GENERAL LEAST-SQUARES DESIGN OF ALLPASS TRANSFORMED DFT FILTER-BANKS

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ABSTRACT

The allpass transformation is a common approach to design a frequency warped analysis-synthesis filter-bank (AS FB) with non-uniform time-frequency resolution. Such filter-banks are beneficial, e.g., for speech and audio subband processing, but their synthesis filter-bank design is more complex as for uniform filter-banks.

This problem is tackled by a new least-squares error (LS) synthesis filter-bank design. Magnitude and phase distortions are minimized with the constraints for complete aliasing cancellation and a restricted signal delay. The coefficients of the synthesis filters are determined by a linear set of equations with linear constraints which can be easily solved. The obtained FIR synthesis subband filters exhibit a frequency selectivity similar to that of the IIR analysis filters. The new concept is very general and can be applied in case of an allpass transformation of first or higher order as well as for polyphase network (PPN) filter-banks where the prototype filter length exceeds the number of channels. These properties make the proposed design of interest for various subband processing systems requiring non-uniform frequency bands.

1. INTRODUCTION

The design and application of allpass transformed analysis-synthesis filter-banks (AS FBs) has been the subject of steady research over the past decades [1–14]. One advantage of such frequency warped filter-banks is that they can achieve a frequency resolution very close to that of the human auditory system, which is exploited, e.g., for speech enhancement [6, 9, 10] or subband coding [7]. Another benefit is the lower algorithmic signal delay and complexity in comparison to tree-structured (QMF or wavelet) filter-banks. However, the non-uniform bandwidths of the analysis (subband) filters also complicate the synthesis filter-bank design, which has been addressed by different proposals.

The 'classical' approach is to apply the allpass transformation to the analysis *and* synthesis filter-bank [3, 6]. The reconstruction error can be minimized by the prototype filter design [9, 11]. However, this allows only a very restricted reduction of the reconstruction error due to the limited degrees of freedom offered by the prototype filter coefficients.

More degrees of freedom are obtained by considering FIR polyphase synthesis filters. In [10], a numerical least-squares error (LS) design is proposed which minimizes aliasing and amplitude distortions but not phase distortions caused by the warped analysis filter-bank. These phase distortions are reduced by the closed-form design of [8] which, however, does not employ an explicit error criterion. In [13], it is shown that a significantly lower signal reconstruction error can be achieved by a closed-form design derived by a LS criterion.

A more general approach to design a warped filter-bank is to apply an allpass transformation of higher order where the delay elements of the analysis filter-bank are substituted by allpass filters of higher order [4]. Several authors have derived an analytical closed-form design for the synthesis filter-bank to achieve perfect reconstruction (PR) [5, 7, 12]. In general, these designs provide IIR synthesis filters which are not necessarily stable. An exception is the special case of an allpass transformation of first order where these approaches provide FIR synthesis filters. Another severe drawback of these closed-form solutions is that the synthesis subband filters have not necessarily a bandpass characteristic, cf., [12]. This can cause high signal distortions if spectral modifications of the subband signals are performed.

In this paper, a new LS FIR synthesis filter-bank design for a warped DFT analysis filter-bank is presented where the amount of linear distortions is minimized with the constraints for complete aliasing cancellation and a specified signal delay. The new design is very general as it considers an allpass transformation of first and higher order as well as a polyphase network (PPN) filter-bank structure where the subband filter degrees can exceed the number of channels.

2. ALLPASS TRANSFORMED DFT FILTER-BANK

A uniform DFT filter-bank with M channels is taken as basis, where the analysis subband filters are complex modulated versions of a prototype lowpass filter with finite impulse response (FIR) $h(n)$ of length L according to

$$H_i(z) = H\big(z\,W_M^i\big) = \sum_{n=0}^{L-1} h(n)\,W_M^{-in}\,z^{-n}\;\forall\; i \in \{0, 1, \dots, M-1\} \tag{1}$$

with $W_M = \exp\{-j\,2\pi/M\}$, cf., [15]. A non-uniform time-frequency resolution can be obtained by an *allpass transformation* where the delay elements of the analysis subband filters are replaced by allpass filters of first order [1–3].

A more general approach is to replace the delay elements by allpass filters of *higher order* [4, 5]

$$z^{-n} \rightarrow A^n(z) \cdot B^{L-1-n}(z) \tag{2}$$

where $A(z)$ and $B(z)$ represent the transfer functions of stable, causal allpass filters of order K and $K-1$:

$$A(z) = \prod_{k=1}^{K} \frac{1 - a^*(k)\,z}{z - a(k)}$$
$$|a(k)| < 1\,;\quad a(k) \in \mathbb{C}\,;\quad \max_k\{|a(k)|\} < |z| \tag{3}$$

$$B(z) = \prod_{k=1}^{K-1} \frac{1 - b^*(k)\,z}{z - b(k)}$$
$$|b(k)| < 1\,;\quad b(k) \in \mathbb{C}\,;\quad \max_k\{|b(k)|\} < |z| \tag{4}$$

with $K \in \mathbb{N}$ and * marking the conjugate complex. The frequency responses of these allpass filters are denoted by

$$A\big(e^{j\Omega}\big) = e^{-j\,\varphi_A(\Omega)} \;\text{ and }\; B\big(e^{j\Omega}\big) = e^{-j\,\varphi_B(\Omega)}\,. \tag{5}$$

Applying the allpass transformation of Eq. (2) to Eq. (1) leads to the new transfer functions

$$\widetilde{H}_i(z) = B^{L-1}(z) \sum_{n=0}^{L-1} h(n)\, W_M^{-i\,n} \left(\frac{A(z)}{B(z)}\right)^n \qquad (6)$$

$$= \Psi(z) \sum_{n=0}^{L-1} h(n)\, W_M^{-i\,n}\, \Theta^n(z) \qquad (7)$$

$$\text{with} \quad \Psi(z) = B^{L-1}(z) \quad \text{and} \quad \Theta(z) = \frac{A(z)}{B(z)}$$

to ease the notation. The common allpass transformation of *first order* is included as special case for $K = 1$ so that

$$\Psi(z) = 1 \quad \wedge \quad \Theta(z) = \frac{1 - a^*\, z}{z - a}\,. \qquad (8)$$

The frequency responses for the uniform analysis subband filters of Eq. (1) and the non-uniform analysis subband filters given by Eq. (6) are related by

$$\widetilde{H}_i\left(z = e^{j\,\Omega}\right) = e^{-j\,(L-1)\,\varphi_B(\Omega)}\, H_i\left(e^{j\,\varphi_\Theta(\Omega)}\right) \qquad (9)$$

$$\text{with} \quad \varphi_\Theta(\Omega) = \varphi_A(\Omega) - \varphi_B(\Omega)\,. \qquad (10)$$

The phase difference of Eq. (10) ensures that the allpass transformation causes a *frequency warping* where a frequency interval of $\Delta\Omega = 2\,\pi$ is mapped onto an interval of $2\,\pi$ on the warped frequency scale

$$[0, 2\,\pi] \rightarrow [0, 2\,\pi] \quad : \quad \Omega \mapsto \varphi_\Theta(\Omega)\,. \qquad (11)$$

In contrast, the allpass transformation $z^{-n} \rightarrow A^n(z)$ maps the frequency interval of $[0, 2\,\pi]$ onto an interval of $\Delta\Omega = 2\,\pi\,K$ which causes an undesirable comb-filter effect for $K > 1$, cf., [4].

Eq. (9) reveals that the warping characteristic is solely determined by $\varphi_\Theta(\Omega)$ and thus the transfer function $\Theta(z) = A(z)/B(z)$. However, dependent on the choice for $B(z)$, the transfer function $\Theta(z)$ can become either unstable or non-causal. Therefore, the additional filter with transfer function $\Psi(z) = B^{L-1}(z)$ is employed so that the warped subband filters of Eq. (6) are always stable and causal.

The function of Eq. (11) is bijective if the continuous (unwrapped) phase response $\varphi_\Theta(\Omega)$ is monotonically increasing, which is guaranteed by a positive group delay

$$\frac{\partial\, \varphi_\Theta(\Omega)}{\partial\, \Omega} > 0\, \forall\, \Omega\,. \qquad (12)$$

This property is required to ensure a *unique* mapping so that a comb-filter effect is avoided. The choice

$$B(z) = z^{-(K-1)} \qquad (13)$$

is of special interest as it reduces the implementation cost for the filter-bank and simplifies the design procedure. With Eq. (13), the requirement of Eq. (12) can be written

$$\sum_{k=1}^{K} \frac{1 - \alpha(k)^2}{1 - 2\,\alpha(k)\, \cos\left(\Omega - \gamma(k)\right) + \alpha(k)^2} > K - 1\, \forall\, \Omega \qquad (14)$$

where the allpass poles are expressed by $a(k) = \alpha(k)\, e^{j\,\gamma(k)}$.

As an example, a DFT AS FB is considered with $M = 8$ channels and analysis and synthesis prototype filter given by

$$h(n) = g(n) = \frac{\sqrt{R}}{L} \left(1 - \sqrt{2}\, \cos\left(\frac{\pi}{M}\, n + \frac{1}{2}\right)\right) \qquad (15)$$

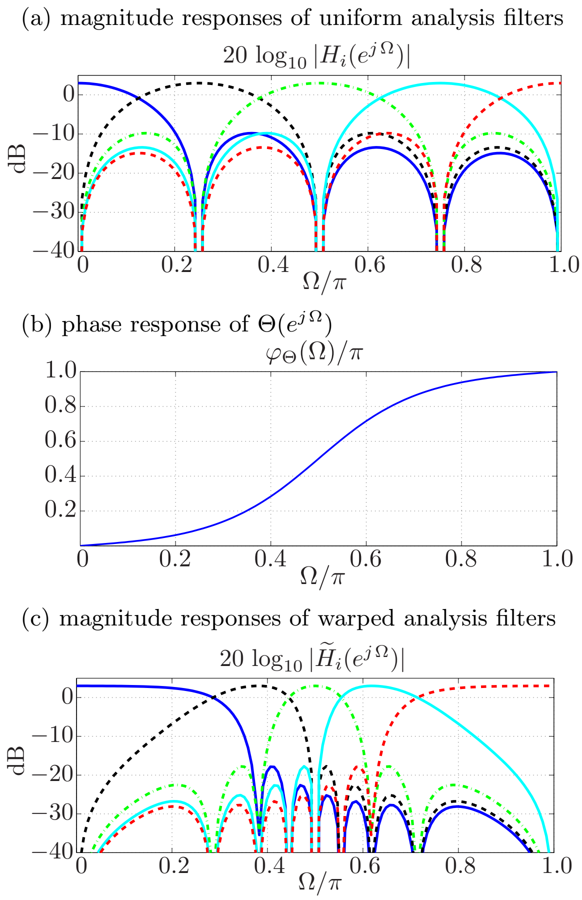

Figure 1: *Non-uniform DFT analysis filter-bank designed by an allpass transformation of second order with parameters:* $L = 2\,M = 16,\ a(1) = j\,0.5,\ a(2) = -j\,0.5,\ b(1) = 0.$

where $L = 2\,M$ and $n \in \{0, 1, \ldots, L-1\}$, cf., [13, 16]. The frequency warping effect of an allpass transformation of second order ($K = 2$) with complex allpass poles is illustrated in Fig. 1. It is easily verified (and visible) that the phase response $\varphi_\Theta(\Omega)$ of Fig. 1-b fulfills Eq. (11) and Eq. (12). The bandwidths of the analysis filters shown in Fig. 1-c decrease first and increase afterwards within the interval $\Omega \in [0, \pi]$ since the phase response $\varphi_\Theta(\Omega)$ has an inflection point within this region. In contrast, such an adjustment of the frequency resolution cannot be achieved by an allpass transformation of *first order*.

A particular efficient *polyphase network* (PPN) implementation of the analysis filter-bank is obtained by rewriting Eq. (7) according to

$$\widetilde{H}_i(z) = \Psi(z) \sum_{m=0}^{l_M - 1} \sum_{\lambda=0}^{M-1} h(m\,M + \lambda) \cdot \Theta^{m\,M+\lambda}(z) \cdot W_M^{-\lambda\,i} \qquad (16)$$

where it is assumed without loss of generality that $L = l_M\, M$ with $l_M \in \mathbb{N}$. Fig. 2 illustrates this PPN implementation of the analysis filter-bank.

The following *synthesis* subband filters are used

$$\bar{G}_i(z) = \sum_{n=0}^{L-1} g(n) \cdot W_M^{-i\,(n+1)} \cdot P(z, L-1-n) \qquad (17)$$

with $g(n)$ denoting the FIR of the synthesis prototype filter

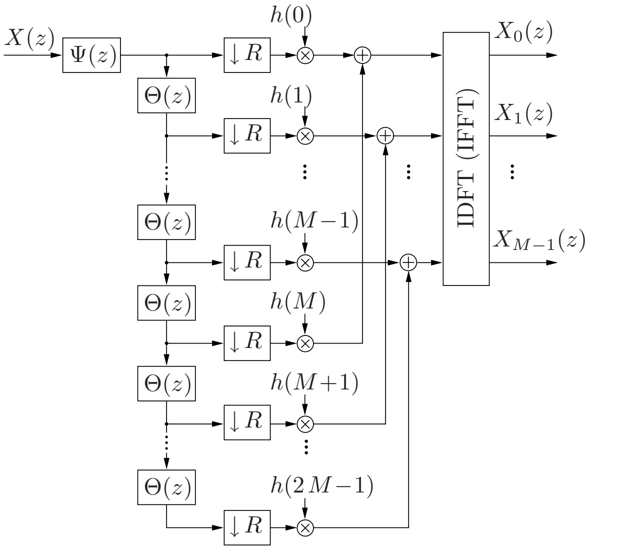

Figure 2: *PPN implementation of the allpass transformed analysis filter-bank with downsampling by R and $L = 2M$.*

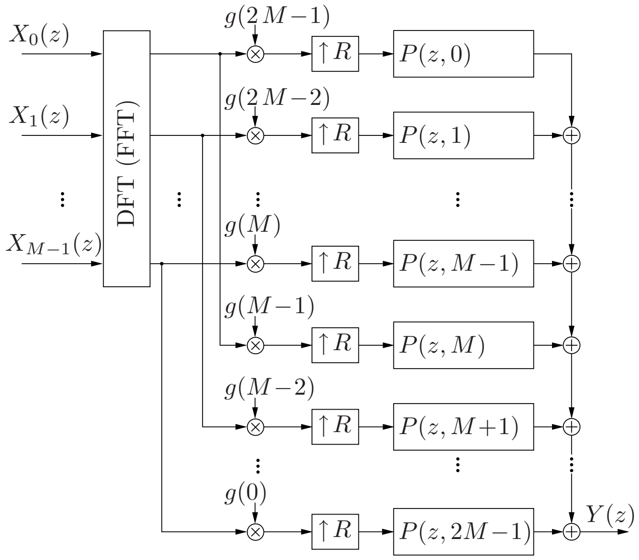

Figure 3: *PPN implementation of the synthesis filter-bank with upsampling by R and $L = 2M$.*

of length L. The coefficients of the L transfer functions

$$P(z,\lambda) = \sum_{n=0}^{N_p-1} p_\lambda(n)\, z^{-n}\,; \quad \lambda \in \{0,1,\ldots,L-1\} \tag{18}$$

with $N_p \in \mathbb{N}$ shall be determined in such a way that a nearly perfect signal reconstruction is achieved.

The FIR synthesis subband filters can be expressed by the PPN representation

$$\bar{G}_i(z) = \sum_{\lambda=0}^{M-1} \bar{G}^{(M)}_{M-1-\lambda}(z) \cdot W_M^{\lambda i};\; i \in \{\,0,1,\ldots,M-1\,\} \tag{19}$$

with 'modified' (type 1) polyphase components

$$\bar{G}^{(M)}_\lambda(z) = \sum_{m=0}^{l_M-1} g(m\,M+\lambda) \cdot P(z,(l_M-m)\,M-1-\lambda)\,. \tag{20}$$

This efficient PPN implementation of the synthesis filter-bank is shown in Fig. 3.

It should be noted that the *uniform* DFT AS FB is included as special case for $\Psi(z) = 1$, $\Theta(z) = z^{-1}$ and $P(z,n) = z^{-(L-1-n)}$ with $n \in \{0,1,\ldots,L-1\}$.

3. SYNTHESIS FILTER-BANK DESIGN

The output signal of the AS FB can be represented in the z-domain (after some calculations) by the expression

$$Y(z) = \frac{1}{R}\sum_{r=0}^{R-1} X(z\,W_R^r) \sum_{i=0}^{M-1} \widetilde{H}_i(z\,W_R^r) \cdot \bar{G}_i(z) \tag{21}$$

with $R \in \{\mathbb{N}\,|\,1 \le R \le M\,\}$. Since $W_R^r = W_R^{r+l\,R}$ for $l \in \mathbb{Z}$, the AS FB with subsampling by R is a *linear periodically time-varying* (LPTV) system with period R. We will take this behavior into account by determining the overall transfer function of the filter-bank for R time-shifted unit sample sequences as input, i.e., $X(z) = z^{-l}$ for $l \in \{0,1,\ldots,R-1\}$. Eq. (21) turns then into the new transfer function

$$\begin{aligned} T_l(z) &= \frac{Y(z)}{z^{-l}} \\ &= \frac{1}{R}\sum_{r=0}^{R-1} W_R^{-r\,l} \sum_{i=0}^{M-1} \widetilde{H}_i(z\,W_R^r) \cdot \bar{G}_i(z)\,. \end{aligned} \tag{22}$$

For our numerical design approach, a matrix representation of the transfer function $T_l(z)$ in dependence of the unknown $L\,N_p$ coefficients $p_\lambda(n)$ of the synthesis polyphase filters is required. In the following, bold lower-case variables denote vectors and matrices are marked by bold upper-case variables. The superscript T indicates the transpose of a vector or matrix. Some manipulations of Eq. (17) lead to

$$\bar{G}_i(z) = \boldsymbol{v}_i^T \cdot \boldsymbol{D}^T(z) \cdot \boldsymbol{p}\,; \quad i \in \{\,0,1,\ldots,M-1\,\} \tag{23a}$$

$$\text{with } \boldsymbol{v}_i = [\,g(L-1),\, g(L-2)\,W_M^{-(L-1)\,i},\, \ldots$$

$$\ldots, g(1)\,W_M^{-2\,i},\, g(0)\,W_M^{-i}\,]^T \tag{23b}$$

$$\boldsymbol{D}(z) = \boldsymbol{I}_L \otimes \boldsymbol{d}_{N_p}(z) \tag{23c}$$

$$\boldsymbol{d}_{N_p}(z) = [\,1,\, z^{-1},\, \ldots,\, z^{-(N_p-1)}\,]^T \tag{23d}$$

$$\boldsymbol{p} = [\boldsymbol{q}_0^T, \boldsymbol{q}_1^T, \ldots, \boldsymbol{q}_{L-1}^T\,]^T \tag{23e}$$

$$\boldsymbol{q}_n = [\,p_n(0), p_n(1), \ldots, p_n(N_p-1)\,]^T \quad \text{for } n \in \{\,0,1,\ldots,L-1\,\}\,. \tag{23f}$$

The $L \times L$ identity matrix is denoted by $\boldsymbol{I}_L$ and the operator $\otimes$ marks the Kronecker product of two matrices.

With Eq. (23), the transfer function of Eq. (22) is now formulated by the matrix notation

$$\begin{aligned} T_l(z) &= \left(\frac{1}{R}\sum_{r=0}^{R-1} W_R^{-r\,l} \sum_{i=0}^{M-1} \widetilde{H}_i(z\,W_R^r) \cdot \boldsymbol{v}_i^T \cdot \boldsymbol{D}^T(z)\right) \cdot \boldsymbol{p} \\ &= \boldsymbol{\xi}(z,l) \cdot \boldsymbol{p}\,; \quad l \in \{\,0,1,\ldots,R-1\,\} \end{aligned} \tag{24}$$

where the complex vector $\boldsymbol{\xi}(z,l)$ is of dimension $1 \times L\,N_p$.

The new formulation of the transfer function according to Eq. (22) allows to express the condition for a *linear time-invariant* (LTI) transfer function (for $R > 1$) as follows

$$T_l(z) \stackrel{!}{=} T_0(z) \;\text{ for }\; l \in \{\mathbb{N}\,|\,0 < l < R\}\,. \tag{25}$$

This condition can now be cast into a matrix notation by means of Eq. (24)

$$\underbrace{\begin{bmatrix} \boldsymbol{\xi}(z,1)-\boldsymbol{\xi}(z,0) \\ \boldsymbol{\xi}(z,2)-\boldsymbol{\xi}(z,0) \\ \vdots \\ \boldsymbol{\xi}(z,R-1)-\boldsymbol{\xi}(z,0) \end{bmatrix}}_{=\,\Xi_\Delta(z)} \cdot \boldsymbol{p} \stackrel{!}{=} \mathbf{0}_{R-1}, \tag{26}$$

where a column vector with R zeros is denoted by $\mathbf{0}_R$. This condition for an LTI system ensures an *aliasing-free* signal reconstruction (if no spectral modifications of the subband signals $X_i(z)$ are performed). In this case, Eq. (22) becomes equal to the *linear* transfer function of the filter-bank which, by means of Eq. (7) and (17), can be written as

$$\begin{aligned} T_{\text{lin}}(z) &= \frac{1}{R}\,\Psi(z)\sum_{i=0}^{M-1}\sum_{n=0}^{L-1}\sum_{\rho=0}^{L-1} W_M^{-i\,(n+\rho+1)} h(n)\,g(\rho) \\ &\qquad\qquad \cdot\,\Theta^n(z)\,P(z,L-1-\rho) &(27)\\ &= \frac{M}{R}\,\Psi(z)\sum_{m\in\mathbb{Z}}\sum_{n=0}^{L-1} h(n)\,g(m\,M-1-n) \\ &\qquad\qquad \cdot\,\Theta^n(z)\,P(z,L-m\,M+n)\,. &(28) \end{aligned}$$

The condition $T_{\text{lin}}(z) \stackrel{!}{=} z^{-d_0}$ avoids linear signal distortions and is fulfilled if

$$\frac{M}{R}\sum_{n=0}^{L-1} h(n)\cdot g(m\,M-1-n) = \begin{cases} 1 & ;\ m = l_M \\ 0 & ;\ m\in\mathbb{Z}\backslash\{l_M\} \end{cases} \tag{29}$$

$$\wedge\ \ \Psi(z)\cdot\Theta^n(z)\cdot P(z,n) = z^{-d_0}\ \forall\ n\in\{0,1,\ldots,L-1\}\,. \tag{30}$$

Eq. (29) states a standard problem in the design of (uniform) AS FBs and can be either solved by numerical design approaches, e.g., [15] or analytical closed-form expressions as given, for instance, by Eq. (15).

The second requirement stated by Eq. (30) can be expressed by means of the matrices introduced in Eq. (23)

$$\left(\left(\mathbf{1}^T_{L\,N_p}\otimes\begin{bmatrix}\Psi(z)\\ \Psi(z)\cdot\Theta(z)\\ \vdots \\ \Psi(z)\cdot\Theta^{L-1}(z)\end{bmatrix}\right)\odot\boldsymbol{D}^T(z)\right)\cdot\boldsymbol{p}\stackrel{!}{=} z^{-d_0}\cdot\mathbf{1}_L \tag{31}$$

with $\odot$ denoting the element-wise multiplication of two matrices of the same dimensions (Hadamard product) and $\mathbf{1}_L$ representing a column vector with L ones. The condition of Eq. (31) is now written by the compact notation

$$\boldsymbol{U}(z)\cdot\boldsymbol{p}\stackrel{!}{=}\boldsymbol{v}(z,d_0) \tag{32}$$

with the complex matrix $\boldsymbol{U}(z)$ being of dimension $L\times L\,N_p$.

The conditions of Eq. (26) and Eq. (31) shall be fulfilled for $\mathcal{N} = L\,N_p$ discrete z-values on the unit circle

$$z = W_{\mathcal{N}}^n;\ \ n\in\{0,1,\ldots,\mathcal{N}-1\}\,. \tag{33}$$

Evaluating the matrix $\boldsymbol{U}(z)$ and vector $\boldsymbol{v}(z,d_0)$ of Eq. (32) at these points can be expressed by the (stacking) notation

$$\boldsymbol{U}^{[\mathcal{N}]} = \begin{bmatrix}\boldsymbol{U}(1)\\ \boldsymbol{U}(W_{\mathcal{N}})\\ \vdots \\ \boldsymbol{U}(W_{\mathcal{N}}^{\mathcal{N}-1})\end{bmatrix},\ \boldsymbol{v}^{[\mathcal{N}]}(d_0) = \begin{bmatrix}\mathbf{1}_L\\ W_{\mathcal{N}}^{-d_0}\cdot\mathbf{1}_L\\ \vdots \\ W_{\mathcal{N}}^{-d_0\,(\mathcal{N}-1)}\cdot\mathbf{1}_L\end{bmatrix} \tag{34}$$

and the matrix $\Xi_\Delta^{[\mathcal{N}]}$ is derived from the matrix $\Xi_\Delta(z)$ of Eq. (26) in the same manner

$$\Xi_\Delta^{[\mathcal{N}]} = \begin{bmatrix}\Xi_\Delta(1)\\ \Xi_\Delta(W_{\mathcal{N}})\\ \vdots \\ \Xi_\Delta(W_{\mathcal{N}}^{\mathcal{N}-1})\end{bmatrix}. \tag{35}$$

The $\mathcal{N}$ synthesis filter coefficients $\boldsymbol{p}$ to fulfill Eq. (26) and Eq. (31) can now be determined by the *equality constrained least-squares error* (CLS) problem

$$\hat{\boldsymbol{p}} = \arg\underset{\boldsymbol{p}}{\text{minimize}}\ \left|\left|\boldsymbol{U}^{[\mathcal{N}]}\cdot\boldsymbol{p}-\boldsymbol{v}^{[\mathcal{N}]}(d_0)\right|\right|_2^2 \tag{36a}$$

$$\text{subject to}\quad \Xi_\Delta^{[\mathcal{N}]}\cdot\boldsymbol{p} = \mathbf{0}_{(R-1)\,\mathcal{N}}\,. \tag{36b}$$

This linear set of equations with linear constraints can be easily solved by means of the function `lsqlin` of the MATLAB optimization toolbox. With this approach, linear signal distortions are minimized with the constraints for complete aliasing cancellation and a given signal delay d_0.

The devised CLS design is a very general concept and contains some previous proposals as *special cases:* The design of [14] is obtained for an allpass transformation of first order according to Eq. (8) with real poles ($a\in\mathbb{R}$) and a prototype filter length restricted to $L = M$. The approach of [13] is obtained for an allpass transformation of first order, and if Eq. (36) is solved without the constraint Eq. (36b). The synthesis polyphase filters with transfer functions $P(z,n)$ act then purely as phase equalizers designed by a LS error criterion.[1] Their coefficients can then of course be determined by closed-form expressions according to [14] instead of solving Eq. (36a) numerically.

4. DESIGN EXAMPLE

The optimization of Eq. (36) has been employed to design the synthesis filter-bank for the analysis filter-bank given by Fig. 1. The obtained vector $\hat{\boldsymbol{p}}$ is rather 'sparse' with about 35.84% of its coefficients having a value of less than $\pm 10^{-7}$. The resulting filter-bank is analyzed in Fig. 4.

The magnitude responses of the FIR synthesis subband filters in Fig. 4-a are similar to that of the IIR analysis filters of Fig. 1-c. Thus, synthesis subband filters with a distinct bandpass characteristic are obtained in contrast to the PR designs of [5, 7, 12]. The frequency response for Eq. (22) $T_0(e^{j\,\Omega}) = |T_0(e^{j\,\Omega})|\,e^{-j\,\varphi_T(\Omega)}$ is analyzed by plotting its magnitude response $|T_0(e^{j\,\Omega})|$ (Fig. 4-b) and phase error $\Delta\varphi_T(\Omega) = \varphi_T(\Omega) - d_0\,\Omega$ (Fig. 4-c) which reveal negligible magnitude and phase distortions of less than ± 0.006 dB and $\pm 0.0002\,\pi$, respectively.[2] The peak aliasing distortions according to [15] are here given by

$$\mathcal{D}_{\text{alias}}(\Omega) = \frac{1}{R}\sqrt{\sum_{\rho=1}^{R-1}\left|\sum_{i=0}^{M-1}\widetilde{H}_i(e^{j\,\Omega}\,W_R^\rho)\cdot\bar{G}_i(e^{j\,\Omega})\right|^2}\,. \tag{37}$$

The plot of this function in Fig. 4-d indicates a complete aliasing cancellation (LTI system), given that calculating the peak aliasing distortions of a closed-form PR design yields a similar amount due to rounding errors.

Finally, it should be noted that a tree-structured (QMF or wavelet) AS FB with a comparable frequency resolution exhibits a higher signal delay and complexity than the presented allpass transformed AS FB, cf., [6].

[1] In this case, $|\bar{G}_i(e^{j\,\Omega})| \to |\widetilde{H}_i(e^{j\,\Omega})|$ for $N_p\to\infty \wedge g(n)=h(n)$.

[2] In practice, a higher signal reconstruction error is usually tolerable so that a lower degree N_p can be taken.

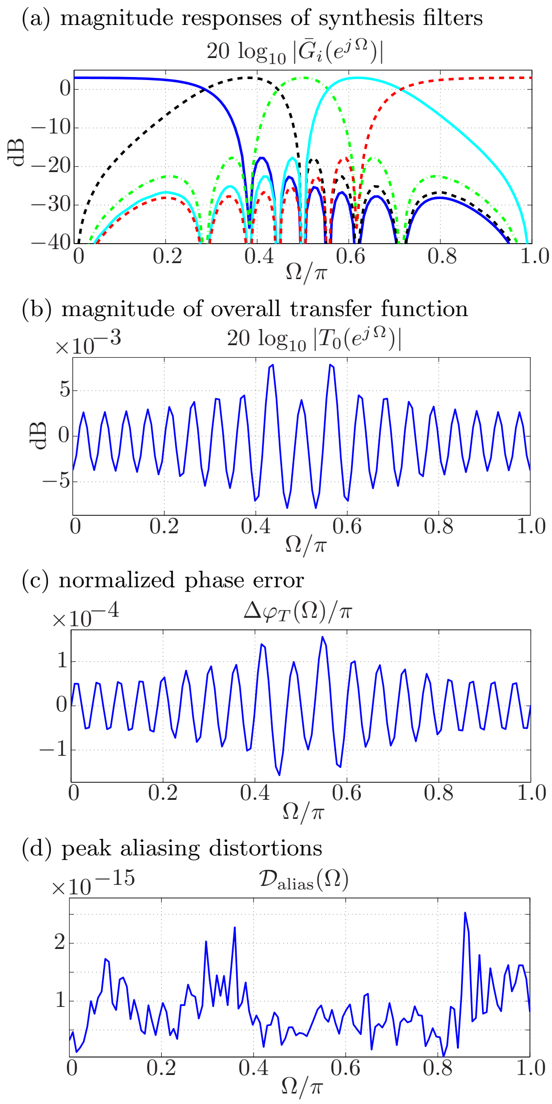

Figure 4: *Analysis of the new synthesis filter-bank design ($N_p = 64$, $d_0 = 60$, $R = M/4 = 2$) designed for the analysis filter-bank of Fig. 1.*

5. CONCLUSION

A general LS design concept for allpass transformed DFT AS FBs is presented. It uses an allpass transformation of first and higher order, and considers a PPN filter-bank where the prototype filter length exceeds the number of channels to achieve an enhanced frequency selectivity for the subband filters. The FIR synthesis filter-bank design is derived by a LS minimization of linear signal distortions with the constraints for complete aliasing cancellation and a specified signal delay. This equality constrained LS optimization consists of a linear set of equations with linear constraints, which is much easier to solve than quadratic optimizations with linear or quadratic constraints, e.g., [9,11]. In addition, the new design minimizes magnitude and phase distortions and can achieve a complete aliasing cancellation in contrast to [8–11]. The synthesis subband filters are inherently stable and have a pronounced bandpass characteristic unlike [5, 7, 12]. The new general design contains our previous proposals [13, 14] as special cases, and it can also be applied to other transformation kernels such as the DCT. These properties make the proposed AS FB of interest for different subband processing systems requiring non-uniform frequency bands as, for example, speech enhancements systems, cf., [6, 9, 10].

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