QUANTITATIVE EVALUATION OF CONCENTRATED TIME-FREQUENCY DISTRIBUTIONS

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ABSTRACT

This work objectively evaluates and presents a quantitative analysis of concentrated time-frequency distributions (TFDs) obtained through Network of Expert Neural Networks (NENNs). The objective methods include the ratio of norms based measures, Shannon & Rényi entropy measures, normalized Rényi entropy measure and Jubisa measure. The introduction of these measures allows quantifying the quality of TFDs instead of relying solely on visual inspection of their plots. Performance comparison with various other quadratic TFDs is provided too.

1. INTRODUCTION

The analysis of time varying signals has important applications in acoustics, speech, communications, geophysics, biomedicine, and many other areas of science and engineering. To analyze the evolving spectra of such signals, the TFDs are used as primary tool [1]. By distributing the signal energy over the time-frequency (t-f) plane, TFDs provide the analyst with information like number of components with their time duration and frequency bands based on type of signal being analyzed, their relative amplitude, phase information, and the instantaneous frequency (IF) laws, which are unavailable from the signal time or frequency domain representations.

Choosing the right TFD to analyze the given signal is not straightforward, even for monocomponent signal, and becomes more complex while dealing with multicomponent signals. Various bilinear distributions (BDs) like the spectrogram (spec), Wigner-Ville Distribution (WVD), Choi-Williams distribution (CWD), and Born-Jordan distribution (BJD) are shown in Fig. 1, which represent a real life multicomponent bat echolocation chirp signal [12] in the t-f domain. A common practice to determine the best TFD for the given signal has been the visual comparison. We can see less interference and better component separation for spec and CWD than the other considered TFDs. However this selection is generally difficult and subjective. The need to objectively compare the plots in Fig. 1 requires the definition of a quantitative performance measure for TFDs. Some theoretical measures that deal essentially with signal concentration have been proposed in literature [10-14].

Concentration of a TFD is one of TFDs' very important and extensively studied properties [1, 2, 8, 13]. However it has been shown that the BDs including the spec results in a blurred version of the true TFD [1, 2]. The spec suffers from the window effect which governs its resulting t-f resolution. Combination of the specs or the spec with adaptive window selection may just reduce the blurring effect. The WVD is a prototype of distribution that is qualitatively different from the spec. It produces ideal concentration along the IF for linear frequency modulated (FM) signals but presence of cross terms reduces their practical effectiveness. Moreover If the IF variations are of a higher order than linear then WVD cannot produce the ideal concentration.

To compute a TFD that is free of any blurring effect, with no initial knowledge of the components, network of expert neural networks (NENNs) are employed in [2]. The method employs Bayesian regularization in training the neural networks to obtain energy concentration along the IF of individual components for unknown blurred TFDs. Fig. 2 is the block representation of the method. Though the visual results were indicative of TFDs' high concentration but the work lacked quantitative analysis. By going further in study and analyzing more complex TFDs, this paper uses some important objective criteria to measure the information content of the output TFDs of NENNs (henceforth the NTFDs) for a practical analysis. The focus is on various existing measures including the *ratio of norms based measures* [13], *Shannon & Rényi entropy measures* [3-4], *normalized Rényi entropy measure* [11] and *Jubisa measure* [9]. The paper includes comparison of the information content captured through these measures of most commonly used BDs with the proposed method.

In this context, it is shown that the NTFDs perform better than other TFDs for multicomponent signals with components closely spaced in the t-f plane. In other signal examples, it is at least as good as others. Also the NTFDs are found to be more informative than the TFDs obtained through simple neural network (SNN), trained without incorporating Bayesian regularization and clustering. The main objective of the paper is to verify the effectiveness of Bayesian regularized NENNs for estimation of highly concentrated, informative and de-blurred TFDs.

2. OBJECTIVE CRITERIA TO MEASURE THE CONCENTRATION OF TFDs

An efficient quantitative criterion to evaluate performances of different distributions can be obtained by TFD concentration measurement. Gabor [5], Vakman [6], Janssen [7], and Cohen [1] made important initial contributions to measure distribution concentration for monocomponent signals. For more complex signals, some quantities in statistics were the inspiration for defining measures for TFDs in various forms such as: the ratio of distribution norms by Jones and Parks [13], the Rényi entropy by Williams et al. [11], and distribution energy for optimal kernel distributions design by Baraniuk and Jones [8]. A simple measure for a distribution's concentration was presented by L. Stankovic [9] based on definition of duration of the time limited signals. A brief overview of these measures is presented next.

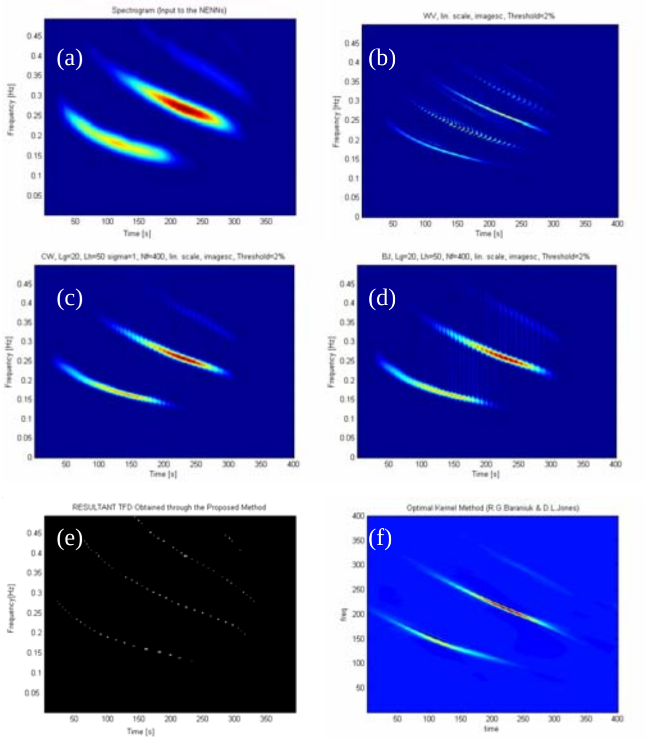

Fig. 1: TFDs of a multicomponent bat echolocation chirp signal. (a) Spec (Test Input to the NENNs)[Hamming window of length L=100], (b) WVD, (c) CWD [kernel width =1], (d) BJD, (e) NTFD [2], (f) optimal kernel TFD [8].

2.1 Ratio of Norms based Measures

An approach to get good t-f concentration [13], adapts the parameters of window to maximize a measure of concentration created by dividing the fourth power norm of TFD $Q(n,\omega)$ by its second power norm, given as:

$$E_{JP} = \sum_{n=0}^{N-1}\sum_{\omega=0}^{W-1}\left|Q(n,\omega)\right|^4 \Bigg/ \left(\sum_{n=0}^{N-1}\sum_{\omega=0}^{W-1}\left|Q(n,\omega)\right|^2\right)^2 \qquad (1)$$

The fourth power in the numerator favors a peaky distribution [13]. To obtain the optimal distribution for a given signal, the value of this measure should be the maximum

2.2 Entropy Measures

A more promising approach to complexity based on entropy functionals has been useful in quantifying the information content of time-varying signals. The peaky TFDs of signals with high concentration would yield small entropy values and vice versa [10].

2.2.1 Shannon Entropy

The well known *Shannon entropy* [3] for TFD of unit energy signals, can be written as

$$E_{Shannon} = -\sum_{n=0}^{N-1}\sum_{\omega=0}^{W-1} Q(n,\omega)\log_2\left(Q(n,\omega)\right) \qquad (2)$$

The negative values taken on by most TFDs prohibit the application of the *Shannon entropy* due to the logarithm in (2). By taking into account the absolute value of the distribution ensures that the integrated logarithm exists.

2.2.2 Rényi Entropy

It is introduced as a more appropriate way of measuring the t-f uncertainty sidestepping the negativity issue, derived from the same set of axioms as the *Shannon entropy* [4, 14], given as

$$E_{RENYI_\alpha} = \frac{1}{1-\alpha}\log_2\left(\sum_{n=0}^{N-1}\sum_{\omega=0}^{W-1} Q^\alpha(n,\omega)\right) \qquad (3)$$

where α is the order of *Rényi entropy*, which for the purpose of this paper has been taken as 3 being the smallest integer value to yield a well-defined, useful information measure for a large class of signals.

2.3 Normalized Entropy Measures

The Rényi entropy measure with $\alpha = 3$ does not detect zero mean cross terms, so normalization either with signal energy or distribution volume is necessary. By definition Rényi entropy normalized by distribution volume is given by:

$$E_{RVNorm_\alpha} = \frac{1}{1-\alpha}\log_2\left(\frac{\sum_{n=0}^{N-1}\sum_{\omega=0}^{W-1} Q^\alpha(n,\omega)}{\sum_{n=0}^{N-1}\sum_{\omega=0}^{W-1}\left|Q(n,\omega)\right|}\right) \text{ with } \alpha \ge 2 \qquad (4)$$

2.4 Jubisa Measure

A simple criterion for objective measurement of TFD concentration is presented by L. Stankovic in [9], which can also be used in automatic determination of some TFDs' parameters. The basic idea comes from the obvious definition of the duration of time limited signals. The *Jubisa concentration measure* in the discrete form is given as:

$$J\left[Q(n,\omega)\right] \equiv \left(\sum_{n=0}^{N-1}\sum_{\omega=0}^{W-1}\left|Q(n,\omega)\right|^{1/\eta}\right)^\eta \qquad (5)$$

The best choice according to this criterion is the distribution that produces the minimal value of $J\left[Q(n,\omega)\right]$.

3. THE REALIZATION OF HIGHLY CONCENTRATED TFDs

The algorithm for the realization of highly concentrated TFDs includes three major steps: pre-processing of the input data, training/testing of NENNs, and post-processing of the output data [2]. Fig. 2 is a block diagram of the method.

3.1 Brief Description of the Algorithm

3.1.1 Step 1

The blurred specs and highly concentrated WVD of various known signals constitute the training set for BRNNs. These training TFDs are converted to vectors of particular length. This procedure is repeated for both input and target TFDs. Input vectors of specified length from blurred image and the mean values of vectors of same length from the corresponding region of the concentrated target image are accumulated.

The vectors obtained from the blurred specs are clustered according to certain underlying image features. The objective is to divide the input vector space into number of sub spaces, S_n, described by directional unit vectors, $\mathbf{v}_n$. A vector will lie in the subspace S_n represented by $\mathbf{v}_n$ that is most similar to this vector with respect to its information content. In our case, $\mathbf{v}_n$ are three directional vectors, used to characterize three types of edges (ascending, descending, triangular) in the image.

3.1.2 Step 2

Two simple signals are used to train the multiple neural networks. The first training signal produces linearly increasing parallel chirps given by:

$$x_1(n) = e^{j(\pi n/4N)n} + e^{j\left(\frac{\pi}{3}+\frac{\pi n}{4N}\right)n} \quad (6)$$

while the second signal is the sinusoidal FM signal given by:

$$x_2(n) = e^{j\pi\left(\frac{1}{2}-\left(0.1\sin\left(2\pi n/N\right)\right)n\right)} \quad (7)$$

here N refers to the number of sampling points.

By keeping track of the network error/performance, accessible via the training record, the best network termed as the expert networks (ENs) can be selected in terms of training performance for each cluster. These ENs constitute the NENNs, which are then fed with the test image vectors.

3.1.3 Step 3

This is exactly the reverse of the first step, which includes de-clustering and formulation of the resultant TFD from the output vectors, by placing them at appropriate pixel positions.

4. EXPERIMENTAL RESULTS

Three examples including both real life and synthetic multicomponent signals are considered. Quantitative analysis is performed for measuring t-f uncertainty using the criteria described in section II. The aim has been to find, based on these measures, the best concentrated and thus an informative TFD.

4.1 Real Life Test Case.

Real life data for the bat echolocation chirp sound provides an excellent test case for it is a multicomponent signal [12]. The nonstationary nature of the signal is only obvious from its TFD, whereas neither the time nor the frequency domain representations convey this information. The spec of the signal referred to as test image 1(TI 1), is shown in Fig. 1(a). The resultant NTFD is depicted as Fig. 1(e) along with other popular BDs and the existing optimal kernel method (OKM) [8] for a visual comparison. On close monitoring, it is revealed that no distribution, except the spec and NTFD, is able to recover the fourth chirp, thus losing some useful information about the signal. Whereas the NTFD is not only highly concentrated along the IF of the individual components present in the signal but also is more informative being able to show all the components.

The slices of the input/output TFDs of NENNs are taken at the time instants n=150 and n=310 (recall that n=1,2,…,400) and the normalized amplitudes of these slices are plotted in Fig. 5. The peaky appearance of the three frequencies present in the signal (see Fig. 1(e)) at these time instants can be seen. There are no cross terms and the results of the proposed method offer better frequency resolution. It is important to highlight that on visual inspection, the NTFD not only has the best resolution i.e. (narrower main lobe and no side lobes) but also successfully recovers the fourth chirp (the weakest) compared to all the other considered distributions in Fig. 1.

4.2 Synthetic Test Cases.

In this paper, two synthetic signals of different nature with specs referred to as test image 2 (TI 2) and test image 3 (TI 3), shown in Fig. 3 are fed as the other two test cases to the trained NENNs. The first one is the synthetic signal consisting of two components sinusoidal FM intersecting each other, given as:

$$X(n) = e^{-j\pi\left(\frac{5}{2}-\left(0.1\sin\left(2\pi n/N\right)\right)n\right)} + e^{j\pi\left(\frac{5}{2}-\left(0.1\sin\left(2\pi n/N\right)\right)n\right)} \quad (8)$$

The second synthetic signal produces multiple nonparallel, nonintersecting chirps once seen on the t-f plane. Mathematically it can be written as:

$$Y(n) = e^{j(\pi n/6N)n} + e^{j\left(\pi+\frac{\pi n}{6N}\right)n} + e^{-j(\pi n/6N)n} + e^{-j\left(\pi+\frac{\pi n}{6N}\right)n} \quad (9)$$

The estimated NTFDs are shown in Fig. 4. The visual results are indicative of their high concentration and good resolution.

However to verify the performance, quantitative assessment by various methods including the *ratio of norms based measures*, *Shannon* & *Rényi entropy measures*, *normalized Rényi entropy measure* and *Jubisa measure* is performed. The resulting values are recorded in Table I for a comparative analysis. The values that represent the best t-f concentration and resolution according to the different criteria have been indicated in bold italics. It is important to mention that the numeric values of the various measures, except simple Rényi entropies, indicate that the NTFDs' performance is better than others for all the examples. Though simple Rényi entropy value is the minimum for the ZAMD but literature [4, 14] indicates the measure's drawbacks and suggests usage of the volume normalized Rényi entropies, which however refer to the NTFDs as the highly concentrated.

To get a clear picture, these measures are independently plotted for various TIs in Fig. 6, which confirm that NTFDs are the most informative. The congruence and regular nature of the curves are very obvious in these plots which ascertain the validity of the selected objective criteria.

5. CONCLUSIONS

In this paper, a quantitative analysis is presented for the evaluation of the Bayesian regularized NENNs for the estimation of informative and highly concentrated TFDs of multicomponent signals whose frequency components vary with time, using both synthetic and real life signals. We have used the objective criteria to compare the concentration performance of TFDs for multicomponent signal analysis thus using a quantitative measure of goodness for TFDs instead of relying solely on the visual inspection of their plots.

The resultant TFDs are compared with some popular distributions known for their high cross terms suppression and high energy concentration in the t-f domain. It has been shown that the TFDs obtained through the proposed method exhibit high resolution, no interference terms between the signal components and are highly concentrated. Also they are found to be better at detecting the number of components in a given signal compared to the conventional distributions. It is however found that the resulting TFDs are not valid energy distributions because they do not observe the signature continuity and marginal characteristics or weak signal mitigation. Due to this reason, the results may not be feasible for certain applications which may have different preferences and requirement to the TFDs. This aspect may be attributed to the discontinuous target data and is expected to improve by a possible use of better pre and post-processing of the data to get TFDs without discontinuities along the IF of the individual components.

Essentially this work merely scratches the surface of potential application of objective criteria in t-f analysis. Worthy of pursuit seems the axiomatic derivation of an application of the *ideal* t-f complexity measure along the lines of Jones and Parks for devising the ratio of distribution norms [13], Baraniuk and Jones's effort in defining optimal kernel distributions' design [8], Rényi's work in probability theory [4] and investigate other possible measures.

A further research direction may be to check the effect of improved clustering methodologies like fuzzy or unsupervised techniques and analyze more complex signals, embedded in additive noise. Also a separate work is needed for the signals that are not linear or sinusoidal chirps to see how performance of the algorithm is affected. To come up with a parametric representation of the IF of individual components of a multicomponent signal through neural networks may be another future work.

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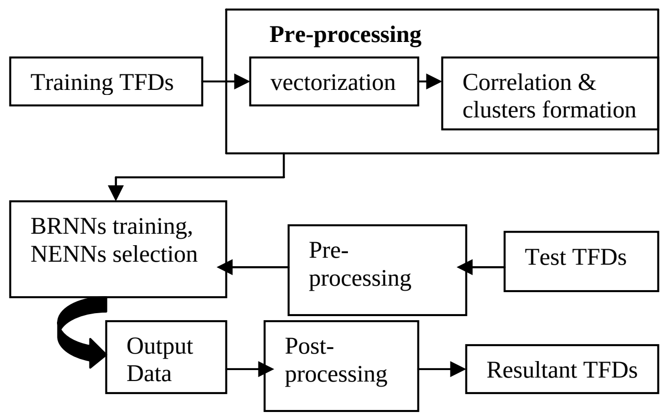

Fig. 2: Block diagram of the method

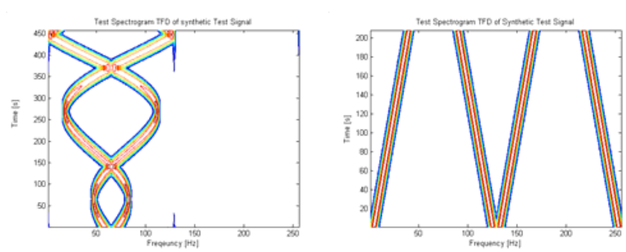

Fig. 3: Test TFDs of two synthetic signals given as input to the trained NENNs

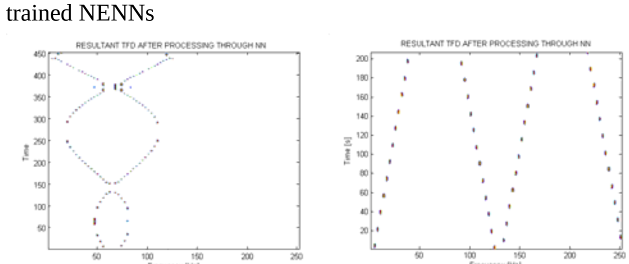

Fig. 4: Resultant NTFDs for the two synthetic test signals

Table I: Comparison of information criteria for various TFDs

Description	*Test TFDs*	*Ratio of Norm based measure ($x10^{4}$)*	*Rényi entropy*	*Normalized Rényi entropy*	*Jubisa measure ($x10^{5}$)*
Spec	*TI 1*	3.81	12.45	12.45	0.2219
	TI 2	1.94	12.98	12.98	0.1600
	TI 3	51.23	17.07	17.07	6.03
WVD	*TI 1*	3.84	10.90	12.02	3.30
	TI 2	1.91	9.95	11.62	4.68
	TI 3	***58.00***	14.01	16.28	47.05
ZAMD	*TI 1*	2.94	***7.00***	9.18	13.14
	TI 2	2.18	***7.56***	9.54	5.62
	TI 3	1.02	8.62	11.35	39.64
MHD	*TI 1*	1.05	11.47	12.75	2.9200
	TI 2	1.10	11.03	12.26	1.18
	TI 3	48.71	14.74	16.70	36.47
CWD	*TI 1*	2.89	12.67	12.93	1.06
	TI 2	3.10	12.06	12.60	1.01
	TI 3	38.53	16.24	16.77	33.08
BJD	*TI 1*	2.73	12.54	12.85	1.01
	TI 2	4.67	11.85	12.38	0.89
	TI 3	26.37	15.84	16.41	29.39
NTFDs	*TI 1*	***66.00***	7.26	***7.26***	***0.0015***
	TI 2	***24.00***	8.74	***8.74***	***0.0024***
	TI 3	44.00	***7.85***	***7.85***	***0.0043***
SNN	*TI 1*	43.88	9.25	12.97	0.0912
	TI 2	18.12	10.89	11.68	0.0145
	TI 3	33.90	12.82	14.49	0.9973
OKM [8]	*TI 1*	8.32	11.65	11.77	0.6300
	TI 2	1.59	13.82	13.98	8.6564
	TI 3	10.26	17.22	17.43	14.73

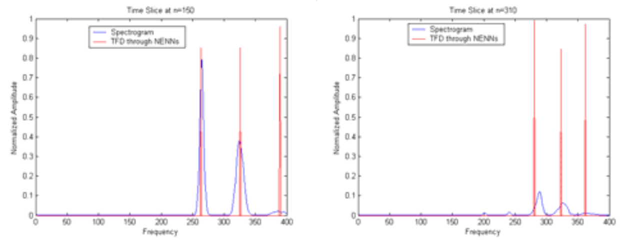

Fig. 5: Time slices for the spec (blue) and the NTFD (red) for bat echolocation chirp signal, at n=150 (left) and n=310 (right)

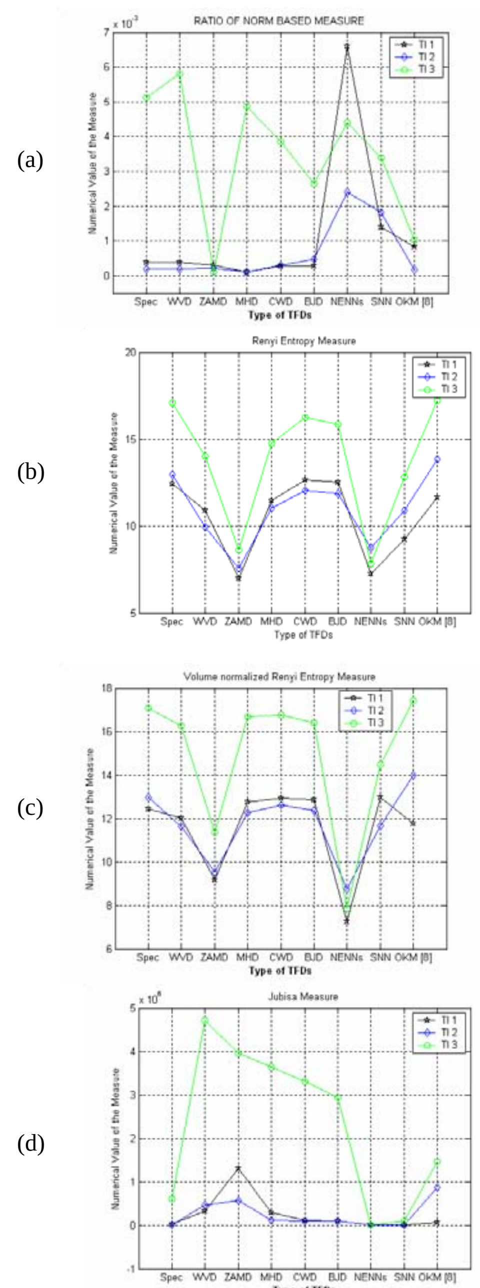

Fig. 6: Comparative plots for the three test images, (a) Ratio of Norm based measure, (b) Rényi entropy measure, (c) Volume normalized Rényi Entropy measure, and (d) Jubisa measure