \approx e^2(n)$, where $E\{\cdot\}$ is the expectation operator; the FxLMS algorithm [2] is given as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_w e(n) \mathbf{x}'(n) \quad (4)$$

where μ_w is the step size parameter, and $\mathbf{x}'(n) = [x'(n), x'(n-1), \dots, x'(n-L+1)]^T$ is filtered reference signal vector, where $x'(n)$ is obtained by filtering reference signal $x(n)$ through the model of secondary path $\hat{S}(z)$ and is given as

$$x'(n) = \hat{s}(n) * x(n) \quad (5)$$

where $\hat{s}(n)$ is impulse response of the secondary path model filter $\hat{S}(z)$. Substituting (3) in (4), the update equation for

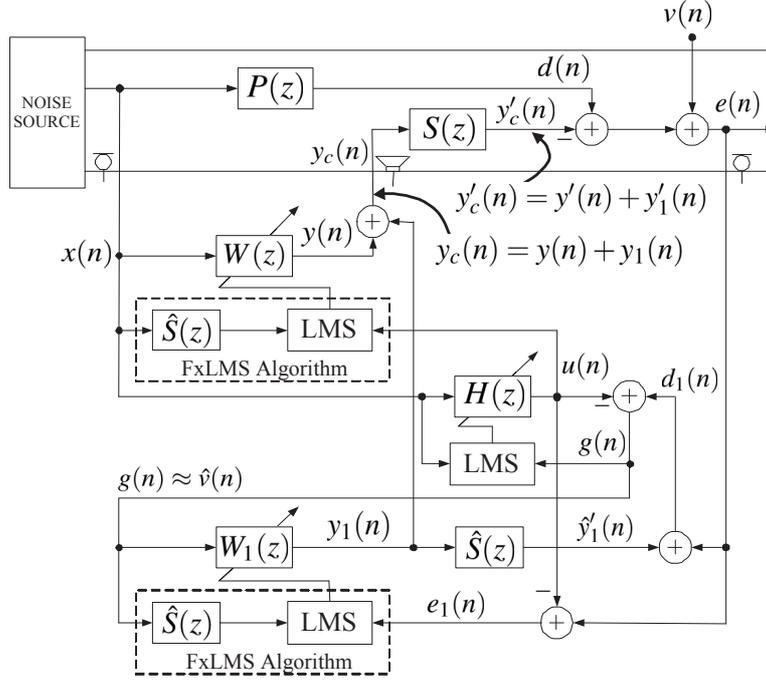


Figure 3: Proposed method for improving performance of ANC system in the presence of uncorrelated disturbance at the error microphone.

FxLMS algorithm can be written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_w e'(n) \mathbf{x}'(n) + \mu_w v(n) \mathbf{x}'(n). \quad (6)$$

It is evident that the adaptation is perturbed by the uncorrelated noise component $v(n)$, and as shown in [6], the steady-state performance of the FxLMS algorithm will be degraded significantly. Furthermore, $v(n)$ appearing uncontrolled at the error microphone degrades the noise reduction performance of the ANC system.

3. PROPOSED METHOD FOR REMOVING THE EFFECT OF UNCORRELATED DISTURBANCE

The main idea of cascading algorithm in [6] is to update the adaptive filter $W(z)$ using desired error signal $e'(n)$ instead of using the disturbed error signal $e(n)$. The block diagram of cascading adaptive filters based ANC system (hereafter called as Sun's method) is shown in Fig. 2, where the adaptive filter $H(z)$ is introduced to estimate the desired error signal $e'(n)$ [6].

It is evident that $H(z)$ is excited by the reference signal $x(n)$, and the error signal $e(n)$ is used as desired response for its adaptation. Thus output of $H(z)$, $u(n)$, converges to that part in $e(n)$ which is correlated with $x(n)$. From (1), (2), and (3), it is clear that in $e(n)$, $e'(n)$ is correlated with $x(n)$ and $v(n)$ is the uncorrelated part. Hence, when $H(z)$ converges, its output converges to $u(n) \approx e'(n) = d(n) - y'(n)$, which is the desired error signal for the adaptation of $W(z)$. Thus FxLMS algorithm for this cascading system is given as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_w u(n) \mathbf{x}'(n). \quad (7)$$

Since a disturbance free error signal is used, Sun's method improves the convergence of the FxLMS algorithm. How-

ever, it cannot mitigate the effect of the uncorrelated disturbance $v(n)$ from the residual noise $e(n)$, and this is motivation for the proposed method as explained below.

The proposed method, as shown in Fig. 3, comprises three adaptive filter: 1) the ANC filter $W(z)$ to cancel the primary noise $d(n)$ correlated with the reference signal $x(n)$, 2) the ANC filter $W_1(z)$ to cancel uncorrelated disturbance $v(n)$, and 3) supporting filter $H(z)$. The $W(z)$ is excited by the reference signal $x(n)$, and the $W_1(z)$ is excited by an internally generated reference signal. Both ANC filters $W(z)$ and $W_1(z)$ are adapted by FxLMS algorithm. The outputs of $W(z)$ and $W_1(z)$ are summed together to get the cancelling signal, $y_c(n)$, being propagated by the cancelling loudspeaker, and is given as

$$y_c(n) = y(n) + y_1(n), \quad (8)$$

where $y(n)$ is the output of $W(z)$ given by (1), and $y_1(n)$ is the output of $W_1(z)$ give as

$$y_1(n) = \mathbf{w}_1^T(n) \mathbf{g}(n) \quad (9)$$

where $\mathbf{w}_1(n)$ is the tap-weight vector for $W_1(z)$, and $\mathbf{g}(n)$ contains samples of internally generated reference signal $g(n)$ for $W_1(z)$. The summed signal $y_c(n)$ is propagated through the secondary path $S(z)$ to acoustically achieve the noise cancellation and the residual error signal $e(n)$ is generated as

$$\begin{aligned} e(n) &= d(n) + v(n) - y'_c(n) \\ &= [d(n) - y'(n)] + [v(n) - y'_1(n)] \end{aligned} \quad (10)$$

where $y'(n) = s(n) * y(n)$ is the cancelling signal for $d(n)$ and $y'_1(n) = s(n) * y_1(n)$ is the cancelling signal for $v(n)$. In (10), the first term is desired error signal for the adaptation of $W(z)$ and second term is desired error signal for $W_1(z)$. To achieve

cancellation [ideally $e(n) = 0$], $W(z)$ needs to be excited by input correlated with $d(n)$ [the reference signal $x(n)$ is indeed that input], and $W_1(z)$ needs to be excited by input correlated with $v(n)$ [such input is not available directly and needs to be generated internally, as explained later].

As shown in Fig. 3, the output of $W_1(z)$, $y_1(n)$ is filtered through secondary path model $\hat{S}(z)$, and added to error signal $e(n)$ to compute the desired response for $H(z)$ as

$$\begin{aligned} d_1(n) &= e(n) + \hat{y}'_1(n) \\ &= [d(n) - y'(n)] + [v(n) - y'_1(n)] + \hat{y}'_1(n) \\ &= [d(n) - y'(n)] + v(n) + [\hat{s}(n) - s(n)] * y_1(n). \end{aligned} \quad (11)$$

Assuming that the secondary path is perfectly identified, which can be obtained by using offline [2] and/or online modeling techniques [7, 8], (11) simplifies to

$$d_1(n) \approx [d(n) - y'(n)] + v(n). \quad (12)$$

The output of supporting filter $H(z)$ is computed as

$$u(n) = \mathbf{h}^T(n)\mathbf{x}(n), \quad (13)$$

where $\mathbf{h}(n)$ is the tap-weight vector for $H(z)$. The error signal for LMS equation of $H(z)$, $g(n)$, is generated as

$$\begin{aligned} g(n) &= d_1(n) - u(n) \\ &\approx [d(n) - y'(n)] + v(n) - u(n), \end{aligned} \quad (14)$$

and $H(z)$ is adapted using LMS algorithm as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu_h g(n)\mathbf{x}(n), \quad (15)$$

where μ_h is the step-size for $H(z)$. Since $H(z)$ is excited by $x(n)$, minimizing the error signal $g(n)$ means that output of $H(z)$, $u(n)$, would converge to that part in (14) which is correlated with $x(n)$, thus

$$u(n) \rightarrow [d(n) - y'(n)], \quad (16)$$

and hence $g(n)$ converges to an estimate of $v(n)$

$$g(n) \rightarrow v(n). \quad (17)$$

This shows that $u(n)$ can be used as an error signal for adaptation of $W(z)$, and $g(n)$ can be used as reference signal for excitation of $W_1(z)$. The $W(z)$ is adapted using FxLMS algorithm as given in (7).

The output $u(n)$ of $H(z)$ is combined with the residual error signal $e(n)$ to generate the error signal for $W_1(z)$ as

$$e_1(n) = e(n) - u(n) \approx [v(n) - y'_1(n)], \quad (18)$$

and then $W_1(z)$ is adapted using FxLMS algorithm as

$$\mathbf{w}_1(n+1) = \mathbf{w}_1(n) + \mu_{w_1} e_1(n)\mathbf{g}'(n) \quad (19)$$

where μ_{w_1} is the step size parameter for $W_1(z)$, and $\mathbf{g}'(n)$ is filtered reference signal vector obtained by filtering $g(n)$ through $\hat{S}(z)$ as

$$\mathbf{g}'(n) = \hat{s}(n) * g(n). \quad (20)$$

A comparison between the proposed method and Sun's method is as given below:

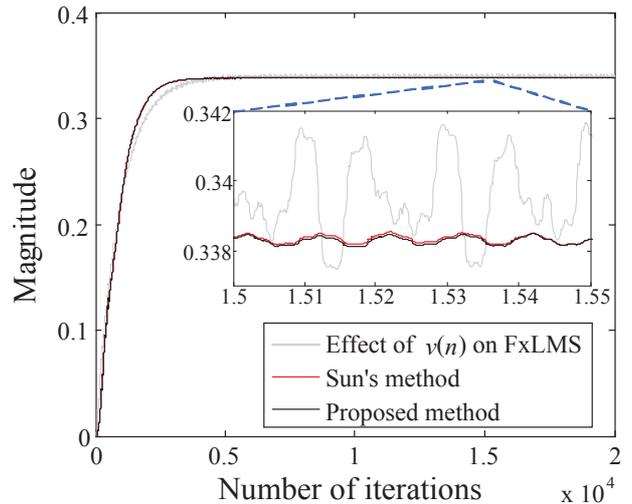


Figure 4: Convergence of norm of adaptive filter tap-weights, $\|\mathbf{w}(n)\|$ for proposed method in comparison with the FxLMS algorithm and Sun's method. Small windows show an enlarged view of curves in steady state.

- The proposed method provides control over both correlated and uncorrelated disturbances, where as Sun's method can only improve the convergence of $W(z)$, but cannot reduce the uncorrelated disturbance.
- In proposed method, the role of $H(z)$ is partly same as that in Sun's method. It generates desired error signal for adaptation of $W(z)$ to provide cancellation for correlated disturbance signal $d(n)$. Furthermore, it is used to generate appropriate signals for adaptation of $W_1(z)$ to cancel uncorrelated disturbance $v(n)$.
- A computational complexity analysis shows that the proposed method would required more number of computations per iteration as compared with the Sun's method.

4. COMPUTER SIMULATIONS

The computer simulations are carried out to demonstrate the effectiveness of the proposed method, in comparison with Sun's method [6]. The acoustic paths are modeled using data provided in the disk attached with [2]. Using this data $P(z)$ and $S(z)$ are modeled as FIR filter of length 256 and 64 respectively. It is assumed that the secondary path modeling filter $\hat{S}(z)$ is exactly identified as $S(z)$. The reference noise signal $x(n)$ is a unit variance narrowband signal composed of three sinusoids with frequencies of 165, 290, and 410 Hz. A white noise with variance 0.001 is added to count for any measurement noise at the reference microphone. The uncorrelated disturbance $v(n)$ is another unit variance narrowband signal comprising three sinusoids with frequencies of 250, 350, and 450 Hz, and a white noise with variance 0.001 is added to count for any measurement noise at the error microphone. The sampling frequency is 4 kHz, and the results shown are average of 10 realizations.

In simulations, the adaptive filters $W(z)$, $H(z)$ and $W_1(z)$ are selected as FIR filters of tap-weight lengths 192, 32, and 192, respectively. All adaptive filters are initialized by null vectors of an appropriate order. The step-size parameters

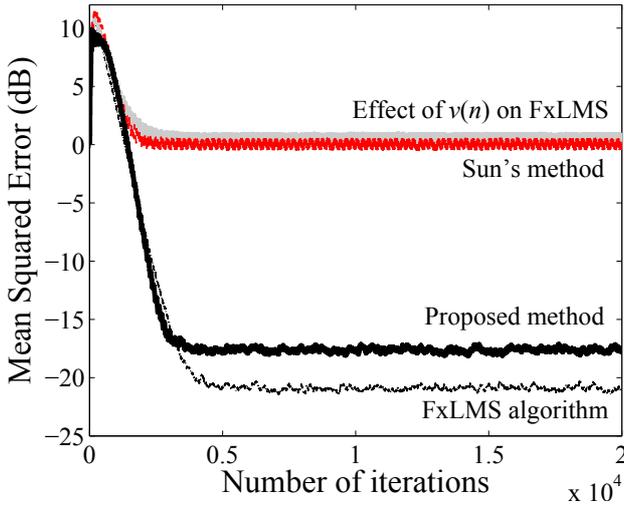


Figure 5: Curves for mean squared error (MSE) for various methods.

are selected experimentally, such that fast and stable performance is achieved. For FxLMS algorithm-based ANC filters, the step-size is selected as 1×10^{-5} , and for supporting filter $H(z)$, the step size is selected as 1×10^{-3} .

The adaptation of adaptive filter $W(z)$, in terms of $\|\mathbf{w}(n)\|$, which is Euclidean norm of weight vector $\mathbf{w}(n)$, is shown in Fig. 4. The presence of uncorrelated disturbance severely affects convergence of the $W(z)$, and results in oscillations even in steady state of adaptive filter (see small window in Fig. 4). We see that both Sun's method and the proposed method can remove the effect of uncorrelated periodic disturbance from the adaptation of $W(z)$, and result in smooth steady state convergence.

The noise reduction performance, in terms of means squared error (MSE), for various methods is shown in Fig. 5. We see that uncorrelated disturbance $v(n)$ appearing at the error microphone degrades the noise reduction performance of the FxLMS algorithm and Sun's method. The proposed method, incorporating a separate ANC filter $W_1(z)$ for uncorrelated disturbance, gives significantly improved noise reduction performance. To further highlight the improved performance of the proposed method, Fig. 5 shows the power spectral density of the residual error signal $e(n)$ in steady state for Sun's method and the proposed method. Here spectrum of the primary disturbance $d(n)$ is also shown as a reference, and location of frequency components in uncorrelated disturbance is indicated by small arrows. We see that, as compared with the Sun's method, the proposed method is more effective in removing the uncorrelated disturbance. From these simulation results we conclude that the proposed method keeps the good performance of Sun's method is improving convergence of ANC filter $W(z)$, and furthermore, gives much better noise reduction performance.

5. CONCLUDING REMARKS

In this paper, we have presented a method to improve the performance of ANC systems in the presence of uncorrelated periodic disturbance at the error microphone. The existing method [6] can make the adaptation of the noise control fil-

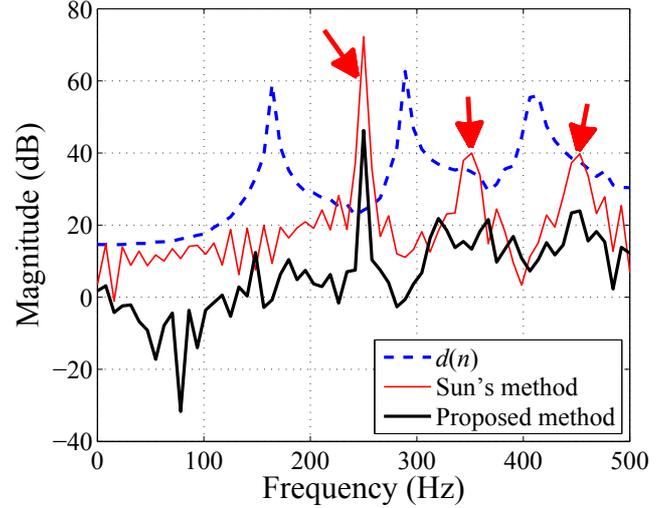


Figure 6: Magnitude spectrum of desired error signal in steady-state for various methods in comparison with the magnitude spectrum of primary disturbance $d(n)$.

ter independent of the uncorrelated disturbance, but cannot remove its effect from the residual noise and results in high steady state MSE as shown in Fig. 5. We have developed a hybrid structure that can simultaneously control the correlated and uncorrelated noise sources, and hence provides significantly improved noise reduction performance. This improved performance is achieved at an increased computational complexity, which is due to introduction of a separate ANC filter for uncorrelated disturbance.

REFERENCES

- [1] S. J. Elliot, *Signal Processing for Active Control*, London, U.K.: Academic Press, 2001.
- [2] S. M. Kuo, and D. R. Morgan, *Active Noise Control Systems- Algorithms and DSP Implementations*, New York: Wiley, 1996.
- [3] B. Widrow, and S.D. Stearns, *Adaptive Signal Processing*, Prentice Hall, New Jersey, 1985.
- [4] S. M. Kuo and W. S. Gan, "Active noise control systems with optimized secondary path," in *Proc. IEEE Int. Conf. Control Applic.*, Sep. 2004, pp. 765770.
- [5] S. M. Kuo and M. Ji, "Passband disturbance reduction in periodic active noise control systems," *IEEE Trans. Speech Audio Process.*, vol. 4, no. 2, pp. 96103, Mar. 1996.
- [6] X. Sun and S. M. Kuo, "Active narrowband noise control systems using cascading adaptive filters," *IEEE Trans. Audio Speech Language Process.*, vol. 15, no. 2, pp. 586592, Feb. 2007.
- [7] M. T. Akhtar, M. Abe, and M. Kawamata, "A New structure for feedforward active noise control systems with improved online secondary path modeling," *IEEE Trans. Speech Audio Process.*, vol. 13, no. 5, pp. 1082–1088, Sep. 2005.
- [8] M. T. Akhtar, M. Abe, and M. Kawamata, "A New Variable Step Size LMS Algorithm-Based Method for Improved Online Secondary Path Modeling in Active Noise Control Systems," *IEEE Trans. Audio Speech Language Process.*, vol. 14, no. 2, pp. 720–726, Mar. 2006.