

# RJMCMC POINT PROCESS SAMPLER FOR SINGLE SENSOR SOURCE SEPARATION: AN APPLICATION TO ELECTRIC LOAD MONITORING

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## ABSTRACT

*This paper presents an original method to separate the residential electric load into its major components. The method is explained in the particular case of space-heating, which is the most consuming electric end-use in France<sup>1</sup>. This is a source separation problem from a single mixture. The components to be retrieved are square signals characterized by a periodic regulation and a slowly time-varying duty cycles. A point process is used to model the electric load as a configuration of possibly overlapping square signals, given the priors on magnitude, duty cycle variations and the regulation periodicity. This stochastic process is simulated using a Reversible Jump Markov Chain Monte Carlo procedure. A simulated annealing scheme is used to achieve the posterior density maximization. First results on real data provided by Electricité de France are quite encouraging.*

## 1. INTRODUCTION

### 1.1. Background on household electric load monitoring

The electric power industry and consumers recently face many challenges such as energy saving, market deregulation, power quality and greenhouse gas emissions reducing. Accurate and reliable information about the nature and the state of the electric systems will undoubtedly be helpful to meet these challenges. Actually, a good knowledge of the electric load and the targeted appliances help consumers understanding their bills and better control their consumption. It also provides utilities with detailed usage profiles of their customers, which is an efficient means to help in levelling peaks load and in planning future capacity.

A non-intrusive and economical solution may rely on information extracted from electric consumption measured at a centralized easily accessible part of a distribution network, namely the electricity meter.

Non-intrusive electric load monitoring has been subject to several approaches over the last twenty years. General overviews can be found in [1, 2, 3]. The available solutions require measurements of the active and the reactive power, which carry out the finger-prints of the electric appliances. They are mostly made up of three steps. Event detection determines the appliances operating schedule. Load identification uses steady state powers and transient patterns, if available, to recognize the elementary components. Energy estimation provides a breakdown of the daily energy into the major end-uses. As the structure of the electric load is very complex because of the diversity of electrical loads and of the consumers' habits, classical methods requires measurements of three voltages and currents with a specific device plugged in the electricity meter.

We propose a novel approach using only the active power. The measured power is a linear additive mixture of an unknown number of elementary signals. Our aim is to provide the most likely decomposition of the daily active power without any intrusion. We propose to incorporate knowledge on the primitives to be extracted as *a priori* knowledge. Our approach allows going further.

### 1.2. Problem statement

In this paper, we focus on the space-heating load decomposition. The observed signal is the active power  $y(t)$ . It is sampled at the sampling rate  $T_e=2s$ . This observation is a sum of an unknown number  $K$  of periodic square waves  $y_k$  ( $1 \leq k \leq K$ ). Each component is described by its period  $T_{0k}$  and its magnitude  $A_k$ .

The observation is modelled as follows (equation 1):

$$\forall t, y(t) = \sum_{k=1}^K y_k(t) + b(t) \quad (1)$$

where  $b$  is an additive Gaussian noise. Each  $k^{\text{th}}$  convector is defined on a compact support  $[t_{0k}, t_{1k}]$ .

A convector period  $T_{0k}$  is almost constant and is defined as follows (equation 2)

$$T_{0k} = T_{00} + \varepsilon_k \quad (2)$$

where  $T_{00} \in \{40, 80\}$  is the theoretic periodicity and  $\varepsilon$  is an additive noise, modelled with a zero mean Gaussian law.

<sup>1</sup> 70% of the whole households electric consumption in France (ADEME, 2005).

An example of a convector signal is given in **Figure 1**. Notice that the width of the observed squares  $l$  varies over time. Time-variations of these parameters might be analyzed through the variations of the duty-cycle defined as follows  $\rho = l/T_{00}$  (**Figure 1**, where  $n$  represents the index of the squares).

The repartition of the duty-cycle variations (absolute values) obtained for a given convector is illustrated in **Figure 2**. The variation between two successive squares is constant. We propose to use a first autoregressive model to this parameter.

Two samples of space-heating load are presented in **Figure 3**, **Figure 4** and **Figure 5**. In the first case, two components are operating simultaneously. Even if the number of components is very small, the global load cannot be easily decomposed, especially because of the convector saturation  $\rho = 1$  (**Figure 3**, double arrow). Another source of complexity of this problem is the interaction between the elementary components. The start up of a square and the shutdown of another one might occur at the same time. Real data show that this synchronism between convectors is realistic and occurs frequently (**Figure 4**, **Figure 5**).

Moreover, magnitudes and periodicities of different components might have the same value, which makes the source separation problem quite difficult.

We aim at extracting a plausible configuration of periodic square waves given the global consumption and priors. The electric load might be seen as a realization of a marked point process [4] of squares defined by a density function to be designed given priors on the duty cycle variations, the periodicity and the magnitude of a convector. The stochastic process is sampled using a Reversible Jump Markov Chain Monte Carlo (RJ-MCMC) sampler. A simulated annealing algorithm [5] achieves the *posterior* density maximization: an estimation of the model parameters in a Bayesian framework is performed this way.

Some definitions and notations are presented in section 2. In section 3, the estimation problem is presented in a Bayesian framework. The optimization algorithm and some details on the proposition kernels introduced in the MCMC sampler are described in section 4. Finally, first results on real data and future works are given in section 5.

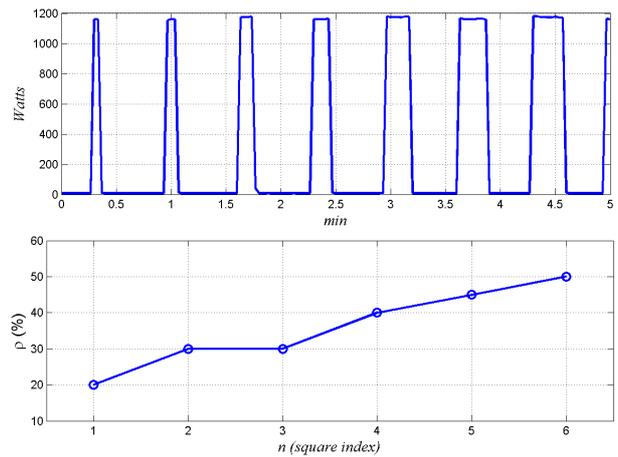


Figure 1- Example of a convector electric load ( $T_{00}=40s$ ,  $A=1150 W$ )

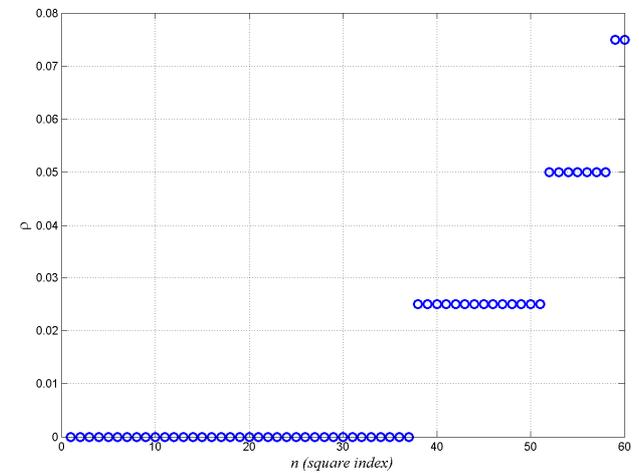


Figure 2- Duty-cycle repartition obtained for one convector (operating during 82 min)

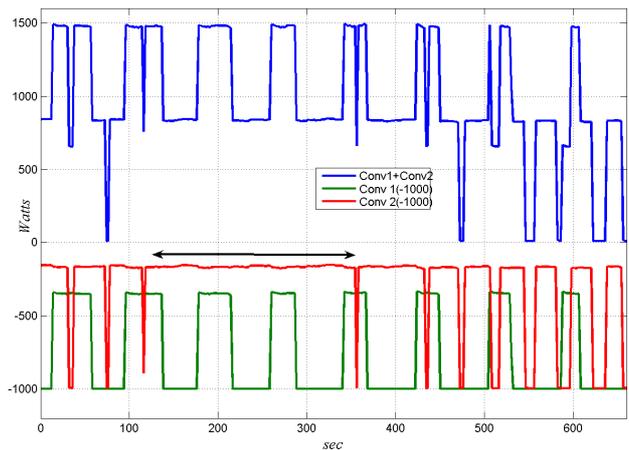


Figure 3- Electric load of two convectors operating simultaneously ( $T_{00}=80s$  for one convector *Conv1* and  $T_{00}=40s$  for the other one: *Conv2*)

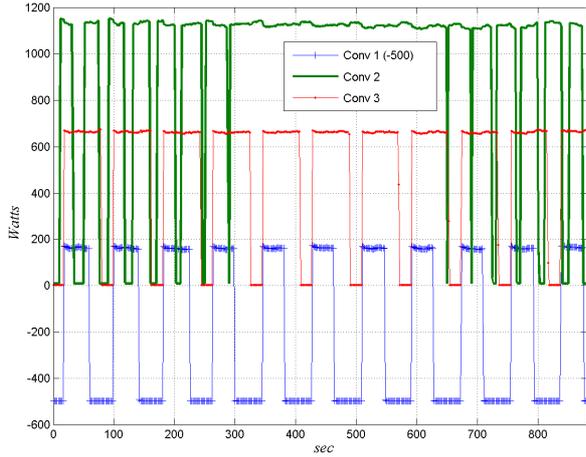


Figure 4- Individual electric loads of three convectors operating simultaneously  $T_{00}=40s$  for *Conv1* and *Conv3*,  $T_{00}=80s$  for *Conv2*.

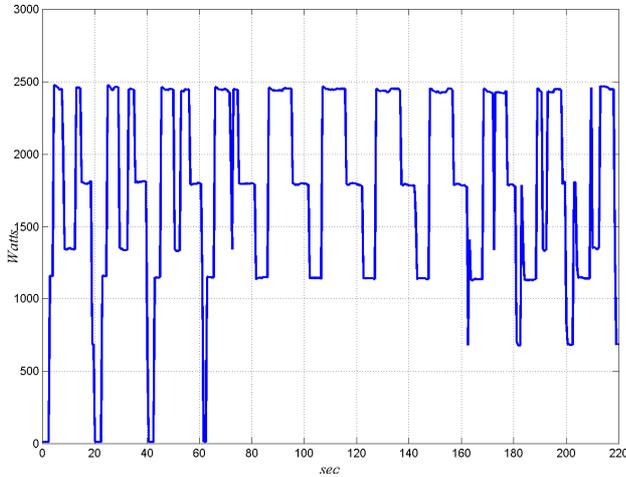


Figure 5- Electric load of three convectors operating simultaneously

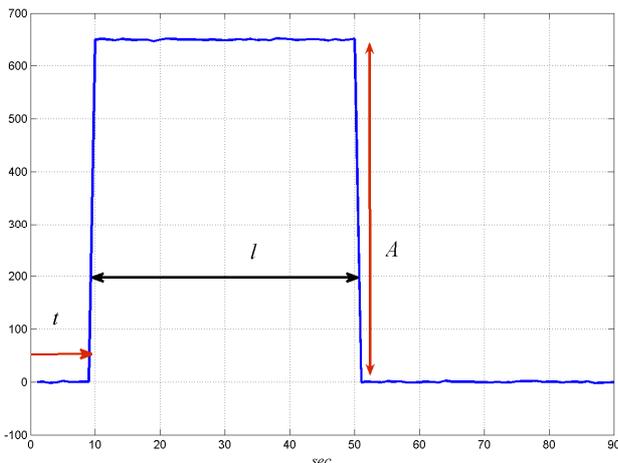


Figure 6 - Model of a one-square convector

## 2. POINT PROCESS FRAMEWORK FOR ELECTRIC LOAD MONITORING

Point processes models have been introduced for object extraction from remotely sensed images [6]. Marked point process models provide a natural setup for the inclusion of prior knowledge on the components of a given observation: parsimony, patterns structure and/or morphology, etc.

These models were widely used in image processing [6,7] to extract complex objects in a scene. They have recently been used in roads [8], buildings [9] or tree crowns [10] extraction. We refer to [6, 11] for mathematic definitions and more details about point processes.

In this work, we are interested in the case study of space-heating electric load. The components to be extracted are the convectors, which might be considered as a set of squares. Notice that the convectors may overlap. Point processes models are adapted in the case of one-dimensional signals where the objects interact with each other and may overlap.

More precisely, the electric load decomposition problem requires to introduce two stochastic processes: a point process where the points are the convectors and a marked point process where the points are the squares and the marks are the square parameters. The proposed model is explained in the following subsections.

### 2.1. The state space

The observed signal  $P$  is defined on  $T = [0, T_{max}]$ . It is the sum of the convectors active powers. Each convector might be considered as a set of squares.

Each square  $u$  of a given convector  $c$  is entirely defined by three parameters  $(t, l, A)$  which are respectively the beginning time, the square width and the square magnitude. These parameters belong to  $M = [l_{min}, l_{max}] \times [A_{min}, A_{max}]$ . The parameters  $l_{min}$  and  $l_{max}$  stand respectively for the minimal and the maximal widths of any square. The magnitudes  $A_{min}$  and  $A_{max}$  are respectively the minimal and the maximal nominal powers of any convector. These parameters are chosen based on information extracted from real data. The model of squares is illustrated in **Figure 6**.

Let  $S = T \times M$  be the state space of squares. Notice that the state space of squares is a subset of  $\mathbb{R}^3$ . A configuration  $c$  of objects in  $S$  is an unordered list of objects in  $S$ , which can be written as

$$c = \{u_1, u_2, \dots, u_n\}, \text{ where } n \geq 1 \text{ and } \forall i, u_i \in S.$$

Let  $C_n$  be the set of all  $n$ -squares configurations, i.e.

$$C_n = \{c \mid c = \{u_1, u_2, \dots, u_n\}\}. \quad (3)$$

The set of all the configurations is  $C = \bigcup_{n=1}^{\infty} C_n$ .

The state space of possible solutions is defined as follows:

$$E = \{x \subset C, 1 \leq |x| < +\infty\}.$$

Each object  $x$  of  $E$  is a finite set of convectors. The electric load components estimation is a large combinatorial problem. It may be solved by minimizing an energy on the state space of convectors. Efficient algorithm helping the state-space exploration are required. In the next sub-section,

two point processes are used to model the squares and the electric load of convectors.

## 2.2. Point processes for electric load modelling

The electric load model is obtained through the definition of a stochastic process that describes the convectors and another one that describes the squares.

Let  $\underline{\mu}$  be the probability distribution of a Poisson point process and  $\underline{\nu}$  be the intensity measure of this process defined on  $C$ .

Consider a point process defined on  $C$ , *i.e.* an  $E$ -valued random variable. Each realization of  $X$  is a set of convectors. Given the reference probability distribution, a realization of  $X$  could be obtained in two steps.

Firstly, one draws the convectors number  $n$  from a Poisson distribution  $p_n = e^{-\underline{\nu}(C)} \frac{\underline{\nu}(C)^n}{n!}$ . Secondly,  $n$  independent convectors are generated with respect to the distribution  $\frac{\underline{\nu}}{\underline{\nu}(C)}$ .

Given this distribution, one defines a Poisson process of probability distribution  $\mu$  on  $S$ . Let us consider a marked point process where the points belong to  $T = [0, T_{max}]$  and the marks are stochastic parameters that belong  $M = [l_{min}, l_{max}] \times [A_{min}, A_{max}]$ . A realization of this process is a finite set of squares (points in  $S$ ), in other words a point in  $C$ .

The observation  $y(t)$  (the space-heating active power) is a realization of a point process  $X$  defined by its density  $f(\cdot)$  with respect to the Poisson process probability distribution  $\mu$ . One might then build a Markov chain which converges in law to  $X$ .

The process density satisfies the following equation

$$f(\mathbf{x}) = \frac{1}{Z} \exp(-U(\mathbf{x})), \quad (4)$$

where  $Z$  is a normalizing constant and  $U(\mathbf{x})$  stands for the energy of the configuration  $\mathbf{x}$ .

## 3. APPLICATION TO ELECTRIC SPACE-HEATING DECOMPOSITION

Our aim is to estimate the configuration  $\mathbf{x}$  that maximizes the posterior density  $f(\mathbf{x}|y) = f(\mathbf{x})$  given the active power  $Y$ . In a Bayesian framework, we need to build both an *a priori* density and a likelihood term.

### 3.1. Posterior density

As mentioned above, in a Bayesian framework [12], the marked process density is given as follows

$$f(\mathbf{x}) \propto f_{prior}(\mathbf{x}) f_{data}(y|\mathbf{x}), \quad (5)$$

where the *a priori* density  $f_{prior}(\mathbf{x})$  includes priors on the parameters to be estimated, and the likelihood  $f_{data}(y|\mathbf{x})$  of a configuration given the observed signal.

The energy term is split into a likelihood term and a prior (regularization) term as follows:

$$U(\mathbf{x}) = U_{prior}(\mathbf{x}) + U_{data}(\mathbf{x})$$

The relationship between energy terms and density functions is given by equations 6.

$$\begin{cases} f_{prior}(\mathbf{x}) \propto \exp(-U_{prior}(\mathbf{x})) \\ f_{data}(P_h|\mathbf{x}) \propto \exp(-U_{data}(\mathbf{x})) \end{cases} \quad (6)$$

### Prior energy

The regularization energy is defined by the following equation:

$$U_{prior}(\mathbf{x}) = \sum_{c \in \mathbf{x}} (U_p(c) - \phi_p(c)), \quad (7)$$

where  $U_p(c) = \alpha^A \psi_p^A(c) + \alpha^T \psi_p^T(c) + \alpha^\rho \psi_p^\rho(c)$  is a linear mixture of three regularization terms respectively on the magnitude  $A$ , the periodicity  $T_0$  and the duty cycle  $\rho$ . Given real data analysis results, these parameters are modelled with Gaussian laws.

Let  $\bar{T}$  and  $\bar{A}$  be respectively the period and the magnitude means of the convector  $c$  and  $\sigma_T$ ,  $\sigma_A$  and  $\sigma_\rho$  the standard deviations. The functions  $\psi_p$  are defined as following:

$$\begin{cases} \psi_p^A(c) = \sum_{u \in c} \frac{(A(u) - \bar{A})^2}{2\sigma_A^2} \\ \psi_p^T(c) = \sum_{u \in c} \frac{(T(u) - \bar{T})^2}{2\sigma_T^2} \\ \psi_p^\rho(c) = \sum_k \frac{(\rho(k+1) - \rho(k) - 1)^2}{2\sigma_\rho^2} \end{cases}$$

The function  $\phi$  is introduced to maximize the number of squares per convector and penalize great number of one-square convectors. The chosen function is defined as below:

$$\phi(c) = \begin{cases} \delta & \text{if } |c| = 1 \\ -\beta |c|^2 & \text{otherwise} \end{cases}$$

where  $\delta$  and  $\beta$  are pre-defined parameters.

### Likelihood energy

This term measures the probability of data given a configuration of objects. Given the chosen model for space-heating load, we propose to define this energy by the sum of each likelihood objects:

$$U_{data}(\mathbf{x}) = \sum_{c \in \mathbf{x}} \sum_{u \in c} U_l(u), \quad (8)$$

The energy term  $U_l$  is designed as following:

- The energy of a plausible square is negative. Otherwise, the energy of the proposed square is a positive convex function.
- The gradient (*on* and *off* events) of a plausible square corresponds to a gradient of the observation in a pre-defined neighbourhood.

The proposed likelihood energy definition requires a pre-processing of the electric load. In fact, the events (start up and trip) of the whole electric load have to be matched.

### 3.2. RJMCMC sampler and optimization algorithm

The space-heating load is modelled by a point process  $X$  defined by its density  $f(\mathbf{x})$  with respect to the Poisson process of intensity  $\underline{\nu}$ . A Markov chain  $(X_m)$  that converges to the distribution  $P_X$  is built using a RJ-MCMC samplers [13].

In this work, some jumping kernels (birth and death of a square) and non-jumping kernels (dilatation of the

magnitude or of the width of a given square, translation) are used to manage the chain transitions have been defined. The maximum *a posteriori*, given by equation 9,

$$x_{MAP} = \operatorname{argmax}_x f(x) \quad (9)$$

is obtained using a simulated annealing algorithm [5].

#### 4. RESULTS ON REAL DATA

The proposed model has been tested on real data gathered at two customers houses. In each case, the active powers of the elementary components have been also measured to evaluate the algorithms. Tests were carried on synthetic mixtures to evaluate the proposed model. Figure 7 presents results obtained for two convectors having nearly different

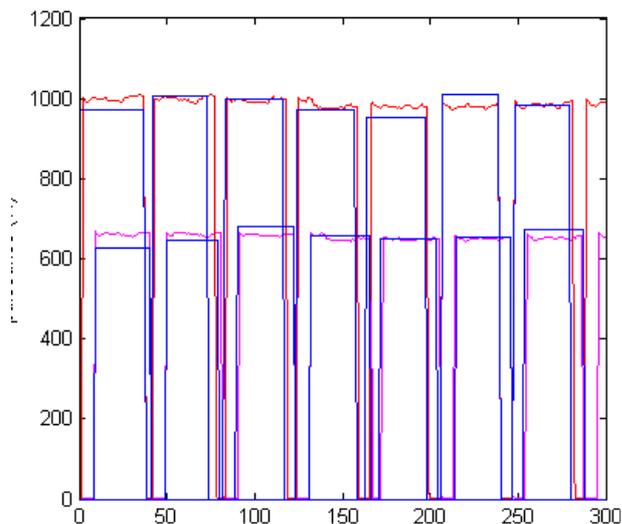


Figure 7- Decomposition of two convectors operating simultaneously ( $T_{01} = T_{02} = 40s$ ): the measured signals (blue), the estimates (red and cyan).

magnitude and the same periodicity of regulation. Their duty-cycles are also very close.

This signal is decomposed into two components (Figure 4-blue) where the periodicity is well estimated. The squares widths are quite well estimated.

Other tests have been carried out during this study taking into account convector saturation and several objects interactions. The obtained results, which are quite satisfying, and more details on the convergence of the MCMC sampler will be given in future works. We notice that some verifications have been carried out to ensure the convergence of the implemented sampler.

#### 5. CONCLUSIONS AND FUTURE WORKS

The electric load decomposition given the active power sampled at one second is a single source separation problem. The load components are some square waves characterized by some priors. In this work, a novel method to estimate these components using the marked point

processes framework is proposed. These models were widely used in image processing and in model selection.

In our application, such processes provide a unique model to any electric appliance avoiding a constrained approach based on a specific model per class of electric appliances. In this paper, the proposed model in the case of space-heating load is described and the *posterior* density is defined. The RJMCMC sampler embedded with a simulated annealing scheme is implemented using some transition kernels. First results on real data are satisfying and show that the model is suitable to deal with objects interactions (events in our application).

Some improvements could be proposed, such the hyper-parameters of the prior energy estimation. In future works, more details on the implemented sampler and on the optimization algorithm will be detailed. The proposed model will be extended to other electric appliances.

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