DIRECT STATE DETERMINATION OF MULTIPLE SOURCES WITH INTERMITTENT EMISSION

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ABSTRACT

This paper investigates the direct state determination problem from passive measurements made with a moving antenna array in the case of a time-varying number of emitting sources. We derive the Cramér-Rao Bound (CRB) for the estimation problem and find an approximation that is applicable for a large number of observations. We use two Subspace Data Fusion (SDF) approaches to solve the estimation problem. Therein, subspaces are formed in the pre-processing step from the raw antenna outputs at all positions of the moving array. Then the state parameters of interest (e.g. position, velocity) are estimated directly from a cost function that results from fusing all subspaces. The SDF approaches are based on the Multiple Signal Classification (MUSIC) and on the Subspace Fitting (SSF) method using a low- and high-dimensional optimization, respectively. In simulations, we find that the SSF-SDF approach outperforms the MUSIC-SDF approach.

1. INTRODUCTION

Target Motion Analysis (TMA) of multiple narrowband sources using passive antenna arrays is a fundamental task encountered in various fields like communication, radar, and sonar. We consider a scenario with a single moving observer equipped with an antenna array. At N different points in space, the sensor receives signals of Q moving sources and collects batches of antenna outputs. The scenario is assumed to be stationary during one batch and non-stationary from batch to batch.

According to the traditional approach to solving the TMA problem, first of all, the Directions of Arrival (DOAs) of all sources are estimated with a direction-finding (DF) estimator like the subspace-based MUSIC method [1]. Then a data association step follows to partition the DOAs into sets of measurements belonging to the same source. Finally, the DOAs for each source are used to determine its state with the help of a suitable bearings-only tracking algorithm.

Recently, some direct position determination (DPD) methods based on the antenna outputs have been proposed without computing intermediate parameters like DOAs. The basic idea for a subspace-based DPD approach goes back to the pioneering work of Wax and Kailath [2]. They noted that in this way the data association step is avoided. Moreover, this kind of approach was used for a multiarray network in order to estimate the positions of multiple sources without explicitly computing DOAs and Times of Arrival [3]. Maximum Likelihood (ML) methods can be found e.g. in [4], but they are more computationally demanding in the case of multiple sources. In our previous work, we proposed a subspace-based DPD approach for a single moving array [5]. Moreover, we have shown that the DPD approach can be extended to estimate the target state (e.g. position, velocity) [6] or adapted to estimate DOAs and DOA rates [7].

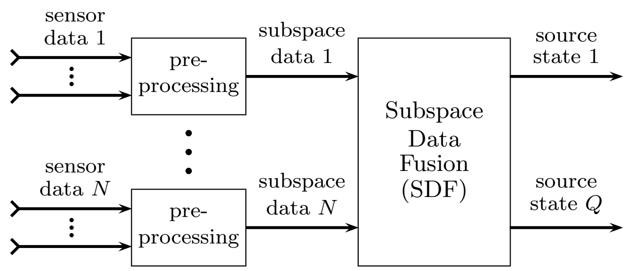

Figure 1: Basic steps of the SDF approach

In all these Subspace Data Fusion (SDF) approaches, the parameters of interest are obtained by minimizing a single cost function into which all subspaces at all sensor positions enter jointly (Fig. 1). Moreover, the estimation accuracy of the source state is much better than the traditional TMA approach in situations where the variance of DOA estimates deviates from the CRB.

However, the methods presented so far only investigate the case where the number of emitting sources is constant. Farina *et al.* derived the CRB for the general case that the probability of detection is smaller than unity [8]. We extend the results to the case of multiple sources with intermittent emission and adapt them to derive the CRB for the direct state determination problem. We give a brief review of the SDF approach based on the MUSIC method [1] and propose an extension by using the Subspace Fitting (SSF) method described in [9]. We show that the state estimation accuracy of the SSF-SDF approach is much better compared to the previous MUSIC-SDF approach in situations of a time-varying number of emitting sources.

This paper is organized as follows: Section 2 presents the problem formulation. In Section 3 we derive the CRB for the described TMA problem, and in Section 4, we outline the considered SDF approaches. Section 5 presents Monte Carlo simulation results that demonstrate the estimator's performance. The conclusions are given in Section 6.

The following notations are used throughout this paper: $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively; $\mathbf{I}_n$ and $\mathbf{0}_n$ denote the $n \times n$-dimensional identity and zero matrix, respectively; and $\mathrm{E}\{\cdot\}$ denotes the expectation operation.

2. PROBLEM FORMULATION

For N observations, 2^N possible emitting/non-emitting sequences per source can be formed. The κ-th possible sequence reads $S_{q,\kappa} : (b_{1,q})_\kappa, ..., (b_{N,q})_\kappa$, $\kappa = 1, ..., 2^N$, where $b_{n,q}$ is a binary variable that corresponds to the case where the q-th source is emitting or non-emitting at time t_n. For a given emitting probability $P_{\mathrm{e},q}$, which is constant over the number of observations, the probability of occurrence of a particular emitting/non-emitting sequence is given by

$$P(S_{q,\kappa}) = P_{\mathrm{e},q}^{N-\bar{\triangle}_\kappa} (1 - P_{\mathrm{e},q})^{\bar{\triangle}_\kappa} , \tag{1}$$

where $\bar{\triangle}_\kappa$ is the number of observations where the q-th source does not emit. For Q sources, 2^{NQ} collections of independent emitting/non-emitting sequences called events are possible. The probability of occurrence of a particular event $E_\ell : (\kappa_1)_\ell, ..., (\kappa_Q)_\ell$, $\ell = 1, ..., 2^{NQ}$, is given by

$$P(E_\ell) = \prod_{q=1}^{Q} P(S_{q,(\kappa_q)_\ell}) . \tag{2}$$

We consider an antenna array composed of M elements mounted on a moving platform and Q inertially moving sources in the far field of the antenna array. The sources are assumed to radiate narrowband signals with wavelengths centered around a common wavelength λ. The q-th source state $\mathbf{x}_q$ is given by the source position $\mathbf{p}_{0,q} = (x_{0,q}, y_{0,q}, z_{0,q})^T$ at reference time t_0 and the constant velocity $\dot{\mathbf{p}}_q = (\dot{x}_q, \dot{y}_q, \dot{z}_q)^T$ for $q = 1, ..., Q$. The source position $\mathbf{p}_n(\mathbf{x}_q)$ at some time t_n is related to the source state $\mathbf{x}_q = (\mathbf{p}_{0,q}^T, \dot{\mathbf{p}}_q^T)^T$ by

$$\mathbf{p}_n(\mathbf{x}_q) = \mathbf{p}_{0,q} + (t_n - t_0)\dot{\mathbf{p}}_q . \tag{3}$$

Fig. 2 shows the geometry for the scenario of Q inertially moving sources and a sensor moving along an arbitrary but known trajectory. During the movement of the array, N batches of data are collected at the positions $\mathbf{r}_n$, $n = 1, ..., N$. For the sake of simplicity, we assume that the antenna attitude does not change with time, i.e. the orientation of the sensor-fixed coordinate system is fixed during the batches. The distance between the q-th source and the observer at the n-th time slot, $\triangle r_{n,q}$, is given by the length of the relative vector

$$\triangle\mathbf{r}_n(\mathbf{x}_q) = \mathbf{r}_n - \mathbf{p}_n(\mathbf{x}_q) . \tag{4}$$

Let $s_{n,k,q}$ denote the complex envelope of the k-th sample, $k = 1, ..., K$, of the q-th source signal measured at time t_n if this source emits, i.e. $b_{n,q} = 1$, and let $\mathbf{z}_{n,k} \in \mathbb{C}^{M\times 1}$ denote the complex envelopes formed from the signals received by the array elements. This received vector can be expressed as

$$\mathbf{z}_{n,k} = \sum_{q=1}^{Q} \mathbf{a}_n(\mathbf{x}_q)\, b_{n,q}\, s_{n,k,q} + \mathbf{w}_{n,k} , \tag{5}$$

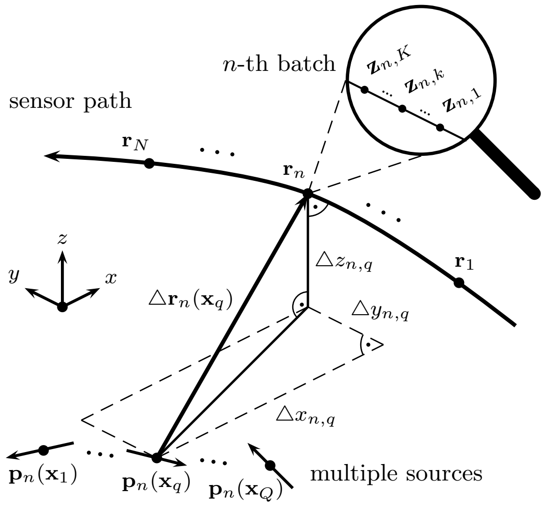

Figure 2: Geometry for the scenario of multiple inertially moving sources and a single moving sensor

where $\mathbf{w}_{n,k} \in \mathbb{C}^{M\times 1}$ is the complex envelope of the noise. Let the array be sampled sequentially at K different mutually exclusive time slots, and assume that the array transfer vectors can be considered quasistatic in each slot, i.e. the sensor's displacement during each time slot is negligible. The array transfer vector expresses its complex response at time t_n to a planar wavefront arriving from the direction of the relative position $\triangle\mathbf{r}_n(\mathbf{x}_q)$ (Eq. 4). We assume that the antenna array is perfectly calibrated for which the array transfer vector is a known function of the source states:

$$\mathbf{a}_n(\mathbf{x}_q) = \left(\mathrm{e}^{\mathrm{i}\,\mathbf{k}_n^T(\mathbf{x}_q)\mathbf{d}_1}, ..., \mathrm{e}^{\mathrm{i}\,\mathbf{k}_n^T(\mathbf{x}_q)\mathbf{d}_M}\right)^T \tag{6}$$

The array transfer vector depends on the position $\mathbf{d}_m$ of the m-th antenna element, $m = 1, ..., M$, relative to the position $\mathbf{r}_n$, and the wavenumber vector

$$\mathbf{k}_n(\mathbf{x}_q) = \frac{2\pi}{\lambda}\frac{\triangle\mathbf{r}_n(\mathbf{x}_q)}{\triangle r_{n,q}} = \frac{2\pi}{\lambda}\frac{1}{\triangle r_{n,q}}\begin{pmatrix}\triangle x_{n,q}\\ \triangle y_{n,q}\\ \triangle z_{n,q}\end{pmatrix} . \tag{7}$$

Eq. 5 can be written more compactly as

$$\mathbf{z}_{n,k} = \mathbf{A}_n(\boldsymbol{\rho}_{\mathbf{x},n})\,\check{\mathbf{s}}_{n,k} + \mathbf{w}_{n,k} , \tag{8}$$

where $\mathbf{A}_n(\boldsymbol{\rho}_{\mathbf{x},n}) = [\mathbf{a}_n(\mathbf{x}_1)\cdots\mathbf{a}_n(\mathbf{x}_{Q_n})] \in \mathbb{C}^{M\times Q_n}$ is the array transfer matrix, and

$$\begin{aligned}\boldsymbol{\rho}_{\mathbf{x},n} &= (\mathbf{x}_1^T, ..., \mathbf{x}_{Q_n}^T)^T \in \mathbb{R}^{6Q_n\times 1}\\ \check{\mathbf{s}}_{n,k} &= (s_{n,k,1}, ..., s_{n,k,Q_n})^T \in \mathbb{C}^{Q_n\times 1}\end{aligned}$$

denote subsets from the complete parameter vectors

$$\begin{aligned}\boldsymbol{\rho}_{\mathbf{x}} &= (\mathbf{x}_1^T, ..., \mathbf{x}_Q^T)^T \in \mathbb{R}^{6Q\times 1}\\ \mathbf{s}_{n,k} &= (s_{n,k,1}, ..., s_{n,k,Q})^T \in \mathbb{C}^{Q\times 1}\end{aligned} \tag{9}$$

w.r.t. the effective number of emitting sources $Q_n = \sum_{q=1}^{Q} b_{n,q}$ at the n-th batch.

Now, we introduce the compact data model

$$\mathbf{z}_k = \boldsymbol{\mathcal{A}}(\boldsymbol{\rho}_\mathbf{x})\,\check{\mathbf{s}}_k + \mathbf{w}_k \tag{10}$$

by stacking the vectors on top and using a block-diagonal matrix:

$$\begin{aligned}
\mathbf{z}_k &= (\mathbf{z}_{1,k}^T, ..., \mathbf{z}_{N,k}^T)^T \in \mathbb{C}^{MN\times 1}\,,\\
\boldsymbol{\mathcal{A}}(\boldsymbol{\rho}_\mathbf{x}) &= \mathrm{diag}[\mathbf{A}_1(\boldsymbol{\rho}_{\mathbf{x},1})\cdots\mathbf{A}_N(\boldsymbol{\rho}_{\mathbf{x},N})] \in \mathbb{C}^{MN\times\sum_n Q_n}\,,\\
\check{\mathbf{s}}_k &= (\check{\mathbf{s}}_{1,k}^T, ..., \check{\mathbf{s}}_{N,k}^T)^T \in \mathbb{C}^{\sum_n Q_n\times 1}\,,\\
\mathbf{w}_k &= (\mathbf{w}_{1,k}^T, ..., \mathbf{w}_{N,k}^T)^T \in \mathbb{C}^{MN\times 1}\,.
\end{aligned}$$

Now, the problem is stated as follows: Estimate all source states $\boldsymbol{\rho}_\mathbf{x}$ from all received data batches $\mathbf{Z} = [\mathbf{z}_1\cdots\mathbf{z}_K] \in \mathbb{C}^{MN\times K}$. To solve the multiple source TMA problem, the following assumptions are made:

1. The noise vectors $\mathbf{w}_k$, $k = 1, ..., K$, (Eq. 10) are zero-mean complex Gaussian. They are temporally and spatially uncorrelated with the covariance

$$\begin{aligned}
\mathrm{E}\left\{\mathbf{w}_k\mathbf{w}_{k'}^H\right\} &= \sigma_\mathrm{w}^2\,\mathbf{I}_{MN}\,\delta_{k,k'}\,,\\
\mathrm{E}\left\{\mathbf{w}_k\mathbf{w}_{k'}^T\right\} &= \mathbf{0}_{MN}\,,
\end{aligned} \tag{11}$$

   where $\delta_{k,k'}$ denotes the Kronecker delta.
2. The signal vectors $\check{\mathbf{s}}_{n,k}$, $k = 1, ..., K$, (Eq. 8) are fixed and need to be estimated (deterministic data model). This does not exclude the possibility that the signals are sampled from a random process. Moreover, we assume that $\sum_{k=1}^K \check{\mathbf{s}}_{n,k}\,\check{\mathbf{s}}_{n,k}^H$ is positive definite.
3. The total number of sources Q and the effective number of sources per batch Q_n, $n = 1, ..., N$, are known. In the past, several methods have been proposed to determine the source number, e.g. in [10].

3. CRAMÉR-RAO BOUND

It is well known that the CRB provides a lower bound on the estimation accuracy and its parameter dependencies reveal characteristic features of the estimation problem. Given a particular event E_ℓ, the target parameters are comprised in the vector

$$\boldsymbol{\rho} = \left(\bar{\check{\mathbf{s}}}_1^T, \tilde{\check{\mathbf{s}}}_1^T, ..., \bar{\check{\mathbf{s}}}_K^T, \tilde{\check{\mathbf{s}}}_K^T, \boldsymbol{\rho}_\mathbf{x}^T\right)^T \in \mathbb{R}^{2K\sum_n Q_n + 6Q\times 1}\,, \tag{12}$$

where $\bar{\check{\mathbf{s}}}_k$ and $\tilde{\check{\mathbf{s}}}_k$ are the real and imaginary part of the source signals. Then, the conditional CRB is related to the covariance matrix $\mathbf{C}$ of the estimation error $\triangle\boldsymbol{\rho} = \boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\mathbf{Z})$ of any unbiased estimator $\hat{\boldsymbol{\rho}}(\mathbf{Z})$ as

$$\mathbf{C} = \mathrm{E}\left\{\triangle\boldsymbol{\rho}\,\triangle\boldsymbol{\rho}^T|E_\ell\right\} \geq \mathrm{CRB}(\boldsymbol{\rho}|E_\ell)\,, \tag{13}$$

where the inequality means that the matrix difference is positive semidefinite. If the estimator attains the CRB then it is called efficient. The CRB is given by the inverse Fisher Information Matrix (FIM)

$$\mathbf{J}(\boldsymbol{\rho}|E_\ell) = \mathrm{E}\left\{\left(\frac{\partial\mathcal{L}(\mathbf{Z};\boldsymbol{\rho})}{\partial\boldsymbol{\rho}}\right)\left(\frac{\partial\mathcal{L}(\mathbf{Z};\boldsymbol{\rho})}{\partial\boldsymbol{\rho}}\right)^T\middle|\,E_\ell\right\}\,, \tag{14}$$

where

$$\mathcal{L} = -KMN\ln(\pi\sigma_\mathrm{w}^2) - \frac{1}{\sigma_\mathrm{w}^2}\sum_{k=1}^K|\mathbf{z}_k - \boldsymbol{\mathcal{A}}(\boldsymbol{\rho}_\mathbf{x})\,\check{\mathbf{s}}_k|^2\,, \tag{15}$$

is the log-likelihood function and the parameters refers to the event E_ℓ. In this log-likelihood function $\mathbf{z}_k$, $k = 1, ..., K$, are random variables due to the random variables $\mathbf{w}_k$, $k = 1, ..., K$, and the expectation operation in Eq. 14 is w.r.t. these random variables.

Performing all calculations analog to [11], we obtain the deterministic CRB for all source states after some algebra (Assumption 1):

$$\mathrm{CRB}(\boldsymbol{\rho}_\mathbf{x}|E_\ell) = \frac{\sigma_\mathrm{w}^2}{2}\left[\sum_{k=1}^K \mathrm{Re}\left\{\boldsymbol{\mathcal{S}}_k^H\,\mathbf{D}^H\,\mathbf{P}_{\boldsymbol{\mathcal{A}}}^\perp\,\mathbf{D}\,\boldsymbol{\mathcal{S}}_k\right\}\right]^{-1} \tag{16}$$

with

$$\begin{aligned}
\boldsymbol{\mathcal{S}}_k &= \mathbf{I}_{6Q}\otimes\check{\mathbf{s}}_k \in \mathbb{C}^{6Q\sum_n Q_n\times 6Q}\,,\\
\mathbf{D} &= [\mathbf{D}_1\cdots\mathbf{D}_Q] \in \mathbb{C}^{MN\times 6Q\sum_n Q_n}\,,\\
\mathbf{D}_q &= \left[\frac{\partial\boldsymbol{\mathcal{A}}}{\partial x_{0,q}}, \frac{\partial\boldsymbol{\mathcal{A}}}{\partial y_{0,q}}, \frac{\partial\boldsymbol{\mathcal{A}}}{\partial z_{0,q}}, \frac{\partial\boldsymbol{\mathcal{A}}}{\partial\dot{x}_q}, \frac{\partial\boldsymbol{\mathcal{A}}}{\partial\dot{y}_q}, \frac{\partial\boldsymbol{\mathcal{A}}}{\partial\dot{z}_q}\right]\,,\\
\mathbf{P}_{\boldsymbol{\mathcal{A}}}^\perp &= \mathbf{I}_{MN} - \boldsymbol{\mathcal{A}}\left(\boldsymbol{\mathcal{A}}^H\boldsymbol{\mathcal{A}}\right)^{-1}\boldsymbol{\mathcal{A}}^H \in \mathbb{C}^{MN\times MN}\,,
\end{aligned}$$

where $\otimes$ denotes the Kronecker product.

The bound in Eq. 16 is conditioned on the particular event E_ℓ. The unconditional CRB is obtained by taking expectation and using Eq. 2

$$\mathrm{CRB}(\boldsymbol{\rho}_\mathbf{x}) = \sum_{\ell=1}^{2^{NQ}} P(E_\ell)\,\mathrm{CRB}(\boldsymbol{\rho}_\mathbf{x}|E_\ell)\,. \tag{17}$$

Observe that although the number of possible events grows exponentially with the number of batches N and sources Q, the probabilities of the vast majority of events (Eq. 2) are negligible.

The cumulative distribution function (cdf) ϕ_q of the number of batches $\bar{\triangle} = 0, ..., N$ with the non-emitting q-th source is given by [8, Eq. 23]

$$\phi_q(\bar{\triangle}) = \sum_{\bar{\delta}=0}^{\bar{\triangle}}\binom{N}{\bar{\delta}}P_{\mathrm{e},q}^{N-\bar{\delta}}\,(1-P_{\mathrm{e},q})^{\bar{\delta}}\,. \tag{18}$$

By definition, $\phi_q(N) = 1$. Above a certain threshold value $\bar{\triangle}_{\mathrm{thr},q}$, all events corresponding to this sequences can be safely ignored in the calculation of the CRB in Eq. 17 for all practical purposes. The threshold value $\bar{\triangle}_{\mathrm{thr},q}$ can be determined by progressively computing Eq. 18 for $\bar{\triangle} = 0, 1, 2, ...$ until its value is greater than some cdf threshold ϕ_thr, which should be chosen to be marginally less than 1, e.g. $\phi_\mathrm{thr} = 0.99$, i.e. [8, Eq. 24]

$$\bar{\triangle}_{\mathrm{thr},q} = \min\{\bar{\triangle}\,,\ \mathrm{s.t.}\ \phi_q(\bar{\triangle}) > \phi_\mathrm{thr}\}\,. \tag{19}$$

This strategy ensures that only events are considered which contribute significantly. Eq. 20 gives the number of events L_approx to take into account in the approximate calculation of the CRB (Eq. 17). A reduction of ϕ_thr would correspond to less computational load but also a reduced accuracy.

$$L_\mathrm{approx} = \prod_{q=1}^{Q}\sum_{\bar{\delta}=0}^{\bar{\triangle}_{\mathrm{thr},q}}\binom{N}{\bar{\delta}} \ll 2^{NQ} \tag{20}$$

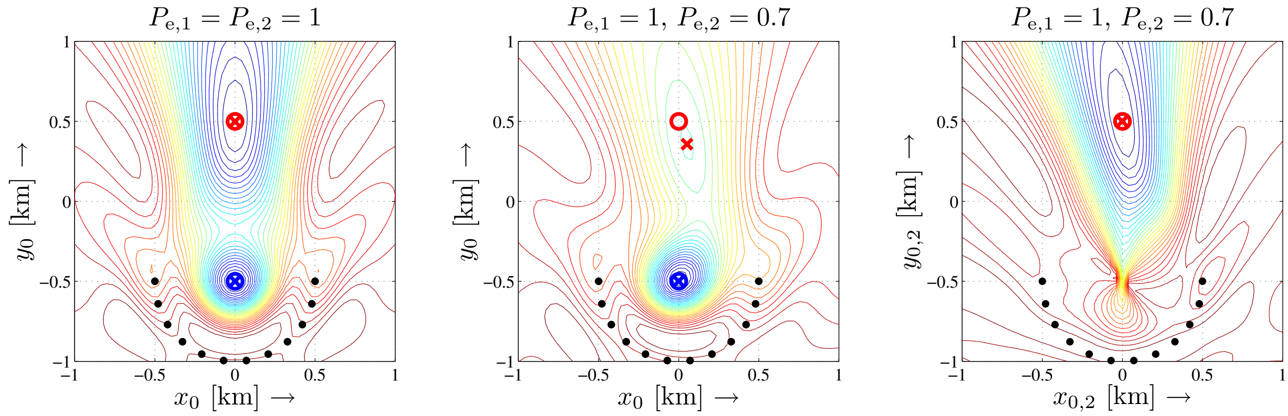

Figure 3: Cuts through cost function of the MUSIC-SDF approach (left, middle) and the SSF-SDF approach (right) with the observation points (black dots), the true (circles) and estimated (crosses) position parameters for the given emitting probabilities

4. SOURCE STATE DETERMINATION

SDF approaches calculate the source states directly in one step from the subspaces at all sensor positions (Fig. 1). The subspaces are calculated in a preprocessing step by performing an eigendecomposition of the covariance matrix (Assumptions 2 and 3):

$$\mathbf{R}_n = \frac{1}{K}\sum_{k=1}^{K} \mathbf{z}_{n,k}\,\mathbf{z}_{n,k}^H = [\mathbf{V}_n\,\mathbf{U}_n]\,\mathbf{\Lambda}_n\,[\mathbf{V}_n\,\mathbf{U}_n]^H\,, \quad (21)$$

where the column vectors of $\mathbf{V}_n \in \mathbb{C}^{M\times Q_n}$ and $\mathbf{U}_n \in \mathbb{C}^{M\times M-Q_n}$ are, respectively, the eigenvectors spanning the signal and orthogonal noise subspaces of the covariance $\mathbf{R}_n$ with the associated eigenvalues in decreasing order on the diagonal of $\mathbf{\Lambda}_n \in \mathbb{R}^{M\times M}$.

Note that the SDF approaches do not use knowledge about the emitting probabilities $P_{e,q}$, because they are not sensor parameters like the probability of detection.

4.1 MUSIC-SDF Approach

This SDF approach uses a MUSIC-type cost function [1], which minimizes the sum of all projections of the array transfer vectors at the sensor positions onto the corresponding noise subspaces. The cost function reads

$$f_{\text{MUSIC-SDF}}(\mathbf{x}) = \sum_{n=1}^{N} \mathbf{a}_n^H(\mathbf{x})\,\mathbf{U}_n\,\mathbf{U}_n^H\,\mathbf{a}_n(\mathbf{x})\,, \quad (22)$$

where the array transfer vector (Eq. 6) is parameterized by the source state $\mathbf{x}$. The cost function shows minima for the proper choice of parameters $\mathbf{x}$, if the subspace at each observer position is orthogonal to the array transfer vector at this position.

4.2 SSF-SDF Approach

The SDF approach can be extended to a SSF-type cost function [9], which fits the full subspace spanned by the array transfer matrix $\mathbf{A}_n(\boldsymbol{\rho}_\mathbf{x})$ to the measurements in a least squares sense by minimizing

$$f_{\text{SSF-SDF}}(\boldsymbol{\rho}_\mathbf{x}) = \sum_{n=1}^{N} \text{tr}\left\{\mathbf{P}_{\mathbf{A}_n(\boldsymbol{\rho}_\mathbf{x})}\mathbf{U}_n\,\mathbf{U}_n^H\right\}\,, \quad (23)$$

where $\text{tr}\{\cdot\}$ denotes the trace operation and

$$\mathbf{P}_{\mathbf{A}_n(\boldsymbol{\rho}_\mathbf{x})} = \mathbf{A}_n\left(\mathbf{A}_n^H\mathbf{A}_n\right)^{-1}\mathbf{A}_n^H \quad (24)$$

is a projection matrix that projects onto the column space of $\mathbf{A}_n(\boldsymbol{\rho}_\mathbf{x})$. This leads to a single search in $6Q$ dimensions instead of Q searches in 6 dimensions, but there are more degrees of freedom available for fitting. Note that the SSF solution can be found by using the iterative Alternating Projection algorithm described in [12], which performs the $6Q$-dimensional minimization by minimizing a sequence of 6-dimensional cost functions.

4.3 Comparison of the SDF Approaches

As an illustration, we consider the DPD problem and a scenario in which the sensor moves along an arc from $\mathbf{r}_1 = (-0.5, -0.5, 0.5)^T$ km to $\mathbf{r}_{12} = (0.5, -0.5, 0.5)^T$ km. Two sources are located on the ground at the positions $\mathbf{p}_{0,1} = (0, -0.5, 0)^T$ km and $\mathbf{p}_{0,2} = (0, 0.5, 0)^T$ km. Furthermore, we consider a 10-element uniform circular array with element positions $\mathbf{d}_m = r\,(\cos\frac{m\pi}{5}, \sin\frac{m\pi}{5}, 0)^T$ and radius $r = \frac{\lambda}{2}(\sin\frac{\pi}{10})^{-1}$.

With the assumption that the sensor lies always above each source ($\triangle z_{n,q} > 0$, $n = 1, ..., N$, $q = 1, ..., Q$), the considered problem has a unique solution, because the condition for unique DF of narrowband sources holds, which implies that $Q < M$ [13], and the observability condition established in [14] is satisfied.

Moreover, we assume $N = 12$ batches with $K = 100$ samples per batch, and the n-th time slot being at $t_n - t_0 = n$ seconds. For the emitted waveforms of each source we assume that they have constant amplitude at the sensor positions: $|s_{n,k,q}| = s$, and we define the signal-to-noise ratio of a single source and single element: $\text{SNR} = s^2/\sigma_\text{w}^2$.

Fig. 3 compares the cost functions of the SDF approaches for a fixed z-coordinate, and SNR = 20 dB. Furthermore, in Fig. 3 (right) are the coordinates of the first source fixed. Firstly, we assume an emitting probability of $P_{e,1} = P_{e,2} = 1$. The MUSIC-SDF cost function displays well-pronounced minima and no further local minima. In the following cases, the emitting probability of the second source is reduced to $P_{e,2} = 0.7$. Then, for a missing subspace component, the MUSIC-SDF cost function introduces significant errors (similar to the known subspace swap effect), while the SSF-SDF cost function can account for missing subspace components. Note that in some cases the MUSIC-SDF cost function displays no minimum at the location of the second source.

5. SIMULATION RESULTS

For the scenario described in Section 4.3, Monte Carlo simulations with 1000 runs have been carried out to study the performance of the estimators given in Section 4.1 and Section 4.2. In our simulations, we use the simplex method to find the minima of all cost functions (Eq. 22 and Eq. 23) and we initialize every search with the true value.

In Fig. 4, we show only the root mean square error (RMSE) of the y_0-coordinate, because the RMSE of the coordinates has a similar form. The RMSE reveals that the SSF-SDF approach performs much better than the MUSIC-SDF approach. For the first source, both estimators attain the CRB, which is the expected asymptotic performance. For the second source, the SSF-SDF estimator approaches the CRB, where the MUSIC-SDF does not reach the CRB, because the state estimates are biased.

Note that we find a similar performance for both approaches in the cases of a single source with intermittent emission and multiple sources with continuous emission.

6. CONCLUSIONS

We investigated the direct state determination problem for sources with intermittent emission, and we proposed an SDF approach based on the SSF method to solve the estimation problem. We derived the deterministic CRB and presented a computationally convenient approximation. In simulations, we demonstrated the superior performance of the SSF-SDF approach over the MUSIC-SDF approach.

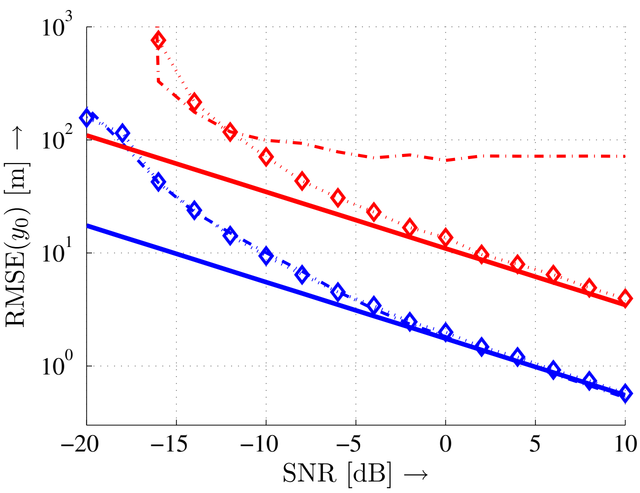

Figure 4: Square-root of the CRB (solid lines) and the estimated RMSE for MUSIC-SDF (dash-dot lines) and SSF-SDF (dotted diamond lines) versus SNR for y_0-coordinate; source 1 (blue lines), source 2 (red lines)

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