BLIND CHANNEL SHORTENING IN ZP-OFDM SYSTEMS WITH CONTROLLED TIR QUALITY

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ABSTRACT

We consider a zero padding OFDM system to which we apply channel shortening to cope with channel responses larger than the guard interval (GI) redundancy. In this article, we propose a blind channel shortening algorithm based on the restoration of zero padding (ZP) redundancy. A main contribution consists in the introduction of a new optimization constraint that allows us to control the target impulse response (TIR) quality and to improve significantly the system performance. Finally, we present a performance comparison between this new technique and similar existing techniques from the literature: MERRY and FRODO algorithms.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) system uses the guard interval (GI) to combat the inter symbol interference (ISI) and the intercarrier interference (ICI). The presence of the GI and Fourier transform in OFDM system ensures a low equalization complexity compared to single carrier systems. For this reason, many digital communication standards adopt OFDM system such that DVB, DAB, ADSL, etc. In OFDM system, the GI length must be higher than the channel size, otherwise, the ISI and ICI persist. To preserve a high effective flow and to eliminate the ICI and ISI, we must reduce the channel length by using channel shortening techniques. From the literature, different algorithms of blind channel shortening are known: the second order statistics based methods in [1, 2], Multicarrier Equalization by Restoration of Redundancy (MERRY) [3, 4], the Carrier Nulling Algorithm (CNA) [5] and the hybrid shortening method in [6]. In this paper, we consider the OFDM system where the GI is formed by zero padding (ZP). We start by proving that the restoration of the ZP redundancy allows us to achieve the desired channel shortening. However, our key result is that, considering the structure of ZP-OFDM symbols, we are able to choose an appropriate constraint that guarantees a good TIR quality. Simulation results are provided to illustrate the gain we obtain with the new optimization constraint.

We use in this paper the following notations: ^{T}, $*$ and ^{H} stand for transpose, conjugate and transconjugate, respectively. $\mathbf{0}_{a,b}$ and $\mathbf{I}_a$ are the $a \times b$ zero matrix and the $a \times a$ identity matrix. $E(.)$ denotes the mathematical expectation and $*$ represents the convolution operator.

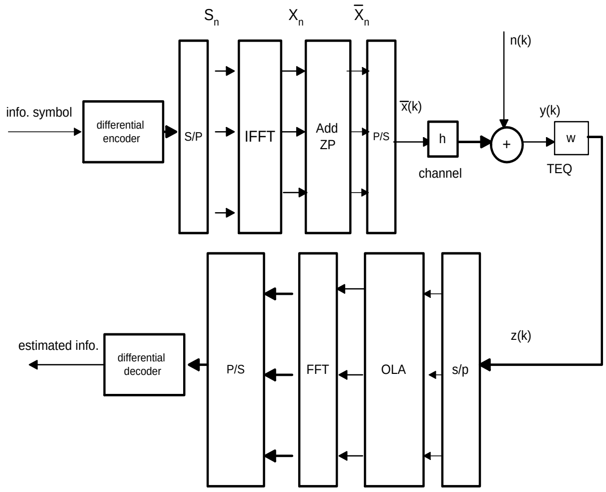

Figure 1: System model.

2. SYSTEM MODEL

In OFDM systems, the transmitted signal $s(k)$ is segmented into blocks of length N (N is the number of frequency bins):

$$\mathbf{s}_n = [s(nN), s(nN+1), \cdots, s(nN+N-1)]^T. \quad (1)$$

The block $\mathbf{s}_n$ is transformed into vector $\mathbf{x}_n$ by Inverse Fast Fourier Transform (IFFT), as shown in Fig. 1.

$$\mathbf{x}_n = \mathbf{F}^H \mathbf{s}_n \quad (2)$$

where $\mathbf{F}$ represents a $N \times N$ normalized Fourier matrix. Then, the GI redundancy (here, we consider zero padding) is added to $\mathbf{x}_n$ to form a vector

$$\bar{\mathbf{x}}_n = \left[\mathbf{x}_n^T, \mathbf{0}_{1,\nu}\right]^T, \ \nu \text{ being the size of the GI.}$$

Due to channel and noise effects, the received signal is given by :

$$y(i) = \sum_{l=0}^{L} h(l)\bar{x}(i-l) + b(i) \quad (3)$$

where $\mathbf{h} = [h(0), h(1), \cdots, h(L)]^T$ represents a finite impulse response channel, $\bar{x}(i)$ is the transmitted symbol and $b(i)$ is the observation noise. In this work, the channel memory is larger than the GI (i.e, $L > \nu$) and hence a shortening is needed to reduce the size of the channel and eliminate the

ISI. Thereafter, the received data is filtered by time domain equalizer (TEQ) $\mathbf{w} = [w(0), w(1), \cdots, w(q-1)]^T$ of degree $q-1$ to obtain the following equalized data:

$$z(i) = \sum_{l=0}^{q-1} w(l) y(i-l). \qquad (4)$$

We assume that:

- $\mathbf{h}$ is unknown.
- the data $s(k)$ and noise $b(k)$ are uncorrelated, zero mean and i.i.d. processes with variance σ_s^2 and σ_b^2, respectively.
- q does not exceed $N-L$.

The combined equalizer-channel impulse response is denoted by $c(n) = h(n) * w(n)$. The goal of the channel shortening is to approximately obtain:

$$\mathbf{c} = [\mathbf{0}_{1,d}, \mathbf{v}^T, \mathbf{0}_{1,(L+q-\nu-d)}]^T$$

where

$$\mathbf{c} = [c(0), c(1), \cdots, c(\underbrace{L+q}_{L_c}-1)]^T$$

$$\mathbf{v} = [v(0), v(1), \cdots, v(\nu-1)]^T$$

is the target impulse response (TIR) and d is the equalizer delay. Before introducing our shortening method, we recall first its counterpart for CP-based OFDM systems.

3. CP-BASED CHANNEL SHORTENING

3.1 MERRY and FRODO algorithms

These algorithms are dedicated for OFDM system where the GI is formed by Cyclic Prefix (CP). So, in this case, vector $\bar{\mathbf{x}}_n$ is given by:

$$\bar{\mathbf{x}}_n = [x_n(N-\nu), \cdots, x_n(N-1), x_n(0), \cdots, x_n(N-1)]^T.$$

MERRY algorithm consists in minimizing the square of the difference between one sample in the CP and its counterpart at the end of the equalized symbol. This allows us to achieve the channel shortening by restoring the CP-redundancy. In [3], the authors propose the following cost function:

$$J_{merry} = E\left[|z(nP+d+\nu-1) - z(nP+d+P-1)|^2\right]. \qquad (5)$$

where $P = N + \nu$. J_{merry} can be rewritten as:

$$J_{merry} = \mathbf{w}^H E\left[\check{\mathbf{y}}_n^* \check{\mathbf{y}}_n^T\right] \mathbf{w} \qquad (6)$$

where

$$\check{\mathbf{y}}_n = \begin{bmatrix} y(nP+d+\nu-1) - y((n+1)P+d-1) \\ y(nP+d+\nu-2) - y((n+1)P+d-2) \\ \vdots \\ y(nP+d+\nu-q) - y((n+1)P+d-q) \end{bmatrix}. \qquad (7)$$

Equation (6) is minimized subject to the unit norm constraint, i.e. $\|\mathbf{w}\| = 1$ or to the unit energy constraint on the combined channel $\|\mathbf{c}\| = 1$ [4] to avoid the trivial solution $\mathbf{w} = \mathbf{0}_{q,1}$.

An extension of MERRY algorithm called FRODO (Forced Redundancy with Optional Data Omission) was proposed recently in [4]. The main idea of this algorithm is to compare more than one sample in the CP to their counterparts at the end of the symbol. The cost function associated to FRODO algorithm is given by:

$$J_{frodo} = \sum_{i \in S_f} E\left[|z(nP+d+i) - z(nP+d+i+N)|^2\right] \qquad (8)$$

where $S_f \subset \{0, \cdots, \nu-1\}$ is appropriate an index set. In [4] , the authors prove that minimizing (8) allows to reduce the channel at length $\kappa = \nu - card(S_f) + 1$ where *card* means the cardinal function.

4. ZP-BASED CHANNEL SHORTENING (ZP-MERRY)

We consider here the OFDM system described in section 2 where the GI is formed by ZP. The proposed technique is a trivial extension of MERRY algorithm, consisting of the restoration of the "zero energy" of the last sample in each block. We refer to this method as Zero Padding Multicarrier Equalization by Restoration of Redundancy (ZP-MERRY) algorithm. Hence, we define the following cost function:

$$J_{zp}^1 = E\left[|z(nP+d+P-1)|^2\right]. \qquad (9)$$

We start first by proving theoretically that minimizing criterion (9) leads to the desired channel shortening.

Lemma 1: Assume that the assumptions made in section 2 are verified . Then, in noiseless case, criterion J_{zp}^1 satisfies:

$$J_{zp}^1 = 0 \quad \Leftrightarrow \quad \mathbf{c} = [\mathbf{0}_{1,d}, \mathbf{v}^T, \mathbf{0}_{1,(L_c-\nu-d)}]^T \qquad (10)$$

where $\mathbf{v} = [c(d), c(d+1), \cdots, c(d+\nu-1)]^T$.

Proof: In the noiseless case, the equalized signal after filtering by TEQ $\mathbf{w}$ can be written as:

$$z(n) = \mathbf{w}^T \begin{bmatrix} y(n) \\ \vdots \\ y(n-q+1) \end{bmatrix} = \mathbf{c}^T \begin{bmatrix} \bar{x}(n) \\ \vdots \\ \bar{x}(n-L_c+1) \end{bmatrix} \qquad (11)$$

Let $\bar{x}_n(i) = \bar{x}(nP+P+i)$. Then, the last equalized sample $z(nP+d+P-1)$ in the $n-$th block preceded by a delay d can be written as:

$$z(nP+d+P-1) = \mathbf{c}^T \begin{bmatrix} \bar{x}_n(d-1) \\ \vdots \\ \bar{x}_n(0) \\ \bar{x}_n(-1) \\ \vdots \\ \bar{x}_n(-\nu) \\ \bar{x}_n(-\nu-1) \\ \vdots \\ \bar{x}_n(d-L_c) \end{bmatrix} = \mathbf{c}^T \begin{bmatrix} \bar{x}_n(d-1) \\ \vdots \\ \bar{x}_n(0) \\ 0 \\ \vdots \\ 0 \\ \bar{x}_n(-\nu-1) \\ \vdots \\ \bar{x}_n(d-L_c) \end{bmatrix}. \qquad (12)$$

In the case where $\mathbf{c} = [\mathbf{0}_{1,d}, \mathbf{v}^T, \mathbf{0}_{1,(L_c-\nu-d)}]^T$, using the previous equation, we have in noiseless case:

$$z(nP+d+P-1) = 0 \qquad (13)$$

and hence $E\left[|z(nP+d+P-1)|^2\right] = 0$. This proves the first part of the equivalence:

$$\mathbf{c} = [\mathbf{0}_{1,d}, \mathbf{v}^T, \mathbf{0}_{1,(L_c-\nu-d)}]^T \Longrightarrow J_{zp}^1 = 0.$$

Since $\mathbf{x}_n$ is zero mean and i.i.d. with variance σ_s^2. We can rewrite J_{zp}^1 as:

$$\begin{aligned} J_{zp}^1 &= E\left[|z(nP+d+P-1)|^2\right] \\ &= \sigma_s^2 \mathbf{c}^H \mathbf{T} \mathbf{c} \end{aligned} \qquad (14)$$

where $\mathbf{T}$ is $L_c \times L_c$ matrix defined as:

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_d & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_\nu & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{L_c-\nu-d} \end{bmatrix}, \qquad (15)$$

which shows clearly that:

$$J_{zp}^1 = 0 \Longrightarrow \mathbf{c} = [\mathbf{0}_{1,d}, \mathbf{v}^T, \mathbf{0}_{1,(L_c-\nu-d)}]^T.$$

We conclude that the restoration of the ZP property is equivalent to channel shortening at length ν. ◇

Remark: Similarly to [4], ZP-MERRY can be extended by forcing more than one sample of the GI to be zero. We propose to refer to this method as ZP-FRODO algorithm. So, we consider the following cost function:

$$J_{zp}^2 = \sum_{i \in S_f} E\left[|z(nP+d+P-i)|^2\right] \qquad (16)$$

where $S_f = \{1, 2, \cdots, \kappa\}$, with $1 \leq \kappa \leq \nu$.
Lemma 2: Using the same assumptions made in section 2, in the noiseless case, criterion J_{zp}^2 satisfies:

$$J_{zp}^2 = 0 \Leftrightarrow \mathbf{c} = [\mathbf{0}_{1,d}, \mathbf{v}^T, \mathbf{0}_{1,(L_c-\nu-d+\kappa-1)}]^T \qquad (17)$$

where $\mathbf{v} = [c(d), c(d+1), \cdots, c(d+\nu-\kappa)]^T$.
Proof: the proof is similar to that of lemma 1.

5. OPTIMIZATION WITH CONTROLLED TIR QUALITY

Using (11) and (12), the expression of J_{zp}^1 can be written in function of $\mathbf{w}$ as:

$$J_{zp}^1 = \mathbf{w}^H E\left[\hat{\mathbf{y}}_n^{d-1}(d-1)\hat{\mathbf{y}}_n^T(d-1)\right]\mathbf{w} \qquad (18)$$

where

$$\hat{\mathbf{y}}_n(i) = [y(nP+P+i), \cdots, y(nP+P+i-q+1)]^T. \qquad (19)$$

As in [3, 4], to avoid the trivial solution $\mathbf{w} = \mathbf{0}_{q,1}$, the previous quadratic criterion can be minimized subject to unit-norm constraint $\|\mathbf{w}\| = 1$ or the unit-energy constraint on the combined channel, $\|\mathbf{c}\| = 1$. However, by doing so, we cannot control the quality (i.e. flatness) of the TIR frequency spectrum. It is known that the ideal case of a flat spectrum corresponds to a filter with only one non-zero tap. Also, note that after channel shortening and thanks to the ZP, the first sample in OFDM symbol is proportional (in the noiseless case) to $v(0)$ (the first TIR tap), i.e.:

$$z(nP+d) = v(0)x(nP). \qquad (20)$$

Hence, by maximizing its averaged power, we maximize the first tap amplitude, getting closer to the ideal filter. This constraint optimization problem is equivalent to solving the following Rayleigh quotient:

$$\begin{aligned} \mathbf{w}_{opt} &= \arg\min_{\mathbf{w}} \frac{\mathbf{w}^H E\left[\hat{\mathbf{y}}_n^*(d-1)\hat{\mathbf{y}}_n^T(d-1)\right]\mathbf{w}}{E\left[|z_n(d)|^2\right]} \\ &= \arg\min_{\mathbf{w}} \frac{\mathbf{w}^H E\left[\hat{\mathbf{y}}_n^*(d-1)\hat{\mathbf{y}}_n^T(d-1)\right]\mathbf{w}}{\mathbf{w}^H E\left[\hat{\mathbf{y}}_n^*(d)\hat{\mathbf{y}}_n^T(d)\right]\mathbf{w}}. \end{aligned} \qquad (21)$$

Solving (21) consists in estimating the least generalized eigenvector of matrices $\mathbf{R}_1 = E\left[\hat{\mathbf{y}}_n^*(d-1)\hat{\mathbf{y}}_n^T(d-1)\right]$ and $\mathbf{R}_2 = E\left[\hat{\mathbf{y}}_n^*(d)\hat{\mathbf{y}}_n^T(d)\right]$. This estimation can be achieved in a fast and effective way by using the algorithm in [7].

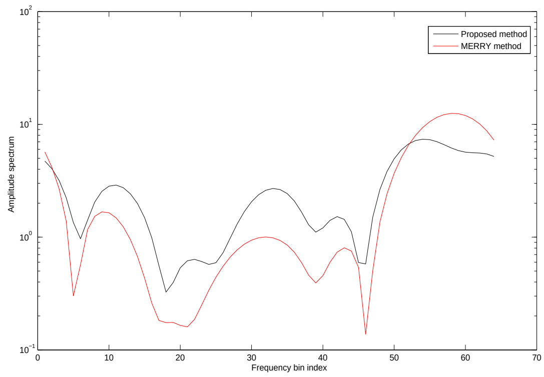

Figure 2: Amplitude spectrum of shortened channel for proposed and MERRY methods.

Remarks:

1. Note that equality (20) holds only in the ZP-OFDM case. Hence, the proposed constraint cannot be used for CP-OFDM systems. This is a main advantage, w.r.t. blind channel shortening, for the ZP-OFDM as compared to CP-OFDM.
2. In the proposed criterion, we do not force the "'flatness"' of the target channel shortening but simply improve it via the use of the constraint. This is illustrated by Fig. 2. where we observe clearly that the frequency selective strong channel fading are mitigated thanks to the new optimization criterion in (21).

6. SIMULATION

In this section, a series of simulations is conducted to study and compare the performance of the considered algorithms. We consider an OFDM system with $N = 64$ subcarriers. $\nu = 8$ represents either CP or ZP samples. A channel of length $L+1 = 20$, is generated randomly such that its taps are zero-mean complex Gaussian variables with variances $\sigma_l^2 = \lambda exp(-\alpha l)$, $l = 0 \cdots L$, where $\alpha = 0.4$ and λ ensures the unit energy of $\mathbf{h}$. A TEQ with $q = 24$ taps is used. $N_b = 200$ OFDM symbols are generated at each run using a scalar differential phase-shift keying (DPSK) encoder. Fig. 3 illustrates the original channel and the combined equalizer-channel impulse response obtained using ZP-MERRY and ZP-FRODO, respectively at SNR $= 20$dB and with delay $d = 5$. It is clear that ZP-FRODO algorithm reduces more

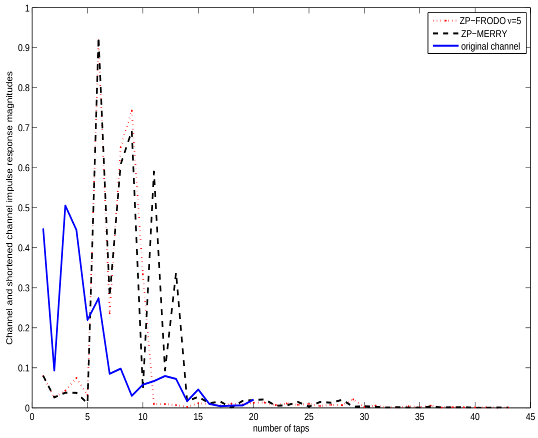

Figure 3: Channel and Shortened channel at 20dB using ZP-MERRY and ZP-FRODO with new constraint at delay $d = 5$.

the length of TIR than ZP-MERRY algorithm. Fig. 4 displays the overall BER performance corresponding to an SNR range of $[5, 25]$ dB for scalar differential encoding using the channel given in Fig. 3. At each SNR, the BER is averaged using 200 realizations and the delay $d = 0$ is used. We note that the proposed method with the new constraint has a much lower BER as compared to the other methods. We note also that the proposed method (ZP-MERRY) with unit energy constraint $||\mathbf{w}|| = 1$ offers the same performance as MERRY algorithm.

In Fig. 5, the channel changes every 200 OFDM symbols and the equalization delay is $d = 0$. At each SNR, the BER is averaged over 200 random channel realizations. We note again that ZP-MERRY with the new constraint has a lower BER than the other techniques especially at high SNR. Fig. 6, compares the performance in terms of BER of MERRY with unit-energy constraint of combined channel $||\mathbf{c}|| = 1$ and MERRY with the constraint $||v(0)|| = 1$[1]. The later constraint is considered here to illustrate its usefulness but is not practical in this context since it relies on the knowledge of $\mathbf{h}$. As we can see, a significant performance gain is observed thanks to the new considered constraint. In Fig. 7, we compare FRODO and ZP-FRODO algorithms for different TIR sizes. This figure presents the performance of these algorithms at 20 dB as function of the length of TIR when using the fixed channel in Fig.2. We noted that ZP-FRODO with new constraint has almost the same performance as ZP-MERRY (i.e ZP-FRODO with $v = 8$) for the range $[4, 8]$. So, we can conclude in this context that using ZP-FRODO is not necessary because this algorithm increases the computation complexity without significant gain in terms of BER. Similarly, we can observe that FRODO and ZP-FRODO algorithms with unit energy constraint $||\mathbf{w}|| = 1$ offer lower BER than MERRY or ZP-MERRY with the same constraint.

[1]Minimizing criterion (6) under the quadratic constrain $\|\mathbf{v}(\mathbf{0})\| = 1$, is equivalent to minimizing the Rayleigh quotient criterion $\frac{J_{merry}}{\|v(0)\|^2}$.

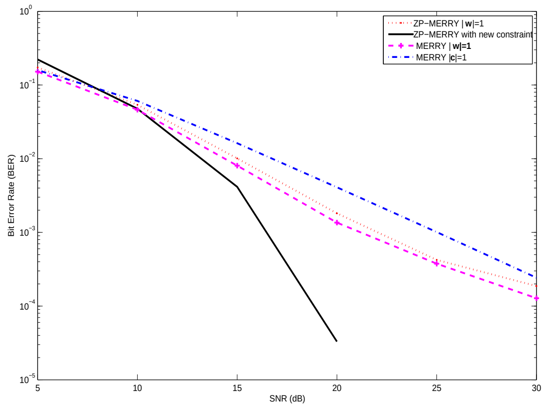

Figure 4: BER vs. SNR using scalar differential encoding with a fixed channel at delay $d = 0$.

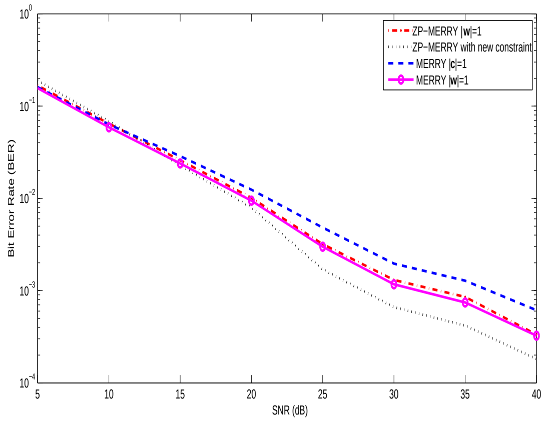

Figure 5: BER vs. SNR using scalar differential encoding with 200 random channel realizations.

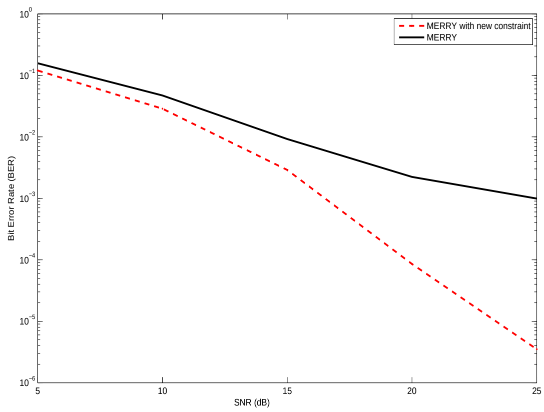

Figure 6: BER vs. SNR using scalar differential encoding with with a fixed channel.

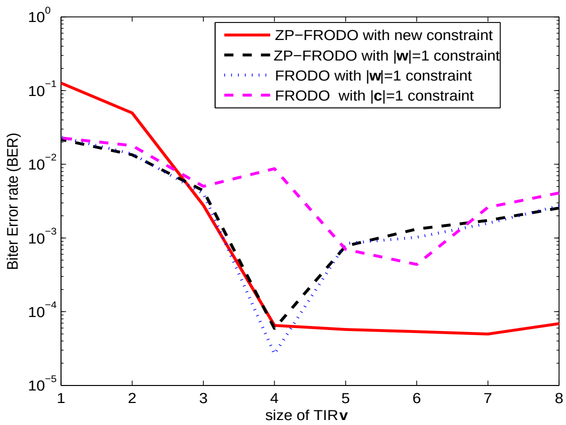

Figure 7: BER vs. size of the TIR using a FRODO and ZP-FRODO with a fixed channel at SNR = 20 dB.

7. CONCLUSION

In this article, we introduce a blind channel shortening technique for ZP-OFDM system with controlled TIR quality. We first prove theoretically that the proposed method leads to the desired channel shortening. Then, we introduce an appropriate constraint that allows us to better control the TIR quality in terms of frequency spectrum flatness. Simulation results and comparison with MERRY and FRODO algorithms confirm the effectiveness of the new channel shortening method.

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