# FREQUENCY DOMAIN EQUALIZATION FOR OFDM SYSTEMS WITH INSUFFICIENT GUARD INTERVAL USING NULL SUBCARRIERS

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## **ABSTRACT**

Frequency domain equalizers (FEQs) have been applied extensively in multicarrier systems to enhance transmission rate by reducing transmit redundancy in the form of guard interval. The proposed equalization algorithm is able to remove intersymbol and intercarrier interference (ISI and ICI) incurred by the reduction or the absence of this redundancy by properly exploiting null subcarriers that are inherent in standardized multicarrier systems. Unlike previous proposed schemes, the proposed algorithm does not require additional temporal nor spatial diversity at the receiver to mitigate the channel-induced interferences. Simulation results show that our approach outperforms those of other schemes in terms of bit error rate.

Index Terms— OFDM, insufficient guard interval, FEQ, null subcarrier

### 1. INTRODUCTION

Multicarrier modulation techniques, such as Orthogonal Frequency Division Multiplexing (OFDM), have been widely deployed in various communication systems because of their ability to achieve high data rate using low-complexity transceiver. This is achieved by injecting sufficient amount of redundancy, known as guard interval, into the transmit bitstream such that it converts the frequency-selective fading channel into a set of flat-fading channels, which allows for ISI/ICI-free transmission by utilizing only an array of one-tap frequency domain equalizers (FEQs). For OFDM based systems, the guard interval usually is in the form of cyclic-prefix (CP).

The CP is obtained by taking the last v samples from the length N block of OFDM symbols, and it is appended at the start of the symbol block. As a result, the transmitted OFDM symbol block is of length N+v. For each OFDM symbol to be independent and to avoid any ISI and ICI, the length v of the CP should be at least equal to the channel order. Hence the distortion caused by the channel impulse response only affects the samples within the CP. Therefore the receiver can truncate the received signal sequence by discarding the CP, and retains the last N samples for ISI-free decoding. The effects of the channel on the remaining N samples can be easily equalized by the FEQ.

Although CP improves the robustness of OFDM, it reduces the transmission efficiency by a factor of  $N/(N+\nu)$ . One way to increase the efficiency is to increase the FFT size N. However, this increases the complexity of the sys-

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tem and reduces the intercarrier spacing of the subcarriers which subsequently makes the system more susceptible to frequency offset and oscillator phase noise. Also, a higher number of subcarriers will increase the Peak-to-Average Power Ratio (PAPR), demanding the use of linear and consequently inefficient power amplifiers. The alternative is to use a time domain equalizer (TEQ) [1]-[5] preceding the FFT demodulator at the receiver in order to constrain the length of the effective channel impulse response to be shorter than the selected CP duration. This permits the use of a much shorter CP than could otherwise be employed and thus raises the transmission efficiency. However, computational complexity of TEQs grows exponentially with channel length and requires very long FIR filters when the channel is highly dispersive; rendering conventional TEQs to be unsuitable for future broadband wireless systems.

A low computational complexity FEQ was proposed in [6] that can mitigate ISI and ICI caused by insufficient CP by modifying the frame structure of existing standard OFDM systems. The deviance from standard format, however, limits its practicality. [7] proposed a zero-forcing FEQ that exploits null subcarriers (NSCs) which exists in conventional OFDM based systems to cancel the channel-induced interferences. However, the equalizer is susceptible to channel noise amplification and the existence of the equalizer requires that the number of NSCs plus the length of the CP be greater than or equal to channel order, which can prevent CP-free transmission when the channel delay spread is relatively long. The same constraint can be seen in [8] since both FEQs are derived from zero-forcing criterion to cancel ISI/ICI.

A beamforming approach was taken by [10] to suppress delayed signal which are beyond the CP. However, [10] made the unrealistic assumption that the array manifold is the same for all subcarriers, which adversely affects the performance of their scheme. A joint transceiver design was proposed in [11] which approximated a target impulse response. However, the approach requires accurate channel state information at the transmitter which is difficult to obtain. In addition, no closed-form solution can be obtained for the design of the transmitter filter, which requires the filter weights to be computed iteratively. Although the algorithm has been proven to guarantee convergence, an iterative approach is not feasible for systems that need to operate in fast-fading channel environment.

A novel FEQ design that is based on the generalized sidelobe canceler (GSC) beamformer was proposed by [12] for CP-free OFDM transmission. The scheme separates the received signal into a "signal + interference + noise" (S+I+N) data group and an "interference + noise" (I+N) data

group. The S+I+N data group is obtained by filtering the received signal through a single-tap FEQ. Since the singletap FEQ only acts as a matched-filter, the output of the FEQ will still be corrupted by residual I+N. The I+N data group is then subtracted from the S+I+N data group after it has been accurately estimated, resulting in an ISI- and ICI-free signal. The I+N data group is obtained by taking the orthogonal complement of the filter in the S+I+N group to form a blocking filter, which is intended to block the desired signal, followed by the design of a second filter that can estimate the residual I+N in the S+I+N group [13]. This approach not only can remove the undesired ISI and ICI, but also does not suffer from noise amplification effects as the aforementioned techniques. Unfortunately, in order to obtain the blocking filter, the received signal either has to be oversampled or multiple antennas will have to be deployed, which increases the overall computational load of the system.

In this paper, we proposed an improved variant of the algorithm in [12] that will not require the additional diversity needed in [12] and works in OFDM systems with insufficient number of CP bits. In addition, we shall show that the proposed algorithm can outperform [12] in terms of BER. The proposed scheme exploits the existing NSCs that are inherent in many standardized OFDM based systems such as IEEE 802.11a [14] and IEEE 802.16e [15].

The paper is organized as follows. A description of the system model and proposed scheme is given in Section 2, followed by simulation results in Section 3. The paper will be concluded in Section 4.

Notation: Upper (lower) bold face letters indicate matrices (column vectors). Superscript  $^H$  denotes Hermitian,  $^T$  denotes transposition.  $E[\cdot]$  stands for expectation. diag(x) denotes a diagonal matrix with x on its main diagonal;  $\mathbf{I}_N$  denotes an  $N\times N$  identity matrix;  $\mathbf{0}_{M\times N}$  denotes an  $M\times N$  all zero matrix.  $\mathscr{N}(\mathbf{A})$  denotes the nullspace of the matrix  $\mathbf{A}$ .

## 2. METHODOLOGY

# 2.1 System Model

Consider a single-input single-output OFDM (SISO-OFDM) system with N subcarriers (assumed throughout to be a power of 2) and a CP of length v, where v is less than the channel order q. After the serial-to-parallel operation at the transmitter, the  $k^{th}$  source signal can be represented in vector form as

$$\mathbf{s}(k) = \left[ s_{-\frac{N}{2}+1}(k), \dots, s_0(k), \dots, s_{\frac{N}{2}}(k) \right]^T, \tag{1}$$

where  $s_{\ell}(k) = s(kN + \frac{N}{2} - 1 + \ell)$ , for  $\ell = -\frac{N}{2} + 1, \dots, \frac{N}{2}$  [16]. s(n) is modeled as an independent, zero-mean random process with unit power, i.e.  $E[s(i)s^*(j)] = \delta(i-j)$ , where  $\delta(\cdot)$  denotes the Kronecker delta function. Denote the  $N \times N$  FFT matrix as  $\mathbf{W}_N$ . The transmitted signal can then be written as

$$\tilde{\mathbf{s}}(k) = \mathbf{T}_{CP} \mathbf{W}_N^H \mathbf{s}(k),$$
 (2)

where  $\mathbf{T}_{CP} = \begin{bmatrix} \mathbf{0}_{v \times (N-v)} & \mathbf{I}_v \\ \mathbf{I}_N \end{bmatrix}$  is the CP insertion matrix.  $\tilde{\mathbf{s}}(k)$  is transmitted through a time-invariant channel of order q, which is assumed to be a priori known or otherwise estimated by the receiver. Denote the additive

noise as  $\eta(k)$  which is assumed to be zero-mean with a priori known covariance matrix  $\mathbf{R}_{\eta(k)\eta(k)}$  and independent with  $\mathbf{s}(k)$ . The  $k^{th}$  received symbol can then be written as

$$\tilde{\mathbf{r}}(k) = \mathbf{H}_0 \tilde{\mathbf{s}}(k) + \mathbf{H}_1 \tilde{\mathbf{s}}(k-1) + \eta(k), \tag{3}$$

where  $\mathbf{H}_0, \mathbf{H}_1 \in \mathbb{C}^{(N+\nu)\times(N+\nu)}$  are lower and upper triangular Toeplitz matrices with its first column being  $[h(0)h(1)\cdots h(q)0\cdots 0]^T$  and first row being  $[0\cdots 0h(q)\cdots h(1)]$ , respectively.

At the receiver, CP removal is performed followed by FFT demodulation, which yields the signal

$$\mathbf{x}(k) = \mathbf{W}_{N} \mathbf{R}_{CP} \left[ \mathbf{H}_{0} \tilde{\mathbf{s}}(k) + \mathbf{H}_{1} \tilde{\mathbf{s}}(k-1) + \eta(k) \right]$$

$$= \mathbf{W}_{N} \mathbf{R}_{CP} \mathbf{H}_{0} \mathbf{T}_{CP} \mathbf{W}_{N}^{H} \mathbf{s}(k)$$

$$+ \mathbf{W}_{N} \mathbf{R}_{CP} \mathbf{H}_{1} \mathbf{T}_{CP} \mathbf{W}_{N}^{H} \mathbf{s}(k-1) + \mathbf{W}_{N} \mathbf{R}_{CP} \eta(k),$$
(4)

where  $\mathbf{R}_{CP} = [\mathbf{0}_{N \times \nu} \ \mathbf{I}_{N \times N}]$  is the CP removal matrix. If  $\nu \geq q$ , then  $\mathbf{R}_{CP} \mathbf{H}_0 \mathbf{T}_{CP}$  will be a circulant matrix and  $\mathbf{R}_{CP} \mathbf{H}_1 \mathbf{T}_{CP} = \mathbf{0}_{N \times N}$ , which makes equalization possible with a single-tap FEQ. However, this is no longer true when  $\nu < q$ . Define the circulant matrix  $\mathbf{C} \triangleq \mathbf{R}_{CP} (\mathbf{H}_0 + \mathbf{H}_1) \mathbf{T}_{CP}$ , the compensation matrix

$$\mathbf{C}_{ICI} \triangleq \left[ \begin{array}{cccc} h(q) & \cdots & h(v+1) \\ & 0 & \ddots & \vdots \\ & & \vdots & \ddots & h(q) & \mathbf{0}_{N \times v} \\ & \vdots & & \ddots & \vdots \\ & & \vdots & & \vdots \\ & & 0 & \cdots & 0 \end{array} \right],$$

and  $\mathbf{C}_{ISI} \triangleq \mathbf{R}_{CP}\mathbf{H}_1\mathbf{T}_{CP}$ . Noting that  $\mathbf{R}_{CP}\mathbf{H}_0\mathbf{T}_{CP} = \mathbf{C} - \mathbf{C}_{ICI}$ , then (4) can be rewritten as

$$\mathbf{x}(k) = \mathbf{W}_{N} \mathbf{C} \mathbf{W}_{N}^{H} \mathbf{s}(k) - \mathbf{W}_{N} \mathbf{C}_{ICI} \mathbf{W}_{N}^{H} \mathbf{s}(k) + \mathbf{W}_{N} \mathbf{C}_{ISI} \mathbf{W}_{N}^{H} \mathbf{s}(k-1) + \mathbf{W}_{N} \mathbf{R}_{CP} \boldsymbol{\eta}(k) = \mathbf{D} \mathbf{s}(k) - \mathbf{H}_{ICI} \mathbf{s}(k) + \mathbf{H}_{ISI} \mathbf{s}(k-1) + \mathbf{W}_{N} \mathbf{n}(k),$$
(5)

where  $\mathbf{D} \triangleq \mathbf{W}_N \mathbf{C} \mathbf{W}_N^H$ ,  $\mathbf{H}_{ICI} \triangleq \mathbf{W}_N \mathbf{C}_{ICI} \mathbf{W}_N^H$  and  $\mathbf{H}_{ISI} \triangleq \mathbf{W}_N \mathbf{C}_{ISI} \mathbf{W}_N^H$ . Therefore, the second and third term in (5) have to be eliminated in order to achieve ISI- and ICI-free transmission.

## 2.2 Proposed Scheme

Similar to the scheme in [12], we proposed to design a FEQ W by decomposing W into two separate sets of filters where one will extract the S+I+N portion from  $\mathbf{x}(k)$  while the other set will extract the I+N portion from  $\mathbf{x}(k)$ , such that  $\hat{\mathbf{s}}(k) = \mathbf{W}^H \mathbf{x}(k)$  is an approximation of  $\mathbf{s}(k)$ . We shall show in the sequel that the proposed algorithm is able to reduce the channel-induced inteferences and channel noise without intentionally oversampling the received signal nor utilizing additional receive antennas. Moreover, the proposed scheme offers better BER than [12] and other reduced guard interval equalization schemes.

In [12], the equalizer  $\mathbf{W} = \mathbf{D} - \mathbf{B}\mathbf{U} \in \mathbb{C}^{N_r N \times N}$  ( $N_r$  is the number of receive antennas or temporal oversampling factor at the receiver) is composed of three different matrices.

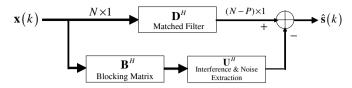


Fig. 1. Block diagram of the proposed FEQ.

 $\mathbf{D} \in \mathbb{C}^{N_r N \times N}$  is the signal signature matrix which is used to match the received signal with the desired signal component,  $\mathbf{B} \in \mathbb{C}^{N_r N \times N_r N}$  is the blocking matrix, consisting of orthogonal basis that span the left nullspace of  $\mathbf{D}$ , and  $\mathbf{U} \in \mathbb{C}^{N \times N}$  is designed to minimize, in the mean-squared sense, the signal output from  $\mathbf{B}^H$  with the residual interference plus noise at the output of  $\mathbf{D}^H$ . Clearly, in order for  $\mathbf{B}$  to be non-zero,  $N_r$  has to be greater than 1. However, this will increase the computational and hardware complexity at the receiver, which greatly inhibits its deployment. Therefore, we propose to eliminate this requirement by exploiting the NSCs that are inherent in standardized OFDM based systems.

In the present scheme, as illustrated in Figure 1,  $\mathbf{D}$  is now a  $N_rN \times (N-P)$  matrix, with P denoting the number of NSCs, which is an even number as it is assumed in [14], [15], though  $N_r$  can now be as small as 1. That is, in (5), we eliminate those columns of  $\mathbf{D}$  which correspond to the frequency bands of NSCs. In present systems, such as IEEE 802.11a/g, the NSCs are usually located at the bandedge of the spectrum and also at the  $0^{th}$  subcarrier in order to mitigate out-of-band radiation and eliminate DC offset caused by unknown complex constant that is added to the received signal prior to the analog-to-digital conversion [14], [15]. Thus,

$$\mathbf{D} = \begin{bmatrix} \vdots & \cdots & \cdots & \cdots & 0 \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{-\frac{N-P}{2}} & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ddots & 0 & \vdots & \vdots & \vdots \\ \vdots & \vdots & d_{-1} & \vdots & \vdots & \vdots \\ \vdots & \vdots & d_{1} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & d_{\frac{N-P}{2}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots \\ 0 & \vdots \\ 0 & \vdots \\ 0 & \vdots & \vdots \\ 0 & \vdots$$

where the subscript in the diagonal elements of  $\mathbf{D}$  denotes the subcarrier index. Therefore,  $\mathbf{B} = \mathcal{N}(\mathbf{D}^H) \in \mathbb{C}^{N_r N \times P}$  consists of elements preset to 0 or 1 in order to block all data subcarriers and to select all NSCs. At the transmitter, the NSCs (by definition) carry no signal of interest and hence, any energy at these NSCs at the receiver must be interference or additive noise.

Since  $\mathbf{D}^H \mathbf{H}_{ISI} \neq \mathbf{0}$  and also  $\mathbf{D}^H \mathbf{H}_{ICI} \neq \mathbf{0}$ , the output

from the matched filter  $\mathbf{D}^H$  will be corrupted by residual interference plus noise; making it necessary to estimate and cancel this I+N by properly designing  $\mathbf{U}$  as indicated in Figure 1.  $\mathbf{U}$  can be computed by minimizing the mean-squared error between the signal output from  $\mathbf{B}^H$  with the residual interference plus noise at the output of  $\mathbf{D}^H$ , i.e.

$$\min_{\mathbf{U}} E\left[\left\|\mathbf{i}(k) - \mathbf{U}^{H} \mathbf{B}^{H} \mathbf{x}(k)\right\|_{2}^{2}\right], \tag{6}$$

where  $\mathbf{i}(k) \triangleq \mathbf{D}^{H} [-\mathbf{H}_{ICI}\mathbf{s}(k) + \mathbf{H}_{ISI}\mathbf{s}(k-1)] + \mathbf{D}^{H}\mathbf{W}_{N}\mathbf{n}(k)$ . (6) can easily be solved by using the principle of orthogonality [17], which yields

$$\mathbf{U} = (\mathbf{B}^H \mathbf{R}_{\mathbf{i}(k)\mathbf{i}(k)} \mathbf{B})^{-1} \mathbf{B}^H \mathbf{R}_{\mathbf{i}(k)\mathbf{i}(k)} \mathbf{D}, \tag{7}$$

where  $\mathbf{R}_{\mathbf{i}(k)\mathbf{i}(k)} \triangleq \mathbf{H}_{ICI}\mathbf{R}_{\mathbf{s}(k)\mathbf{s}(k)}\mathbf{H}_{ICI}^{H} + \mathbf{H}_{ISI}\mathbf{R}_{\mathbf{s}(k-1)\mathbf{s}(k-1)}\mathbf{H}_{ISI}^{H} + \mathbf{W}_{N}\mathbf{R}_{\mathbf{n}(k)\mathbf{n}(k)}\mathbf{W}_{N}^{H},$   $\mathbf{R}_{\mathbf{s}(k)\mathbf{s}(k)} \triangleq E[\mathbf{s}(k)\mathbf{s}^{H}(k)] = \mathbf{I}_{N}, \text{ and } \mathbf{R}_{\mathbf{s}(k-1)\mathbf{s}(k-1)} \triangleq E[\mathbf{s}(k-1)\mathbf{s}^{H}(k-1)] = \mathbf{I}_{N}, \text{ and } \mathbf{R}_{\mathbf{n}(k)\mathbf{n}(k)} = E[\mathbf{n}(k)\mathbf{n}^{H}(k)].$ 

The above proposed scheme has lower computational complexity than the full adaptivity algorithm in [12] because it requires the inversion of only the  $P \times P$  matrix  $\mathbf{B}^H \mathbf{R_{i(k)i(k)}} \mathbf{B}$  instead of the  $N \times N$  channel matrices  $\mathbf{H_0}$  and  $\mathbf{H_1}$  in [12], where typically  $P \ll N$ . In addition, the proposed algorithm has comparable computational complexity with that of the partial adaptivity algorithm in [12] since the partial adaptivity algorithm requires performing matrix inversion on a  $q \times q$  matrix, where  $q \approx P$ . Besides the computational complexity, we will show in the next section that our proposed scheme also outperforms [12] in terms of BER

## 3. SIMULATION RESULTS

Monte-Carlo simulations were used to demonstrate the efficacy of the proposed scheme. N=64 subcarriers (including the NSCs) were used in all the simulations, with P=12 and q=10. The complex gain of the channel was randomly generated with Rayleigh distribution with an average power decaying exponentially [14]

$$\sigma_{\ell}^2 = (1 - e^{-T_S/T_{RMS}})e^{-\ell T_S/T_{RMS}}, \quad \forall \ell = 0 \, \cdots \, q,$$

where  $T_s$  denotes the sampling period, and  $T_{RMS}$  is the root-mean-square delay-spread of the channel. The ratio  $T_s/T_{RMS} = 1.25$  was used to produce a 11-tap channel according to the criterion in [14]. The source symbol sequence  $\{s(n)\}$  is QPSK modulated with an uniform distribution. The additive noise in (3) is modeled as additive white Gaussian with a priori known variance  $\sigma_{\eta(k)\eta(k)}^2$ , and thus  $\mathbf{R}_{\eta(k)\eta(k)} = \sigma_{\sigma(k)\eta(k)}^2$ .

 $\mathbf{R}_{\eta(k)\eta(k)} = \sigma_{\eta(k)\eta(k)}^2 \mathbf{I}_N.$  Several algorithms were used for performance comparison with our proposed scheme. Besides the GSC-based FEQ in [12], a NSC-based FEQ [7], a two-stage zero-forcing (ZF) equalizer [18] and a 2-stage minimum mean-squared error (MMSE) equalizer [19] were also used in our simulations to show the effectiveness of the proposed algorithm. The 2-stage equalizers first project the received signal onto the left nullspace of the space spanned by the ISI, followed by a second stage which removes the ICI based on either the ZF or MMSE criterion.  $\nu=0$  was used for these equalizers. A single-tap ZF FEQ with sufficient CP, where  $\nu=16$ , is used as a benchmark.

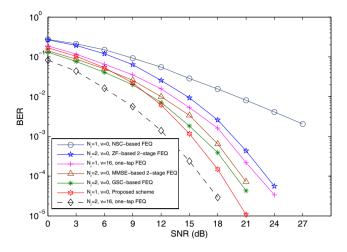


Fig. 2. BER vs. SNR performance for competing equalizers: NSC-based FEQ [7], ZF-based 2-stage FEQ [18], MMSE-based 2-stage FEQ [19], and GSC-based FEQ [12].

Figure 2 shows the BER performance for these various equalizers, with  $N_r = 1$  and 2. Two values are used for  $N_r$  because the 2-stage ZF equalizer, 2-stage MMSE equalizer and the GSC-based equalizer cannot be derived for  $N_r = 1$ . From the figure, it is clear that the proposed scheme outperforms [7] by approximately 10 dB at BER =  $10^{-2}$  because the latter scheme suffers from noise amplification problem. For the other insufficient CP equalizers, the proposed method outperforms all of them when the SNR is greater than 9 dB. In particular, the proposed scheme is shown to have a 1.5 dB advantage over the GSC-based equalizer in [12] at BER =  $10^{-4}$  because the proposed method is able to more accurately estimate the interference and noise component of the received signal by using the NSCs. This is because the NSCs are not contaminated with any of the desired signal, which is not the case for the GSC-based FEO since it relies on added (temporal or spatial) diversity to estimate the interference and noise. In the low SNR region, however, the proposed algorithm performs slightly worse than the GSCbased FEQ (for SNR less than 9 dB) and MMSE-based 2-stage FDE (for SNR less than 6 dB). This is because the extra diversity offers additional input samples to these FEOs such that the additive channel noise can be better smoothed (averaged) out. The proposed scheme is able to outperform the ZF single-tap FEQ with sufficient CP because in the proposed scheme, the channel noise is mitigated by U in (7) and it is also able to better estimate the interference.

The next simulation shows the BER performance of our algorithm as the number of available NSCs is varied while the channel order remained fixed. As Figure 3 shows, as the number of NSCs, P, increases, the BER performance increases as well. This is because the dimension of  $\mathcal{N}(\mathbf{D}^H)$  increases as P increases, which allows the proposed scheme to block out more of the desired signal at the lower branch of the equalizer depicted in Figure 1 and get a better estimate of channel induced interference. Moreover, since no error floor is present, it also indicates that the proposed algorithm is not limited by the same constraint that limits the algorithm in [7], [8], i.e. the FEQ in [7], [8] only exists if v+P>q. Therefore, as long as NSC exists, no matter what the amount is nor where they are located, the

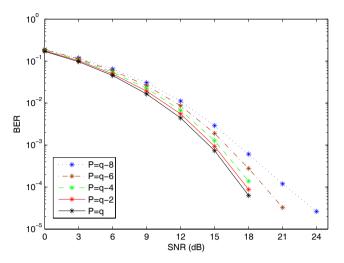


Fig. 3. BER vs. SNR for different number of NSC (P) for the proposed FEQ (no CP).

proposed algorithm will be able to mitigate the channel-induced interferences and channel noise.

### 4. CONCLUSION

This work investigates the channel-induced interference problem caused by insufficient CP in OFDM based systems. Insufficient CP scenario may occur when channel delay spread is extremely long, when the transmitter deliberately shortens the guard interval to reduce transmission overhead in order to increase system throughput, or in multiuser environment when the propagation delay differences among different users are significant. A null subcarrier based frequency domain equalizer is proposed to mitigate the adverse effects caused by the shortened guard interval as well as channel noise. The proposed algorithm has shown to have less computational complexity than a similar approach proposed by [12] and it also outperforms [12] in terms of BER by 1.5 dB. Furthermore, the proposed scheme is not limited by the number of cyclic prefix and the number null subcarriers that exists, nor by their location. Future work will be to extend this FEQ to MIMO systems and to incorporate Space Frequency Block Coding (SFBC) into the proposed scheme to deal with time-varying channels.

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