A STOCHASTIC MODEL FOR THE DEFICIENT LENGTH PSEUDO AFFINE PROJECTION ADAPTIVE ALGORITHM

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ABSTRACT

This paper presents a statistical analysis of the deficient length Pseudo Affine Projection (PAP) adaptive algorithm. The PAP algorithm is obtained by introducing a step size control parameter in the weight update equation of the unity step size Affine Projection (AP) algorithm assuming autoregressive input signals. The deficient case occurs when the number of adaptive coefficients is smaller than the necessary to whiten the error signal. Deterministic recursive equations are derived for the mean weight and mean-square error behaviours. Monte Carlo simulations show excellent agreement with the theoretically predicted behaviour in steady-state conditions. It is shown that the PAP coefficients converge in the mean to the initial plant coefficients, producing an unbiased solution even for correlated inputs.

1. INTRODUCTION

The Affine Projection (AP) adaptive algorithm [1] is nowadays recognized as a good alternative to speed up the convergence of the gradient based or Least Mean Square (LMS) algorithm family [2], [3]. The AP algorithm applies weight updates in directions that are orthogonal to the last P input vectors. This decorrelates the input signal and speeds up convergence [4]. The improved transient performance comes at the cost of an increased computational complexity.

Assuming autoregressive (AR) input signals, the estimation error vector entering the AP weight update equation becomes a scalar (i.e. the past errors are zero) for step size μ=1 [4]. Step sizes μ<1 allow a tradeoff between steady-state misadjustment and convergence speed. However, for μ<1, the AP weight update equation becomes a function of the last P error signal samples. Thus, the scalar error becomes P-dimensional, increasing both the algorithm complexity and the analysis difficulties.

A simplified version of the AP algorithm has been recently proposed in [5]. The so-called Pseudo-AP (PAP) algorithm replaces the input signal with its autoregressive prediction in determining the weight update direction vector. The resulting weight update equation for μ=1 reduces to the simple update equation of the AP algorithm derived in [4] for an AR input process. PAP uses this same simplified AP weight update equation even for μ<1. The weight update is a function only of the present (scalar) error, but the algorithm is no longer AP.

Results presented in [6] indicate that PAP can lead to a smaller steady-state mean square error (MSE) than AP at the price of a reasonable increase in convergence time for medium values of μ. The behaviour of both algorithms becomes very similar for large values of μ. The PAP algorithm becomes the AP algorithm for μ=1 if the input is AR(P) (autoregressive process of order P) [4]. This behaviour and the results in [5] indicate that PAP could be a reasonable alternative to AP in practical situations where some increase in convergence time can be traded for an improved steady-state performance. Thus, the statistical behaviour of the PAP algorithm becomes of interest.

A statistical model for the behaviour of PAP has been presented in [6] for an adaptive filter of sufficient order. However, if the plant length is underestimated, the adaptive system will not reach acceptable identification accuracy or canceling efficiency [7]-[9]. Such a situation is not rare in practice, as designers often have to deal with computational limitations. Recent studies on the insufficient order LMS [8] and AP [9] algorithms have shown that they exhibit convergence behaviors which are significantly distinct from those of the sufficient length adaptive filters. Such a study is not available for the PAP algorithm.

This work extends the results in [9] to analyze the deficient length PAP adaptive algorithm. Recursive equations are derived for the mean and mean-square error behaviors for white and AR inputs. It is shown that, for an M-tap filter and an N-tap unknown impulse response ($M<N$) the mean weights converge to the first M plant coefficients even for correlated inputs. The derived analytical model for the mean-square error is shown to be very accurate in steady-state conditions and provides acceptable estimations as σ_z^2 tends to $(\mathbf{w}_N^{\mathbf{o}T}\mathbf{R}_{\mathbf{uu}}\mathbf{w}_N^{\mathbf{o}})/(\mathbf{w}_N^{\mathbf{o}T}\mathbf{w}_N^{\mathbf{o}})$ (whose variables are defined in Section 2).

This paper is organized as follows. Section 2 introduces the input signal model and the notation used. Section 3 presents the PAP weight update equation. Section 4 presents the derivation of the analytical model for the algorithm behavior. Section 5 presents Monte Carlo simulations to validate the theoretical models. Section 6 concludes the work.

In this work scalars are denoted by plain lowercase or uppercase letters, vectors are denoted by lowercase boldface letters and matrices by uppercase bold letters. The superscript T denotes transposition. The letter n represents discrete time.

2. DEFICIENT LENGTH ADAPTIVE FILTER

Consider the classical adaptive identification problem where the input signal $u(n)$ is described by an AR process of order P. Then,

$$u(n) = \sum_{i=1}^{P} a_i u(n-i) + z(n) \quad (1)$$

where $z(n)$ is the innovation, modelled by a zero-mean white Gaussian signal with variance σ_z^2; and a_i are the AR coefficients. The desired signal $d(n)$ is related to $u(n)$ through the following linear model

$$d(n) = \mathbf{w}_N^{\mathbf{o}T} \mathbf{u}_N(n) + r(n) \quad (2)$$

where $\mathbf{w}^{\mathbf{o}}_N=[w^o_0 \; w^o_1 \ldots w^o_{N-1}]^T$ of length N is the impulse response of the unknown system (plant), $\mathbf{u}_N(n)=[u(n) \; u(n-1) \ldots u(n-N+1)]^T$ is the plant input vector with correlation matrix $\mathbf{R_{uu}}=E\{\mathbf{u}_N(n)\mathbf{u}_N^T(n)\}$, and $r(n)$ is a zero-mean white Gaussian noise, independent of $u(n)$. The output of the M-tap adaptive filter is given by

$$y(n) = \mathbf{w}^T(n)\mathbf{u}(n) \quad (3)$$

where $\mathbf{w}(n)=[w_0(n) \; w_1(n) \ldots w_{M-1}(n)]^T$; $\mathbf{u}(n)=[u(n) \; u(n-1) \ldots u(n-M+1)]^T$. Here we assume the deficient length case defined by the condition $M<N$.

The instantaneous error is given by

$$\begin{aligned} e(n) &= d(n) - y(n) \\ &= \mathbf{w}_N^{\mathbf{o}T}\mathbf{u}_N(n) - \mathbf{w}^T(n)\mathbf{u}(n) + r(n) \\ &= r(n) - \mathbf{v}^T(n)\mathbf{u}(n) + b(n) \end{aligned} \quad (4)$$

where $b(n)=\bar{\mathbf{w}}^{\mathbf{o}T}\bar{\mathbf{u}}(n)$, $\bar{\mathbf{w}}^{\mathbf{o}}=[w^o_M \; w^o_{M+1} \ldots w^o_{N-1}]^T$, $\bar{\mathbf{u}}(n)=[u(n-M) \; u(n-M-1) \ldots u(n-N+1)]^T$, and $\mathbf{v}(n)=\mathbf{w}(n)-\mathbf{w}^{\mathbf{o}}$ is the weight-error vector where $\mathbf{w}^{\mathbf{o}}=[w^o_0 \; w^o_1 \ldots w^o_{M-1}]^T$. The term $b(n)$ in (4) describes the part of the output due to the exceeding N-M coefficients in $\mathbf{w}^{\mathbf{o}}_N$.

3. WEIGHT-ERROR UPDATE EQUATION

The weight-error update equation of the PAP algorithm with AR input can be written as [6]

$$\mathbf{v}(n+1) = \mathbf{v}(n) + \mu \frac{\mathbf{\Phi}(n)}{\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)} e(n) \quad (5)$$

Vector $\mathbf{\Phi}(n)$ defines the update direction, and is given by:

$$\mathbf{\Phi}(n) = \mathbf{u}(n) - \mathbf{U}(n)\hat{\mathbf{a}}(n) \quad (6)$$

where $\mathbf{U}(n)=[\mathbf{u}(n-1) \; \mathbf{u}(n-2) \ldots \mathbf{u}(n-P)]$, and $\mathbf{U}^T(n)\mathbf{U}(n)$ is assumed of rank P. The least squares estimate of the AR coefficients (a_i) is given by:

$$\hat{\mathbf{a}}(n) = [\mathbf{U}^T(n)\mathbf{U}(n)]^{-1}\mathbf{U}^T(n)\mathbf{u}(n) \quad (7)$$

where $\hat{\mathbf{a}}(n)=[\hat{a}_1(n) \; \hat{a}_2(n) \ldots \hat{a}_P(n)]^T$. Using (4) and (6) in (5) we have

$$\begin{aligned} \mathbf{v}(n+1) = \mathbf{v}(n) &- \mu \frac{\mathbf{\Phi}(n)}{\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)}\mathbf{u}^T(n)\mathbf{v}(n) \\ &+ \mu \frac{\mathbf{\Phi}(n)}{\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)}\left[r(n)+b(n)\right] \end{aligned} \quad (8)$$

Note from (8) that the effect of $M<N$ is the increase of the additive noise $r(n)$ by a term equal to $b(n)$, which is zero-mean, correlated in time and correlated with $u(n)$.

4. ANALYSIS

The following statistical assumptions are used in the analysis, and were initially presented and fully discussed in [10]:

- Assumption A1: The order P of the AP algorithm is equal to the order of the AR process.
- Assumption A2: The statistical dependence between $z(n)$ and $\mathbf{U}(n)$ can be neglected for $M>>P$.
- Assumption A3: The vector $\mathbf{\Phi}(n)$ is orthogonal to the columns of $\mathbf{U}(n)$.
- Assumption A4: The vectors $\mathbf{\Phi}(n)$ and $\mathbf{w}(n)$ are statistically independent.

4.1 Mean Weight-Error Vector Behaviour

Pre-multiplying (8) by $\mathbf{u}^T(n)$ and $\mathbf{U}^T(n)$, using (6), and using Assumption 3 ($\mathbf{U}^T(n)\mathbf{\Phi}(n)=\mathbf{0}$) it can be shown that

$$\begin{cases} \mathbf{u}^T(n)\mathbf{v}(n+1) = (1-\mu)\mathbf{u}^T(n)\mathbf{v}(n) + \mu r(n) + \mu b(n) \\ \mathbf{U}^T(n)\mathbf{v}(n+1) = \mathbf{U}^T(n)\mathbf{v}(n) \end{cases} \quad (9)$$

Combining these results ([6],[9]) we obtain

$$\mathbf{u}^T(n)\mathbf{v}(n) = \frac{1}{\gamma}\mathbf{\Phi}^T(n)\mathbf{v}(n) + \frac{\mu}{\gamma}\mathbf{a}^T\left[\mathbf{r}(n-1)+\mathbf{b}(n-1)\right] \quad (10)$$

where $\mathbf{r}(n-1)=[r(n-1) \; r(n-2) \ldots r(n-P)]^T$; $\mathbf{b}(n-1)=[b(n-1) \; b(n-2) \ldots b(n-P)]^T$ and $\gamma=1-(1-\mu)\Sigma^P_{i=1} a_i$. Substituting (10) into (8) results in

$$\begin{aligned} \mathbf{v}(n+1) = \mathbf{v}(n) &- \mu\frac{\mathbf{\Phi}(n)\mathbf{\Phi}^T(n)}{\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)}\mathbf{v}(n) \\ &+ \mu\frac{\mathbf{\Phi}(n)}{\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)}\left[r_a(n)+b_a(n)\right] \end{aligned} \quad (11)$$

and

$$\begin{cases} r_a(n) = r(n) - \dfrac{\mu}{\gamma}\mathbf{a}^T\mathbf{r}(n-1) \\ b_a(n) = b(n) - \dfrac{\mu}{\gamma}\mathbf{a}^T\mathbf{b}(n-1) \end{cases} \quad (12)$$

where $r_a(n)$ is the filtered noise sequence [4] and $b_a(n)$ corresponds to the correlated additive noise filtered by the same all-pole filter [9].

Taking the expected value of (11) and using Assumption A4, we obtain

$$\begin{aligned} E\{\mathbf{v}(n+1)\} &= \left[\mathbf{I} - \frac{\mu}{\gamma}E\left\{\frac{\mathbf{\Phi}(n)\mathbf{\Phi}^T(n)}{\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)}\right\}\right]E\{\mathbf{v}(n)\} \\ &+ \mu E\left\{\frac{\mathbf{\Phi}(n)r_a(n)}{\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)}\right\} + \mu E\left\{\frac{\mathbf{\Phi}(n)b_a(n)}{\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)}\right\} \end{aligned} \quad (13)$$

Under Assumption A2, the first expected value of (13) was already solved in [10]. The second expected value is a null vector since $r(n)$ and $\mathbf{\Phi}(n)$ are zero-mean and independent of any other signal. The last expectation in (13) was already solved in [9] and is a null vector. Using these results in (13) leads to a deterministic recursive equation for the deficient length PAP algorithm mean weight-error vector:

$$E\{\mathbf{v}(n+1)\} = \frac{\mu}{\gamma}\frac{N-P-3}{N-P-2}E\{\mathbf{v}(n)\} \quad (14)$$

Assuming convergence, the steady-state mean weight-error vector can be obtained from (14), resulting in

$$\lim_{n\to\infty} E\left\{\mathbf{v}(n)\right\} = \mathbf{0} \tag{15}$$

Eq. (15) shows that the mean weights of the deficient length PAP algorithm converge to the actual first M plant coefficients even in the correlated case.

4.2 Mean-Square Error Behaviour

Squaring (4) and taking the expected value leads, after some algebraic manipulation, to

$$E\left\{e^2(n)\right\} = \left(1+\frac{\mu^2}{\gamma^2}\left(\mathbf{a}^T\mathbf{a}+\sigma_z^2 tr\left\{E\left\{[\mathbf{U}^T(n)\mathbf{U}(n)]^{-1}\right\}\right\}\right)\right)\sigma_r^2 + E\left\{b_a^2(n)\right\} + tr\left[\mathbf{R}_{\phi\phi}\mathbf{K}(n)\right] \tag{16}$$

where $\mathbf{K}(n)=E\{\mathbf{v}(n)\mathbf{v}^T(n)\}$ is the weight-error correlation matrix. In determining (16), it was assumed that the algorithm has sufficient order (equal to $\boldsymbol{P}$). Thus, $\hat{\mathbf{a}}(n)\cong\mathbf{a}$ was used.

The first term in (16) is a function of the input statistics. The second and third terms need to be determined. Squaring the second equation in (12) and using $\bar{\mathbf{u}}(n)=\bar{\mathbf{U}}(n)\mathbf{a}+\bar{\mathbf{z}}(n)$ leads to

$$E\left\{b_a^2(n)\right\} = \left(1-2\frac{\mu}{\gamma}+\frac{\mu^2}{\gamma^2}\right)\bar{\mathbf{w}}^{\mathbf{o}T}\mathbf{R}_{\bar{\mathbf{u}}\bar{\mathbf{u}}}\bar{\mathbf{w}}^{\mathbf{o}} + \frac{\mu}{\gamma}\left(2-\frac{\mu}{\gamma}\right)\bar{\mathbf{w}}^{\mathbf{o}T}\mathbf{R}_{\bar{\mathbf{u}}\bar{\mathbf{z}}}\bar{\mathbf{w}}^{\mathbf{o}} - \frac{\mu^2}{\gamma^2}\bar{\mathbf{w}}^{\mathbf{o}T}\mathbf{R}_{\bar{\mathbf{z}}\bar{\mathbf{u}}}\bar{\mathbf{w}}^{\mathbf{o}} + \frac{\mu^2}{\gamma^2}\sigma_z^2\bar{\mathbf{w}}^{\mathbf{o}T}\bar{\mathbf{w}}^{\mathbf{o}} \tag{17}$$

Substituting (17) into (16), results in

$$\begin{aligned} E\left\{e^2(n)\right\} &= \left(1+\frac{\mu^2}{\gamma^2}\left(\mathbf{a}^T\mathbf{a}+\sigma_z^2 tr\left\{E\left\{\left[\mathbf{U}^T(n)\mathbf{U}(n)\right]^{-1}\right\}\right\}\right)\right)\sigma_r^2 \\ &+ \sigma_\Phi^2 tr\left[\mathbf{K}(n)\right] + \left(1-2\frac{\mu}{\gamma}+\frac{\mu^2}{\gamma^2}\right)\bar{\mathbf{w}}^{\mathbf{o}T}\mathbf{R}_{\bar{\mathbf{u}}\bar{\mathbf{u}}}\bar{\mathbf{w}}^{\mathbf{o}} \\ &+ \frac{\mu}{\gamma}\left(2-\frac{\mu}{\gamma}\right)\bar{\mathbf{w}}^{\mathbf{o}T}\mathbf{R}_{\bar{\mathbf{u}}\bar{\mathbf{z}}}\bar{\mathbf{w}}^{\mathbf{o}} - \frac{\mu^2}{\gamma^2}\bar{\mathbf{w}}^{\mathbf{o}T}\mathbf{R}_{\bar{\mathbf{z}}\bar{\mathbf{u}}}\bar{\mathbf{w}}^{\mathbf{o}} + \frac{\mu^2}{\gamma^2}\sigma_z^2\bar{\mathbf{w}}^{\mathbf{o}T}\bar{\mathbf{w}}^{\mathbf{o}} \end{aligned} \tag{18}$$

The weight-error correlation matrix for the deficient length case can be obtained as done in [6] and [10]. Post-multiplying by its transpose, taking its expected value and applying the statistical assumptions A1-A4 yields, after calculations

$$\begin{aligned} \mathbf{K}(n+1) &= \mathbf{K}(n) - \frac{\mu}{\gamma}\frac{2}{(G-2)}\mathbf{K}(n) \\ &+ \frac{\mu^2}{\gamma^2}\frac{1}{M(G+2)}tr\left\{\mathbf{K}(n)\right\}\mathbf{I} \\ &+ \frac{\mu^2}{\gamma^2}\frac{M-G}{GM(G+2)}E\left\{\mathbf{v}^T(n)\right\}E\left\{\mathbf{v}(n)\right\}\mathbf{I} \\ &+ \mu^2\frac{\sigma_r^2}{\sigma_\Phi^2(G-2)(G-4)} \\ &\times\left(1+\frac{\alpha^2}{\gamma^2}\mathbf{a}^T\mathbf{a}+\sigma_z^2 tr\left\{E\left\{[\mathbf{U}^T(n)\mathbf{U}(n)]^{-1}\right\}\right\}\right)\mathbf{I} \\ &+ \mu^2 E\left\{\frac{b_a^2(n)\mathbf{\Phi}(n)\mathbf{\Phi}^T(n)}{[\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)]^2}\right\} \end{aligned} \tag{19}$$

The last expected value in (19) can be approximated by

$$\begin{aligned} &E\left\{\frac{b_a^2(n)\mathbf{\Phi}(n)\mathbf{\Phi}^T(n)}{\left[\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)\right]^2}\right\} \\ &= E\left\{b_a^2(n)\right\}E\left\{\left[\mathbf{\Phi}^T(n)\mathbf{\Phi}(n)\right]^{-2}\right\}E\left\{\mathbf{\Phi}(n)\mathbf{\Phi}^T(n)\right\} \end{aligned} \tag{20}$$

The second and third expect values in the right hand side of (20) were already solved in [10]. Thus, equation (19) can be written as

$$\begin{aligned} \mathbf{K}(n+1) &= \mathbf{K}(n) - \frac{\mu}{\gamma}\frac{2}{(G-2)}\mathbf{K}(n) \\ &+ \frac{\mu^2}{\gamma^2}\frac{1}{M(G+2)}tr\left\{\mathbf{K}(n)\right\}\mathbf{I} \\ &+ \frac{\mu^2}{\gamma^2}\frac{M-G}{GM(G+2)}E\left\{\mathbf{v}^T(n)\right\}E\left\{\mathbf{v}(n)\right\}\mathbf{I} \\ &+ \mu^2\frac{\sigma_r^2}{\sigma_\Phi^2(G-2)(G-4)} \\ &\times\left(1+\frac{\alpha^2}{\gamma^2}\mathbf{a}^T\mathbf{a}+\sigma_z^2 tr\left\{E\left\{[\mathbf{U}^T(n)\mathbf{U}(n)]^{-1}\right\}\right\}\right)\mathbf{I} \\ &+ \mu^2\left[\begin{array}{l}\left(1-\frac{\mu}{\gamma}\right)^2\bar{\mathbf{w}}^{\mathbf{o}T}\mathbf{R}_{\bar{\mathbf{u}}\bar{\mathbf{u}}^T}\bar{\mathbf{w}}^{\mathbf{o}} + \frac{\mu}{\gamma}\left(2-\frac{\mu}{\gamma}\right)\bar{\mathbf{w}}^{\mathbf{o}T}\mathbf{R}_{\bar{\mathbf{u}}\bar{\mathbf{z}}}\bar{\mathbf{w}}^{\mathbf{o}} \\ -\frac{\mu^2}{\gamma^2}\bar{\mathbf{w}}^{\mathbf{o}T}\mathbf{R}_{\bar{\mathbf{z}}\bar{\mathbf{u}}}\bar{\mathbf{w}}^{\mathbf{o}} + \frac{\mu^2}{\gamma^2}\sigma_z^2\bar{\mathbf{w}}^{\mathbf{o}T}\bar{\mathbf{w}}^{\mathbf{o}}\end{array}\right] \\ &\times\frac{M}{\sigma_z^2 G(G-2)(G-4)}\mathbf{I} \end{aligned} \tag{21}$$

Elements of the cross-correlation matrices $\mathbf{R}_{\bar{u}\bar{z}}$ and $\mathbf{R}_{\bar{z}\bar{u}}$ can be defined by the deterministic equation

$$r_{\bar{z}u} = \begin{cases} 0 & to \;\; l < c \\ \sigma_z^2 r(l-c) & to \;\; l \geq c \end{cases} \tag{22}$$

where l is the row number, c the column number and $r(m)$ is obtained through the following recursive equation (assuming $r(0)=1$)

$$r(m) = \sum_{k=1}^{m} a_k r(m-k) \tag{23}$$

Details of the derivation of (23) are omitted due to the lack of space.

5. SIMULATION RESULTS

This section presents simulation results to verify the accuracy of the analytical models given by equations (14), (18) and (21). In all cases, matrix $E\{[\mathbf{U}^T(n)\mathbf{U}(n)]^{-1}\}$ has been numerically estimated using the input process samples. However, its contribution is usually not significant and can be disregarded.

The following parameters have been used in all presented examples: $\sigma_\Phi^2=1$; $\sigma_r^2=10^{-6}$ (additive noise power); the plant to be identified is an acoustical impulse response, measured in a typical office, originally sampled at 8 kHz (N=1000) [11], but resampled here to 2kHz (N=125) in order to fasten simulations (Fig. 1). All the other parameters are informed in the respective figure.

Fig. 2 shows the mean weight behaviour for coefficients 10, 20 and 30 assuming M=40 coefficients, an AR(1) input signal

($u(n)$=0.9$u(n$-1)+$z(n)$) and μ=0.6. Figs. 3 and 4 show results for the mean square error for the same input signal as in Fig. 2 but with step-sizes μ=0.6 and 0.8. The evolution of the MSE is presented for different adaptive filter lengths (M=40, 60 and 80). Fig 5 shows the MSE results for an AR(8) input signal and for M=40, 60 and 80. Excellent agreement between simulations (400 runs) and theory can be verified in steady-state conditions.

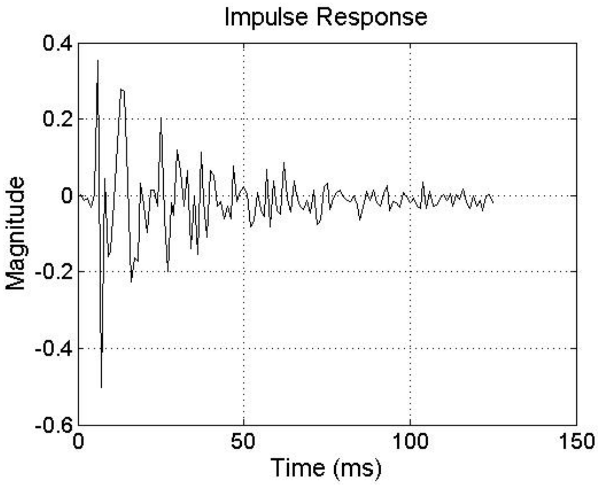

Figure 1 – Impulse response of the plant. Obtained from [11] and resampled to 2 kHz.

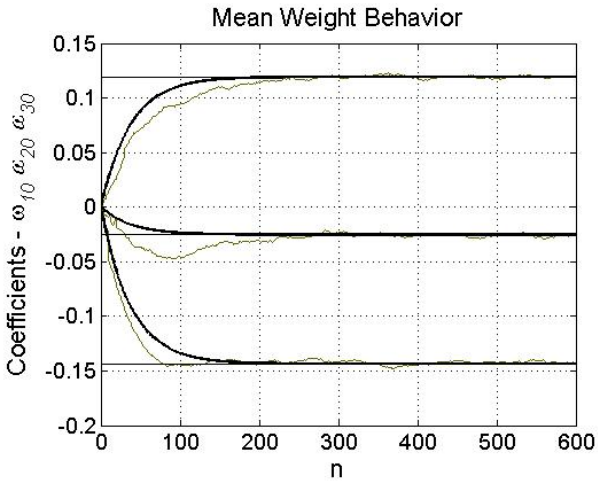

Figure 2 – Mean weight behaviour for weights indexes 10, 20 and 30. μ=0.6, M=40 and AR(1) input signals.

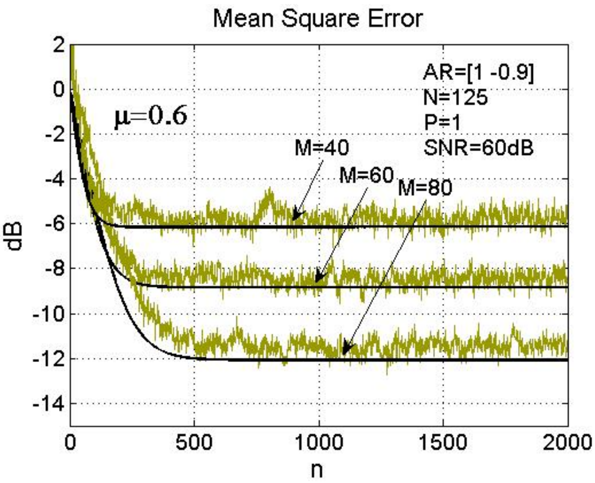

Figure 3 – MSE: Comparisons between Monte Carlo simulations (ragged lines) and proposed analytical model (continuous lines).

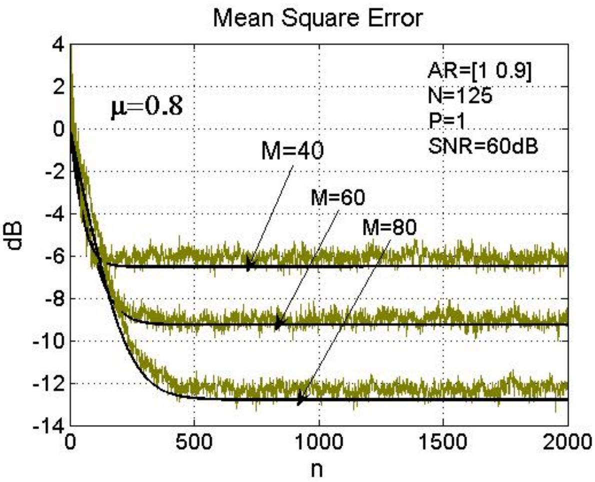

Figure 4 – MSE: Comparisons between Monte Carlo simulations (ragged line) and proposed analytical model (continuous lines).

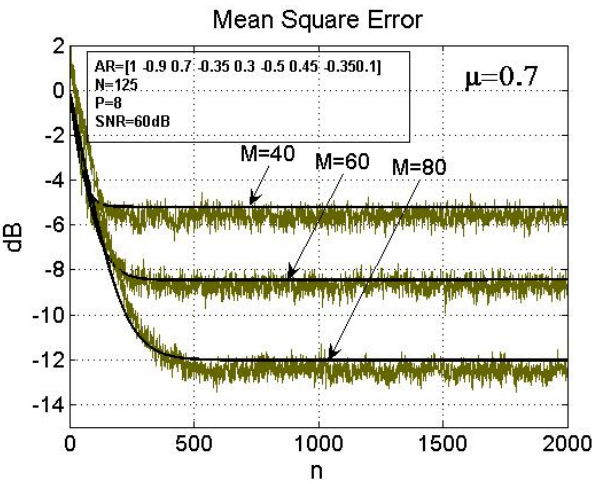

Figure 5 – MSE: Comparisons between Monte Carlo simulations and analytical model proposed.

6. CONCLUSIONS

This paper presented an analytical model for predicting the stochastic behaviour of the deficient length Pseudo Affine Projection algorithm. Deterministic recursive equations were derived for the mean weight and mean square error for an equal or smaller number of adaptive taps compared to the plant length**.** Simulation results have show excellent agreement with theoretical predictions during both the adaptation (transient) and steady- state phases.

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