POLYPHASE MAGNITUDE MODULATION FOR PEAK POWER CONTROL

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ABSTRACT

This paper addresses the problem of controlling the envelope's power peak of single carrier modulated signals, band limited by root-raised cosine (RRC) pulse shaping filters, in order to maximize the efficiency of the transmitter's high power amplifier (HPA). A novel multistage polyphase approach for the magnitude modulation (MM) concept is developed. As opposed to traditional MM solutions, based on look-up-tables (LUT), in this proposal, the MM coefficients are computed in real-time by a low complexity polyphase filter system. The need to use back-off from HPA saturation is almost eliminated (reduction greater than 95%) with just a 2-stage system. Simulation results also show peak-to-average (PAPR) reduction gains similar to the state of the art trellis shaping techniques. However our proposal has the major advantage of allowing this MPMM system to be easily added to any currently working transmitter system to significantly improve its power efficiency.

1. INTRODUCTION

The peak power control of band-limited single-carrier signals has been an issue for decades [1]. This problem is emphasised when, in order to meet the currently growing demands for higher data rates (and so higher spectral efficiency), very low roll-off root-raised cosine (RRC) pulse shaping filters and high-order constellations are used. These lead to an increase on the peak-to-average power ratio (PAPR) of the modulated signal to transmit and also a decrease of the transmitter's high power amplifier (HPA) efficiency. This happens since the amplification of signals with high PAPR require the use of linear HPAs operated with a large back-off and close to saturation. However, only a few publications have considered this problem. Since the main contribution to PAPR results from Nyquist pulse shaping bandwidth limitation, some of the proposed solutions try to decrease the PAPR by optimizing these filters [2, 3]. More recently, trellis coding solutions that avoid critical sequences of modulated symbols were also proposed [4, 5]. However, such solutions are complex, with a high computational overhead and difficult to include on currently operating radio transmission systems.

Our approach to control the envelope's excursion at the RRC filter output (so as to decrease PAPR) uses the concept of magnitude modulation (MM) [1, 6, 7], i.e. the adjustment of amplitude of each data pulse prior to filtering, so as to suppress peaks in the transmitted signal. Traditional MM solutions [6, 7] compute a priori MM coefficients and stored them in a look-up table (LUT). Those techniques proved to be efficient for M-ary constellations with $M \leq 16$, however they can hardly be applicable to higher modulations [7], due to the huge number of symbol combinations involved, even for low memory systems.

In this paper a new multistage polyphase magnitude modulation (MPMM) scheme is proposed. MM coefficients are computed in real-time, using a simple polyphase filter system that predicts undesirable peak excursions at the output of the RRC filter and adjusts accordingly the amplitude of the symbols at its input. Its performance can be further improved by cascading any desired number of these MPMM base-system blocks. Experimental results showed that by using two or three blocks we nearly eliminate the need to use of back-off from the HPA saturation point, even at very low roll-off values, α. Another great advantage of our MPMM scheme is that it is independent from the modulation used, since the polyphase filter structure only depends on the RRC impulse response. An additional very important advantage, from both implementation and economical points of view, is that it can be included in currently operating transmitter systems, without the need to replace the equipment already installed. In fact, it is possible to take advantage of the existing forward-error-control (FEC) capabilities, in order to efficiently compensate the increase sensitivity to noise resulting from MM. By computing the initial receiver's soft-decoding estimates based on the average positions of the transmitted MPMM symbols, a minor BER degradation is observed. Results show that, for the 8-PSK case, net gains above 5dB for $\alpha \leq 0.2$ RRC filtering are obtained.

This paper is organized as follows. Section 2 presents the MM concept and clearly defines PAPR and back-off for system's performance assessment. Section 3 describes the new proposed MPMM scheme. For comparison purposes [5], 8-PSK modulation has been selected. PAPR and back-off gains obtained with our MPMM technique are reported

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in section 4. Finally, the main conclusions are summarized.

2. THE MAGNITUDE MODULATION PRINCIPLE

Fig. 1 illustrates the functional diagram and the basic building blocks of a typical digital communication transmitter system using the MM concept [6, 7]. The MM block adjusts the amplitude of each data pulse, $s[n]$, prior to RRC filtering, taking into account past and future symbols in its neighbourhood, always trying to suppress peaks from the transmitted signal, $x[n] \in \mathbb{C}$. Fig.1 also shows the generic scheme of a MM system, where D past and future symbol neighbours of $s[n]$ are considered in the computation of the best MM coefficient, $m[n] \in]0,1]$, to apply to $s[n]$. The delay DT_{symb} introduced by the MM technique, with T_{symb} denoting the symbol duration time, is usually small. It depends on the time duration in which the RRC impulse response maintains significant non-zero amplitude values.

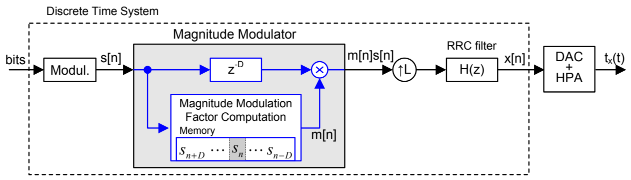

Figure 1 – Generic transmitter system block diagram including MM.

Since the system is multirate, a better understanding of its behaviour is obtained by considering its different operating rates. So, we denote T_{samp} as the sampling period at which the pulse shaping filter operates, which is related to symbol data period, according to $L = T_{symb} / T_{samp}$, with $L \geq 2$ and $L \in \mathbb{N}$.

The equivalent complex digital baseband signal, after filtering, and prior to D/A conversion, is then given by:

$$x[n] = \left[\sum_k m[k]s[k]\delta[n-kL] \right] * h[n], \tag{1}$$

where $h[n]$ is the impulse response of the pulse shaping filter. Its PAPR is defined as

$$PAPR = 10\log_{10}\left[\max|x[n]|^2 \Big/ E\left(|x[n]|^2\right) \right] \quad \text{(dB)}, \tag{2}$$

and results from two main components: $PAPR_{const}$ and $PAPR_{rrc}$, due to the constellation and the RRC filter, respectively.

Denoting the M symbols of an M-ary constellation by S_n, with $n = 1,\cdots,M$, the $PAPR_{const}$ contribution only depends on its geometry and is given by:

$$PAPR_{const} = 10\log_{10}\left[\frac{\max\limits_{n=1,\cdots,M} |S_n|^2}{\frac{1}{M}\sum\limits_{n=1}^{M} |S_n|^2} \right] \quad \text{(dB)}. \tag{3}$$

It is clear from (3) that $PAPR_{const}$ is null for constant amplitude constellations, such as M-PSK. However, for higher order modulations, it contributes with a non-negligible value to the total PAPR value, as is shown in Table 1. In practice, the HPA should be able, at least, to handle $PAPR_{const}$, i.e., if the multirate pulse shaping system (L upsampler followed by filter $h[n]$) has unitary power gain, then maximum amplitude symbols at the output of modulator (Fig. 1) should suffer no distortion if they are directly fed at the HPA input. So, and denoting the maximum amplitude of a modulated symbol by A, i.e., $A = \max\limits_{n=1,\cdots,M} |S_n|$, under ideal DAC operation, the back-off to be applied to the signal $x[n]$, prior to high power amplification, is defined as:

Table 1. – Constellation PAPR Contribution.

M-PSK	16-APSK(a)	16-QAM	32-APSK(b)	32-QAM	64-QAM
0 dB	1.1 dB	2.6 dB	2.1 dB	2.3 dB	3.7 dB

a. 16-APSK DVB-S2 constellation with $\gamma = 3.15$

b. 32-APSK DVB-S2 constellation with $\gamma_1 = 2.84$ and $\gamma 2 = 5.27$

Table 2. – PAPR of an RRC digital filter

Roll-off	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
PAPR (dB)	6.45	5.93	5.15	4.53	4.07	3.64	3.41	3.40

$$Back\text{-}off = 10\log_{10}\left(\max|x[n]|^2 / A^2 \right) \quad (dB). \tag{4}$$

The major contribution to PAPR is due to pulse shaping, $PAPR_{rrc}$. In order to limit the bandwidth of the transmitted signal without inter-symbol interference (ISI), the modulated signal, $s[n]$, is usually filtered by a linear phase RRC filter, $H(z)$. This limitation in the frequency domain results in undesirable envelope variations in the time domain. Let's assume, without loss of generalization, a type I FIR RRC filter whose impulse response spreads over $2N$ symbol intervals, T_{symb}, given by:

$$H(z) = \sum_{n=0}^{2NL} h[n]z^{-n} \quad \text{with} \quad h[n] = h[2NL-n]. \tag{5}$$

The filter contribution, $PAPR_{rrc}$, is evaluated as

$$PAPR_{rrc} = 10\log_{10}\left[\frac{\max\limits_{i=0,\cdots,L-1}\left(\sum\limits_{n=0}^{2N} |h[nL+i]| \right)^2}{\frac{1}{L}\sum\limits_{n=0}^{2NL} |h[n]|^2} \right] \quad (dB), \tag{6}$$

and can be much higher than $PAPR_{const}$, as shown in Table 2, for roll-off values in the range of interest, $\alpha \in [0.15; 0.35]$. The main goal of our MM technique is, therefore, to cancel this undesirable $PAPR_{rrc}$ contribution.

3. POLYPHASE MAGNITUDE MODULATION

The polyphase decomposition of the RRC filter, $H(z)$, is behind the idea of a LUT-less MM approach. In Fig. 2(a) we present a *k*-stage MPMM system, followed by the polyphase representation of the RRC filter. Each stage incrementally adjusts the symbols' amplitudes, in order to optimize the excursion at the RRC's output. Without loss of generalization, we consider that $H(z)$ is given by (5) and that oversampling, L, is even. Similar developments for a type II filter or an odd L can, easily, be inferred.

Pulse shaping can be performed at symbol rate using the polyphase scheme of Fig. 2(a), with

$$E_i(z) = \sum_{n=0}^{2N} e_i[n]z^{-n} = \sum_{n=0}^{2N} h[nL+i+\lambda]z^{-n}, \tag{7}$$

considering the required zero padding as show in Fig. 4 and where $\lambda \in \mathbb{Z}$ is the filter decomposition phase offset.

The output $x[n]$ is given by one of the polyphase filters' output, at instant $n_0 = \lfloor n/L \rfloor$, since

$$x[n] = y_{(n \bmod L)}\left[\lfloor n/L \rfloor \right]. \tag{8}$$

All outputs $y_i[n_0]$, with $i=0,\cdots,L-1$, depend on the same set of inputs symbols $s[k]$, with $k=n_0-2N,\cdots,n_0$, since

$$y_i[n]=\sum_{k=0}^{2N} e_i[k]s[n-k]\,. \tag{9}$$

The MPMM principle consists in computing the best MM coefficient to apply in symbol $s[n_0]$, so as to guarantee that $|x[n]|<A$. This takes into account the RRC output samples, at those sampling instants, for which $s[n_0]$ most contributes. Once the energy of the impulse response $h[n]$ is centred around sample NL, $s[n_0]$ is, according to (8) and (9), the most relevant symbol to the RRC output excursion, $x[n]$, during the interval $[(n_0+N-0.5)L;(n_0+N+0.5)L[$, as shown in Fig. 3. Based on this premise, we have developed the system of Fig. 2(a), considering polyphase decomposition with $\lambda=-L/2$, as sketched in Fig. 4. The system adjusts the amplitude of symbol $s[n_0]$ in order to control the output excursion of all polyphase filters $E_i(z)$ at sampling instant n_0+N (Fig.3), so as to guarantee that $|y_i[n_0+N]|\le A$. The impulse responses of FIR filters $G_{0i}(z)$ and $G_{1i}(z)$ are obtained directly from $e_i[n]$ and they are defined as

$$g_{0i}[n]=\begin{cases} e_i[n] & ,0\le n\le N \\ 0 & ,\text{otherwise}\end{cases}, \tag{10}$$

$$g_{1i}[n]=\begin{cases} e_i[n+N+1] & ,0\le n\le N-1 \\ 0 & ,\text{otherwise}\end{cases}. \tag{11}$$

For a better understanding and without loss of generality, we will focus our attention in Fig. 2(b), which depicts a single branch of the MPMM system. When magnitude modulation is used, (9) is written as

$$y_i[n]=\sum_{k=0}^{2N} e_i[k]m_i[n-k]s[n-k]\,, \tag{12}$$

where $m_i[n]$ is the sequence of MM coefficients which control the output peak power of polyphase filter $E_i(z)$.

Basically, the system tries to anticipate the output of $E_i(z)$ at each instant n_0+N and, accordingly, it computes the MM factor $m_i[n_0]$ to apply into $s[n_0]$, the input sample that most contributes to the output $y[n_0+N]$. Each MPMM stage system introduces a small delay of NT_{symb} during transmission. When computing $m_i[n_0]$, coefficients $m_i[n_0-k]$, with $k=1,\cdots,N$, are already known (past symbols relative to $s[n_0]$), although nothing is known about the MM values that will magnitude modulate symbols $s[n_0-q]$, with $q=-N,\cdots,-1$ (future symbols relative to $s[n_0]$). In order to avoid excessive time variation of the average power of the signal after pulse shaping, we have assumed that future symbols should be MM as,

$$m_i[n_0-q]\simeq m_i[n_0],\quad \text{for}\quad q=-N,\cdots,-1\,. \tag{13}$$

Considering equality in equation (13), a non-negative $f(\cdot)$ function (see Appendix) is defined so as to guarantee (14).

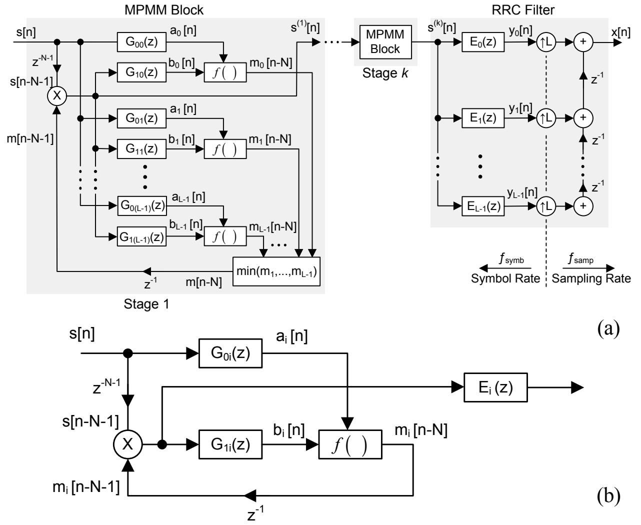

Figure 2 – Polyphase magnitude modulation scheme for controlling the signal's excursion at the RRC output. (a) Global MPMM system followed by an RRC filter block. (b) Detailed MPMM branch.

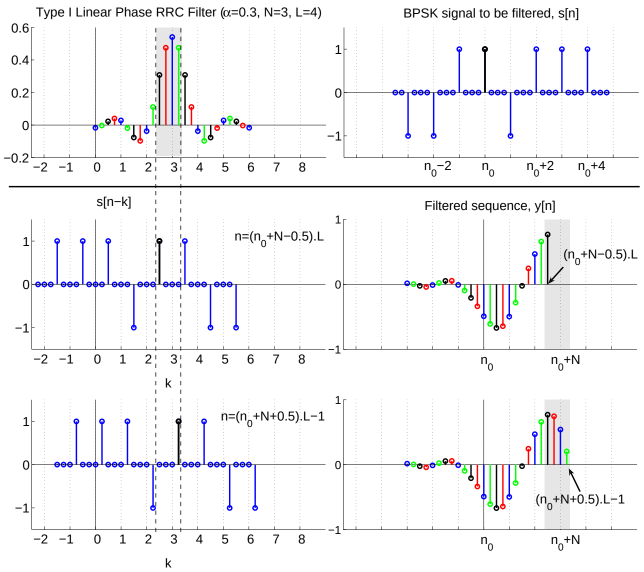

Figure 3 – Controlling the output around sample n_0+N, within a range of L samples.

The procedure just described computes the magnitude modulation factor $m_i[n]$, that multiplies each symbol $s[n]$, in order to limit the excursion at the output of a particular filter $E_i(z)$. However, $m[n]$ has to be unique and has to guarantee that $|y_i[n]|\le A$ at the output of all filters $E_i(z)$ (with $i=0,\cdots,L-1$). We have solved this problem by multiplying each symbol $s[n]$ by the most restricted factor, i.e., by computing

$$m[n]=\min_{i=0,\cdots,L-1}\left(m_i[n]\right). \tag{15}$$

$$|y_i[n_0+N]|\le A \Leftrightarrow \left|\sum_{k=0}^{2N} e_i[k]m_i[n_0+N-k]s[n_0+N-k]\right|\le A \Leftrightarrow \left|m_i[n_0]\sum_{k=0}^{N} e_i[k]s[n_0+N-k]+\sum_{k=N+1}^{2N} e_i[k]m_i[n_0+N-k]s[n_0+N-k]\right|\le A$$

$$\Leftrightarrow \left|m_i[n_0]\underbrace{\sum_{k=0}^{N} g_{0i}[k]s[n_0+N-k]}_{a_i[n_0]\text{ according to Fig. 2}(b)}+\underbrace{\sum_{k=0}^{N-1} g_{1i}[k]m_i[n_0-k-1]s[n_0-k-1]}_{b_i[n_0]\text{ according to Fig. 2}(b)}\right|\le A \Leftrightarrow |m_i[n_0]a_i[n_0+N]+b_i[n_0+N]|\le A \tag{14}$$

$$\Rightarrow m_i[n_0]=f(A,a_i[n_0+N],b_i[n_0+N])$$

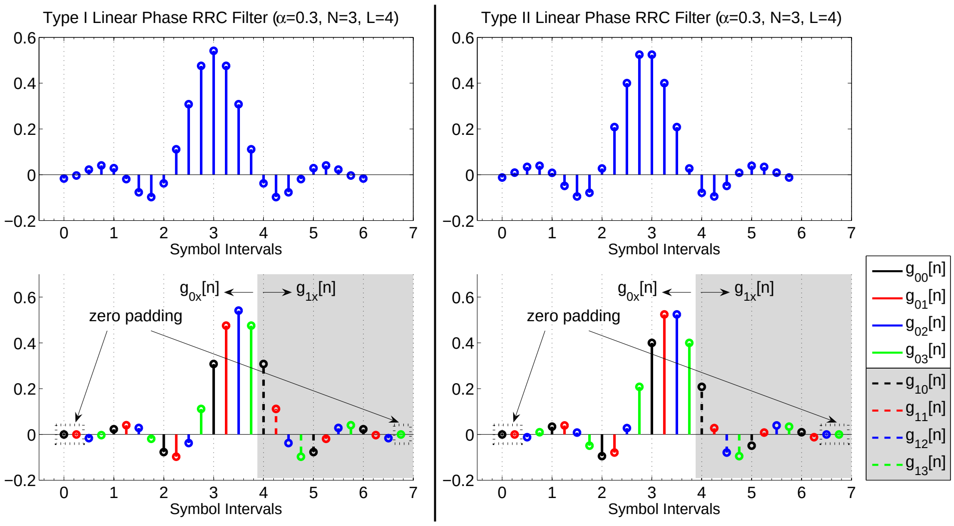

Figure 4 – MPMM component filters, $g_{0x}[n]$ and $g_{1x}[n]$, for polyphase decomposition with phase-offset, $\lambda = -L/2$.

However, a further improvement on $m[n]$ has to be done, since the equality condition in (13) is only assumed when computing $m[n_0]$. When MM coefficient, $m[n_0+1]$ is greater than $m[n_0]$, condition $|y_i[n_0+N]| \leq A$ doesn't hold. However, since symbol $s[n_0+1]$ is more relevant to output $y_i[n_0+N+1]$ than $y_i[n_0+N]$, there must be a compromise for $m[n_0+1]$, that does not decrease $y_i[n_0+N+1]$ too much (which would be undesirable) and still guarantees that restriction $|y_i[n_0+N]| \leq A$ is only slightly violated. A simple procedure to accomplish this was found by using the following time variant filter:

$$m[n+1] > m[n] \quad \Rightarrow \quad m'[n+1] = (m[n+1]+m[n])/2 \,. \quad (16)$$

4. SIMULATION RESULTS

This section reports reduction gains obtained in the PAPR of $x[n]$ and in the required back-off to drive HPA close to saturation. The MPMM method was simulated for typical M-ary constellations (i.e. QAM, PSK, APSK) with $M \leq 32$ and RRC filters [8] with $\alpha \in [0.1; 0.5]$. For comparison purposes with the trellis PAPR reduction technique [5], and also due to space limitations, we illustrate in Fig. 5 and Fig. 6 simulation results only for the 8-PSK case, using the same filter conditions of [5].

Fig. 5(a) shows the complementary cumulative distribution function (CCDF) for the PAPR of the transmitted signal, as well as the required back-off, with and without our new MPMM technique, for the very demanding case of $\alpha = 0.1$ RRC filtering. As previously mentioned, $PAPR_{const}$ is null for constant amplitude constellations, so without MM, the back-off equals the signal PAPR. This equality holds for a 8-PSK trellis shaping [5], given that the symbol's power is preserved. Results from Fig. 5(a) show that the MPMM technique produces a significant reduction, as desired, of the signal's dynamic range. The MPMM PAPR is similar to the trellis shaped method [5], but requires a much lower back-off. This means that, in practice, when restricting the maximum power at the HPA input, the power of a considerable number of MM transmitted symbols is much higher than in [5], where all symbols are transmitted with the same power, although both methods transmit the same average power. Though not obvious, this provides a major advantage: it minimizes degradation of bit error rate (BER) performance and so, it doesn't imply a reduction on the transmitted information rate. By applying this MPMM technique to current radio transmission systems on service, it is possible to take advantage of their systems' FEC capabilities to efficiently compensate the increased sensitivity to noise resulting from MM. This means that no further error control protection is required, so the link's information rate and the quality of service are preserved. Contrarily, the trellis technique [5] requires constellation redundancy, needing 1-bit (only for shaping control) per transmitted symbol, to achieve the better performance. Thus, as opposed to our technique, that method reduces the information rate of 8-PSK signalling by $2/3$, corresponding in practice to the information rate of a 4-ary constellation.

Fig. 5(b) shows the performance of the proposed MPMM scheme on the AWGN channel. Initial receiver's soft-decoding estimates (SDE) should take into account the symbol displacement statistics due to MM. A simply way to accomplish this, without changing the existing receivers, is to compute those estimates based on the expected average MM constellation (EAC). Fig. 5(b) shows that, for both codes, the BER performance loss is less than 0.5 dB at $\text{BER} = 10^{-6}$, considering a 3-stage MPMM scheme. Since the back-off reduction at $\text{CCDF}=10^{-5}$ is 5.8 dB, the power efficiency of the system is improved by more than 5.3 dB, when compared to the non-MM case. Note that, although using different codes, the performance is similar.

Fig. 5(a) also shows that, with just 2-stages, the need to use back-off is reduced by almost 95%. When considering 3-stages, it almost vanishes. The same fact is observed for different roll-off factors, as shown in Fig. 6, that plots back-off and PAPR reduction gains at $\text{CCDF} = 10^{-5}$. Note that the reduction on PAPR decreases for high roll-off values, although the need to use back-off is completely eliminated. This observation is easily understandable, considering the

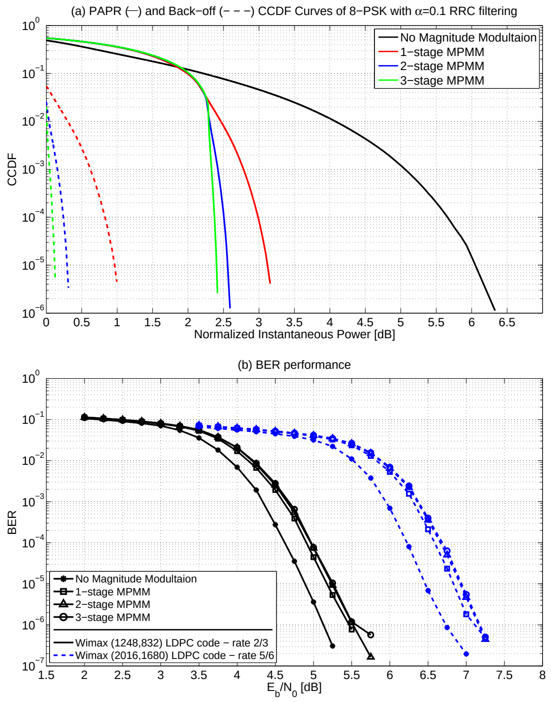

Figure 5 – Simulation results for an 8-PSK constellation with $\alpha = 0.1$ RRC filtering, considering MPMM with different number of stages. (a) CCDF of the signal's PAPR and the required back-off. (b) BER performance for the AWGN channel, when using the $(1248, 832)$ and $(2016, 1680)$ LDPC Wimax802.16e codes.

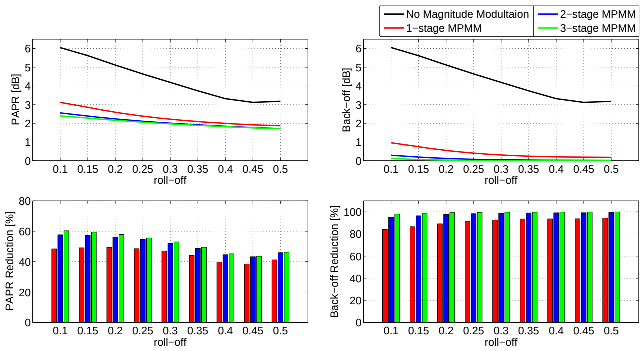

Figure 6 – PAPR and required back-off at CCDF $= 10^{-5}$ vs. roll-off, and resultant reduction gains (%), for 8-PSK signalling.

fact that for less stringent roll-offs, the need to adjust the symbol's amplitudes becomes weaker and therefore the average power doesn't decrease too much.

5. CONCLUSIONS

In this paper a novel multistage polyphase magnitude modulation (MPMM) scheme is proposed to efficiently control the envelope's power peak of single carrier band limited signals. Besides its simplicity and independency from the modulation used, this MPMM base-system block can be cascaded to improve performance, allowing a reduction in the required back-off greater than 95%, with just 2-stages. When compared to state-of-the-art trellis PAPR reduction techniques, this has shown similar PAPR reduction gains, with a major advantage: MPMM doesn't imply a penalty reduction on the information rate. A remarkable advantage of the proposed system is that it can be easily included in currently working transmitter systems, without the need to change existing receivers, while providing a significant improve in their power efficiency. As shown by simulation results, this improvement is greater than 5 dB, for the 8-PSK case exemplified.

APPENDIX

Given $a, b \in \mathbb{C}$ and $A \in \mathbb{R}^+$, we want to find the maximum value for $m \in [0;1]$ that satisfies

$$|ma + b| \leq A \tag{17}$$

- Case $|a + b| \leq A \;\Rightarrow\; m = 1$ (18)
- Case $|a + b| > A$, condition (17) is satisfied with equality when:

$$\begin{aligned} |ma + b| = A &\Leftrightarrow (ma + b)(ma + b)^* = A^2 \\ &\Leftrightarrow |a|^2 m^2 + 2\,\mathrm{Re}\{ab^*\} m + |b|^2 - A^2 = 0 \end{aligned} \tag{19}$$

An $\mathbb{R}^+$ solution for the second degree equation in m exists if $|b| \leq A$. Due to the asymmetrical partition of polyphase filter $E_i(z)$ in filters given by (10) and (11), this condition is always verified in practice. Combining (17) and (18) we define a non-negative function given by:

$$f(A, a, b) = \begin{cases} 1 & , |a + b| \leq A \\ \dfrac{-\mathrm{Re}\{ab^*\} + \sqrt{\mathrm{Re}\{ab^*\}^2 - |a|^2 (|b|^2 - A^2)}}{|a|^2} & , |a + b| > A \end{cases} \tag{20}$$

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