FIXED-COMPLEXITY REGULARIZED VECTOR PRECODING FOR THE MULTIUSER MIMO DOWNLINK CHANNEL

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ABSTRACT

A fixed-throughput vector precoding (VP) approach specially suitable for hardware implementation in field programmable gate array (FPGA) devices is presented in this paper. The algorithm, which is based on the sphere encoder (SE), is divided into two main stages: at the preprocessing stage, the columns of the precoding matrix are ordered following any of the ordering approaches that have been proposed in this paper. Secondly, the search tree is configured so as to yield an appropriate bit error rate (BER) performance. Simulation results show that the BER performance of the proposed algorithm is very close to that of the SE, whereas its complexity is significantly smaller.

1. INTRODUCTION

The performance of wireless networks must be vastly improved if the great challenges of the emerging applications are to be met. One of the most promising approaches is the incorporation of multiple transmit and receive antennas to rise the transmission data rate and to combat the hostility of the radio channels. This is known as multiple-input multiple-output (MIMO) systems. Most of the work published on MIMO so far has been focused on point-to-point communications. However, in the last few years the interest in MIMO has evolved to the development of multiuser schemes which consider more complex albeit realistic scenarios with multiple terminals sharing the time, space, wideband and power resources available in a wireless network.

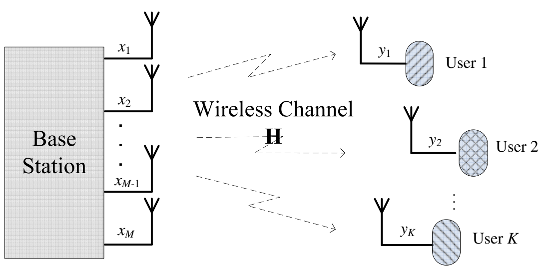

Figure 1: A broadcast multiuser MIMO channel with M transmit antennas and K single-antenna users.

Generally speaking, the MIMO multiuser environment is composed of two channels that communicate the base station with the user terminals: the multiple access channel (MAC), also known as the uplink channel, covers the communication from the terminals to the base station, whereas the broadcast channel, or downlink channel, carries the transmissions that stem from the base station and end at the users' terminals (Figure 1).

Focusing on the latter scenarios, the lack of cooperation between terminals in the signal detection stage is the main cause that prevents those detection techniques designed for single user schemes from being applied into multiuser MIMO environments. However, when performing a preprocessing stage on the signal at the base station, it is possible to avoid interference at the user terminals so that the information can be obtained without any interaction between them [1]. This is referred to as *precoding*, which can be either linear or non-linear.

Among the linear approaches, zero-forcing precoding [2] and regularized precoding [3] are some of the most simple and effective schemes. The first one premultiplies the signal to be transmitted by the inverse of the channel, whereas the latter is the regularized version of zero-forcing precoding. Another popular linear approach is the Wiener filter precoding [2] which achieves minimum mean square error (MMSE) performance.

The non-linear precoding approaches for multiuser scenarios are based on the concept of *writing on dirty paper* introduced by Costa [4]. For instance, Tomlinson-Harashima precoding (THP) cancels the interference between streams in a sequential fashion by means of a feedback filter [5]. However, the most important feature of this precoding approach is the insertion of a modulo operation, in both the transmitter and the receiver, to reduce the unscaled transmit power. The modulo operation at the transmitter can be equivalently replaced by the addition of a perturbing signal. This leads to the idea of vector precoding [6], which optimizes the perturbing signal to be added directly, as opposed to the iterative procedure used in THP. In order to find the perturbation vector lattice techniques must be applied. One of the most popular approaches for the search of the perturbing signal is the SE. This algorithm was successfully applied in [6] to obtain the perturbation signal required for VP. Other approaches employ lattice-reduction techniques to simplify the search for the closest lattice point [7].

This paper aims to introduce a vector precoding system which applies a fixed-complexity SE, or fixed SE (FSE), lattice search technique for the computation of the perturbing vector. The decoder version of the FSE, the fixed-complexity sphere decoder (FSD), was originally developed for MIMO detection single user scenarios [8]. In this reference, simulation results show that the proposed approach achieves nearly-optimum performance with a significant reduction in computational cost. Moreover, being it a fixed-complexity system, it is highly suitable for hardware implementation on FPGAs, where the parallelization and pipelining of resources can be applied to enhance the system throughput.

This contribution is organized as follows: in Section 2 the system model is introduced. The complexity of the SE at the transmitter side in relation to different matrix orderings is analyzed in Section 3, while the fixed-complexity search technique for VP is described in Section 4. Numerical results are given in Section 5 and some concluding remarks are drawn in Section 6.

2. SYSTEM MODEL

Consider a MIMO broadcast channel with M antennas at the transmitter and K single-antenna users, denoted as $M \times K$ (Figure 1). We assume that the channel between the base station and the K users is

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represented by a complex matrix $\mathbf{H} \in \mathbb{C}^{K\times M}$, whose element $h_{k,m}$ represents the channel gain between transmit antenna m and user k. The entries of the channel matrix are such that $E[|h_{k,m}|^2] = 1$, being $E[.]$ the expectation operator. The received data at the K users can be written by using a vector equation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w},$$

where $\mathbf{y} = [y_1, \ldots, y_K]^T$ represents the data received at the K users and $[.]^T$ denotes the transpose operator. The transmitted data vector is contained in $\mathbf{x} = [x_1, \ldots, x_M]^T$ and $\mathbf{w} = [w_1, \ldots, w_K]^T$ is the additive white Gaussian noise (AWGN) vector added to the signal at the user terminals. The transmitted power at the base station E_{tr} is constrained as $E_{tr} = M$. Without loss of generality, this paper assumes $K = M$, i.e. the same number of transmit antennas as users.

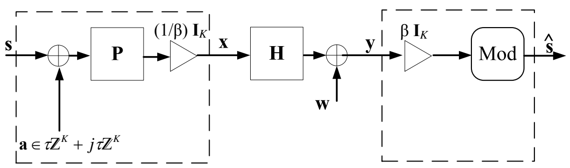

Figure 2: Block diagram of an VP system with $M = K$.

An arbitrary VP system with $M = K$ is depicted in Figure 2. The matrix $\mathbf{I}_K$ is the identity matrix of dimension $K \times K$. The data vector $\mathbf{s} = [s_1, \ldots, s_K]^T$ to be transmitted is perturbed by a complex signal $\mathbf{a} \in \tau\mathbb{Z}^K + j\tau\mathbb{Z}^K$, where τ is the modulo constant. This value has to be chosen large enough so that unambiguous decoding can be performed [10]. Nevertheless, small values of τ yield denser perturbation lattices and hence the ability of the precoding scheme to reduce the norm of the transmitted signal is enhanced. It is suggested in most of the literature on VP that τ is chosen as $\tau = 2(d_{max} + \Delta/2)$, where d_{max} is the absolute value of the constellation symbol with the largest magnitude and Δ is the minimum spacing between constellation points. The precoding matrix $\mathbf{P}$ shapes the signal to be transmitted and a scaling factor β^{-1} is applied prior to transmission to comply with the transmit power constraints. At the user terminals, the received signal is scaled by β again to meet the modulo operation requirements. This non-linear operation at the receivers is essential if the effects of the perturbing signal $\mathbf{a}$ are to be reversed.

The VP approach that achieves the best performance is the Wiener filter VP (WF-VP), which jointly optimizes the perturbing signal, the precoding matrix and the power scaling factor to reach the MMSE solution [5]. A simpler approach to WF-VP can be attained if the transmit power is minimized instead of aiming for the MMSE solution. This approach is referred to as Regularized Vector Precoding and is supported by the following equations:

$$\mathbf{a}_{regVP} = \underset{\hat{\mathbf{a}} \in \tau\mathbb{Z}^K + j\tau\mathbb{Z}^K}{\operatorname{argmin}} \left\| \underbrace{\mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \xi\mathbf{I}_K)^{-1}}_{\mathbf{P}}(\mathbf{s} + \hat{\mathbf{a}}) \right\|_2^2, \quad (1)$$

$$\mathbf{x} = \beta^{-1} \underbrace{\mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \xi\mathbf{I}_K)^{-1}}_{\mathbf{P}}(\mathbf{s} + \mathbf{a}_{regVP}).$$

The conjugate transpose is denoted as $[.]^H$ and $\|.\|_2^2$ represents the squared two-norm. The value ξ represents the inverse of the signal to noise ratio (SNR) and is defined as $\xi = KN_0/E_{tr}$. The real and imaginary parts of the candidates for the perturbation vector belong to the integer set. However, for the sake of simplicity only those candidate points that lie within a set boundary are considered. This selection can be made by either selecting those candidate points whose absolute values are below a certain threshold, or by reducing the number of feasible integers to a reasonable value. The latter is the approach that has been selected for this research work, being B the number of candidate perturbing values.

From Equation (1) one can notice that the computation of the perturbing signal $\mathbf{a}$ entails a search for the closest point in a lattice. One of the most popular methods for the resolution of this type of problems is the SE algorithm, which is explained in detail in the following section.

3. SPHERE ENCODER AND MATRIX ORDERINGS

The sphere search method is a widely used technique in detection and precoding schemes to minimize the complexity that an exhaustive search method would imply. This method consists of searching only over those points that lie within a hypersphere of radius R around the reference signal. Applying the sphere constraint to the computation of the perturbing vector, the cost function described in (1) can be rewritten as

$$\mathbf{a}_{regVP} = \arg\{ \min_{\hat{\mathbf{a}} \in \tau\mathbb{Z}^K + j\tau\mathbb{Z}^K} \left\| \mathbf{P}(\mathbf{s} + \hat{\mathbf{a}}) \right\|_2^2 \leqslant R \}, \quad (2)$$

where $\mathbf{P} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \xi\mathbf{I}_K)^{-1}$ is the precoding matrix. The minimization problem in Equation (2) can be equivalently described as

$$\mathbf{a}_{regVP} = \arg\{ \min_{\hat{\mathbf{a}} \in \tau\mathbb{Z}^K + j\tau\mathbb{Z}^K} \left\| \mathbf{U}(\mathbf{s} + \hat{\mathbf{a}}) \right\|_2^2 \leqslant R \}, \quad (3)$$

where $\mathbf{U}$ equals the upper triangular matrix obtained by the Cholesky decomposition of $\mathbf{P}^H\mathbf{P}$ or, equivalently the QR decomposition of $\mathbf{P}$. The solution to (3) can be recursively obtained by using a tree search approach, starting from the level $i = K$ and working backwards until $i = 1$. At each level, B child nodes originate from each parent node on the tree. Only those child nodes that fulfil (4) are considered as feasible solutions, with Equation (4) given by

$$|a_i + z_i|^2 \leqslant \frac{T_i}{u_{ii}^2}, \quad (4)$$

with

$$z_i = s_i + \sum_{j=i+1}^{K} \frac{u_{ij}}{u_{ii}}(s_j + a_j), \quad (5)$$

and

$$T_i = R^2 - \sum_{j=i+1}^{K} u_{jj}^2 |s_j + z_j|^2. \quad (6)$$

Every time a new leaf node is reached, that is $i = 1$, the path in the tree leading to that leaf node is stored as a feasible solution and the search radius is updated with the Euclidean distance to that leaf node. The SE search is continued with the new SE constraint. On the other hand, if the established radius is exceeded at a certain level in the tree search, such node in the tree and all its child nodes can be discarded. Eventually, the last leaf node that is reached in the tree search represents the solution to the closest vector problem [8].

As stated in [9], it can be seen from Equations (4), (5) and (6) that, on average, the number of parent nodes visited per level is inversely proportional to $E[u_{ii}^2]$. In addition, this effect is more relevant in the first level ($i = K$) since T_i decreases with decreasing i. Therefore, augmenting $E[u_{KK}^2]$ would reduce the average number of nodes visited in the first level, which would lead to a reduction in the total number of paths evaluated during the tree search in the following levels. In the light of these statements, a pre-ordering of the precoding matrix $\mathbf{P}$ that would maximize the value of $E[u_{KK}^2]$ would dramatically enhance the performance in terms of complexity of the SE search. Two ordering approaches that aim in this direction will be discussed in the following subsections.

3.1 Iterative matrix ordering

In this section the performance of several iterative matrix ordering algorithms will be assessed. For this study, the matrix $\mathbf{G} = \mathbf{P}_i^H(\mathbf{P}_i\mathbf{P}_i^H)^{-1}$ will be considered, where $\mathbf{P}_i$ represents the precoding matrix with the previously selected columns set to zero. At each stage of the ordering algorithm ($i = K \ldots i = 2$), the row of $\mathbf{G}$ with the maximum or minimum squared norm, as stated by the ordering strategy, is selected. Once the order of the columns of $\mathbf{P}$ has been set, these are rearranged accordingly. The ordering strategies under consideration are as follows:

- A: FSD ordering proposed in [8] for MIMO detection.
- B: The opposite of the FSD ordering.
- C: Maximization of the norm at each stage (*Max Ordering*).
- D: Minimization of the norm at each stage (V-BLAST).
- E: Alternate maximization and minimization of the norm.
- F: Alternate minimization and maximization of the norm.
- G: Maximization of the norm until the $K/2^{th}$ stage and minimization of the rest.
- H: Minimization of the norm until the $K/2^{th}$ stage and maximization of the rest.

These orderings have also been applied on a reverse fashion, that is from $i = 1 \ldots K$, hence doubling the number of orderings considered in the study. We will refer to these orderings as the *reverse* orderings.

As stated in Section 3, the relevance of $E[u_{KK}^2]$ is paramount in the complexity of the SE. Among all the considered orderings, the V-BLAST ordering yields the highest $E[u_{KK}^2]$ and the smallest complexity of the SE search in terms of average evaluated nodes per level. This result is supported by [**?**], where the authors prove that the aim of the V-BLAST ordering is to find the permutation matrix Π such that the QR decomposition of $\mathbf{P}' = \mathbf{P}\Pi$ has the property that the originally minimum $E[u_{ii}^2]$ is maximized over all column permutations. In Table 1 the experimental values of $E[u_{ii}^2]$ are displayed for the unordered, iteratively ordered (V-BLAST) and non-iteratively ordered (Max Ordering) cases for a 6×6 system with 16-QAM modulation. For the iterative and non-iterative approaches, only the data for the ordering with best performance in terms of searched points per level for the SE have been included in each case. From the data in Table 1 it is clear that the minimum $E[u_{ii}^2]$ in the unordered case is $E[u_{66}^2]$. This value is maximized by the V-BLAST iterative ordering, yielding the highest $E[u_{66}^2]$ value among the non-iterative and the rest of iterative orderings. Therefore, the results provided in Table 1 corroborate the optimality of the V-BLAST ordering to reduce the complexity of the SE.

In addition to this, and based on Equation (6), one can assume that maximizing the sum of the $E[u_{ii}^2]$ at each stage $i = K \ldots 1$ should reduce the T_i value by the maximum possible, and hence allow the SE to discard as many paths as possible in the algorithm. From all the studied orderings, the V-BLAST approach is the only one that yields the highest $E[u_{KK}^2]$ at the first stage, then the highest $E[u_{KK}^2] + E[u_{K-1,K-1}^2]$, and so on.

To sum up, it has been shown in this subsection that the V-BLAST ordering is the optimal ordering, among the ones that have been studied, in terms of complexity of the SE search. The results of the complexity reduction for the selected ordering are shown in Table 2, where the average total number of nodes per level considered by the algorithm as feasible solutions is depicted.

3.2 Non-iterative matrix ordering

Although Section 3.1 proves that the iterative V-BLAST ordering is the one that has a higher impact on the complexity reduction of the SE search, a simpler approach will be described here. This method differs from the iterative one in the pre-processing stage of the precoding matrix, where this process is performed in a non-iterative way. The columns of the precoding matrix $\mathbf{P}$ are ordered according to their norm. The ordering methodology proposed in Section 3.1 will be applied here, except for a minimal change: the D ordering can no longer be related to the V-BLAST ordering since no iterative procedure is performed. Moreover, the number of ordering strategies under consideration is reduced since some of the orderings yield an identical matrix ordering, such as the D and the reverse C orderings.

	$E[u_{11}^2]$	$E[u_{22}^2]$	$E[u_{33}^2]$	$E[u_{44}^2]$	$E[u_{55}^2]$	$E[u_{66}^2]$
Unordered	1.905	0.837	0.478	0.332	0.249	0.197
Iterative	0.739	0.559	0.429	0.360	0.323	0.336
Non-iter.	0.589	0.610	0.510	0.401	0.329	0.284

Table 1: Mean u_{ii}^2 values for several matrix orderings in a 6×6 system with 16-QAM modulation.

	SE	**SE+Iter.Ord.**	**SE+Non-Iter.Ord.**
Level 6	6.558	3.529	4.520
Level 5	20.228	7.654	10.307
Level 4	37.468	11.369	14.763
Level 3	46.294	12.309	14.322
Level 2	37.725	9.923	9.648
Level 1	17.774	5.596	4.729

Table 2: Mean value of the number of visited nodes per level for several matrix orderings in a 6x6 system with 16-QAM modulation.

From all the studied non-iterative orderings, the *Max ordering* is the one that yields the highest $E[u_{KK}^2]$. Nevertheless, the $E[u_{KK}^2]$ value for the non-iterative ordering is smaller, and therefore less optimum, than that of the iterative approach, as shown in Table 1. This is an expected result since, as stated in Section 3.1, the V-BLAST ordering achieves the highest $E[u_{KK}^2]$ value. Moreover, and following the rationale introduced in the previous section, the *Max ordering* is the non-iterative ordering that maximizes the sum $\sum_{n=i}^{K} E[u_{nn}^2]$ at each stage, thus minimizing the number of branches visited by the SE algorithm.

The optimality of the *Max ordering* can be further justified if a closer look at the $\mathbf{U}$ matrix is taken. As previously mentioned, the u_{KK}^2 entry of matrix $\mathbf{U}$ is the most relevant among all the diagonal values as it has a great impact on the performance of the SE search. We henceforth define $\mathbf{A} \triangleq \mathbf{P}^H\mathbf{P}$, and therefore $\mathbf{U} = \mathrm{chol}(\mathbf{A})$. Applying formula (8) for the computation of the Cholesky decomposition of an arbitrary matrix for a 4x4 configuration, the value of u_{44} can be calculated as expression (9).

$$u_{ij} = \frac{1}{u_{ii}}\left(A_{ij} - \sum_{k=1}^{i-1} u_{ki}^* u_{kj}\right) \tag{7}$$

$$u_{ii} = \sqrt{A_{ii} - \sum_{k=1}^{i-1} |u_{ki}|^2} \tag{8}$$

$$u_{44} = \sqrt{A_{44} - \underbrace{(|u_{14}|^2 + |u_{24}|^2 + |u_{34}|^2)}_{W_4}} \tag{9}$$

Clearly from the above formulae, in order to increase the value of u_{44}^2, A_{44} has to be maximized and W_4 minimized. Simulations showed that the best performing non-iterative ordering (*Max ordering*) yielded the highest A_{44} value, whereas the worst performing ordering (D ordering) achieved an A_{44} value 80% smaller than the optimum. This supports the hypothesis that an increase on the value of A_{44} leads to an augmentation in the value of the u_{44}^2 element. Following this observation, and since $\mathbf{A} \triangleq \mathbf{P}\mathbf{P}^H$, the diagonal values of the matrix $\mathbf{A}$ equal the squared value of the norm of the i^{th} column of the precoding matrix $\mathbf{P}$. Therefore, by selecting the column with the highest norm to be the first one to be processed during the tree search (when $i = K$), the value of A_{KK} is maximized, which with a high probability leads to a higher u_{KK}^2 value. Nonetheless, several orderings, such as B, E and G orderings, select the column of $\mathbf{P}$ with the highest norm in the $i = K$ level and still achieve a worse performance than the *Max Ordering*.

The rationale behind this lies on the $u_{K-1,K-1}^2 \dots u_{11}^2$ values of the $\mathbf{U}$ matrix, as only the *Max ordering's* $E[u_{ii}^2]$ values render the highest $\sum_{n=i}^{K} E[u_{ii}^2]$ at each stage. This can be explained by following Equation (8) for the computation of the diagonal elements in a Cholesky matrix. Therefore, the values of u_{33}^2, u_{22}^2 and u_{11}^2 are defined as

$$u_{33} = \sqrt{A_{33} - \underbrace{(|u_{13}|^2 + |u_{23}|^2)}_{W_3}} \;,$$

$$u_{22} = \sqrt{A_{22} - \underbrace{(|u_{12}|^2)}_{W_2}} \;\text{ and }\; u_{11} = \sqrt{A_{11}}.$$

If the $\sum_{n=i}^{K} E[u_{nn}^2]$ is to be maximized at each stage $i = K \dots 1$, the A_{ii} values should have an increasing tendency as i decreases, that is $A_{KK} > A_{K-1,K-1} > \dots > A_{11}$. If the *Max Ordering* is applied, it is clear that the distribution of the diagonal elements of $\mathbf{A}$ fulfils such condition. However, and despite the fact that a decreasing tendency of the W_i values would be desirable so as to maximize $A_{ii} - W_i$ at the early stages of the algorithm, the matrix ordering methodology affects both coefficients in equal terms. Therefore, it is advisable not only to select the column of $\mathbf{P}$ with the highest norm first, but to apply this ordering criteria in the rest of the levels to reduce the complexity of the SE.

To sum up, if at each iteration i the column of $\mathbf{P}$ with the maximum norm is selected, the value of A_{ii} will be maximized, which is very likely result in a higher value for u_{ii}^2. The effect of the increase of u_{ii}^2 at each iteration is a reduction of the number of points to be searched by the SE, which yields a smaller complexity for the algorithm, and thus a more suitable scenario for the implementation of the FSE. The complexity reduction achieved by this ordering method is reflected in Table 2.

4. FIXED SPHERE ENCODER FOR VP

4.1 FSD for detection in MIMO systems

The FSD was originally published by Barbero [9] with the purpose of overcoming the two main drawbacks of the SD detection scheme in single-user $N_{tx} \times N_{rx}$ MIMO systems. On one hand, the complexity of the detector is variable and depends strongly on the noise level and channel conditions. On the other hand, the sequential nature of the algorithm makes it unsuitable for hardware implementation, as the parallelism and pipelining features of such devices cannot be fully exploited.

The main feature of the FSD is the fixed-complexity tree search performed in order to find a quasi-ML solution of the detection process. As can be seen in Figure 3, the search tree is defined by $\mathbf{n} = [n_1, \dots, n_K]$, being n_i the number of child nodes to be considered at each level. At each level, the symbols to be selected are chosen in accordance with the Schnor-Euchner enumeration [9]. The total number of paths visited by the algorithm is calculated as $n_T = \prod_{i=1}^{N_{tx}} n_i$. The suggested distribution of nodes in the original FSD is $n = [1, 1, \dots, 1, T]$, where T denotes the constellation size.

The original FSD presented in [9] performed a preprocessing stage of the channel matrix prior to developing the search for the closest lattice point. The matrix ordering stage iteratively orders the N_{tx} columns of the channel matrix starting from $i = N_{tx}$ and working backwards until $i = 1$. At the first iteration the signal with the largest noise amplification is selected. For the rest of the iterations, the opposite operation is performed, i.e. the signal with the smallest noise amplification is chosen. The square norm of the rows of $\mathbf{H}_i^{\dagger} = (\mathbf{H}_i^H \mathbf{H}_i)^{-1} \mathbf{H}_i^H$ are used as the metric to evaluate the amount of noise

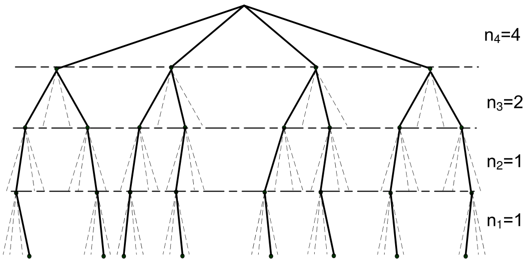

Figure 3: FSD tree search in a 4×4 system with QPSK modulation.

amplification, where $\mathbf{H}_i$ equals the channel matrix with the columns selected in previous iterations set to zero.

However, the search for the closest lattice point that needs to be performed in a VP multiuser system entails a different problem and, therefore, the recommended design rules such as the node distribution pattern ($\mathbf{n}$) or the matrix ordering do no longer apply.

4.2 FSE for VP in multiuser MIMO systems

The implementation of the FSE at the transmitter side is splitted in two stages:

- **Matrix ordering:** The columns of the precoding matrix $\mathbf{P}$ must be rearranged in order to achieve a better performance of the algorithm. To this end, any of the orderings described in Section 3.1 or Section 3.2 can be used. However, the BER performance of the algorithm will be different in systems where distinct orderings have been applied.
  The complexity of the matrix ordering methods described in this paper differ as the iterative ordering involves performing $K - 1$ matrix inversions, while none is required for the non-iterative procedure. However, note that the matrix ordering stage of the algorithm is performed only once per channel realization, and therefore, its complexity is negligible when compared to the processing of the whole data block.

- **FSE tree search:** The design of the search tree is crucial, since it has a strong influence on both the complexity of the algorithm and its performance in terms of BER.
  For every system configuration a trade-off between complexity and performance has to be met. Notice that the higher the number of calculated paths is, the better the performance will be and viceversa. It all depends on the hardware resources available and the target BER. Therefore, no optimum node distribution pattern will be provided in this paper. However, some hints for a proper setup of the search tree will be supplied: first, selecting $n_i > 1$ at high levels in the tree (when $i = K, \dots, i = K/2$) has a positive impact on the BER performance. It is advisable as well to always set $n_1 = 1$, as setting a higher value for this element would not make any difference if the Schnor-Euchner enumeration is followed. Finally, simulations show that those trees with $n_K \geqslant n_{K-1} \geqslant \dots \geqslant n_1$ render a better BER performance than those who do not.

5. SIMULATION RESULTS

For the results in Figure 4 and Figure 5 a channel model with independent and identically distributed unit-variance Rayleigh fading coefficients has been used and perfect channel state information (CSI) is assumed at the transmitter. The channel remains constant during every transmission block which consists of 100 16-QAM symbols and the modulo constant τ has been set according to the recommendations in Section 2. The node distribution vector has been chosen as $\mathbf{n} = [1, 1, 2, 5]$ for the $K = 4$ antenna FSE system, $\mathbf{n} = [1, 1, 1, 2, 3, 4]$ for the $K = 6$ configuration and $\mathbf{n} = [1, 1, 1, 1, 2, 2, 3, 4]$ for the $K = 8$ antenna FSE system.

The simulations in Figure 4 show that the BER performance of the non-iterative ordering FSE approach is very close to that of the

SE. The difference in performance is approximately 0.09 dB at a BER of 10^{-3} for all system configurations. Although the initial results foresaw that the V-BLAST matrix ordering would perform better than the non-iterative one, since it maximized the $E[u_{KK}^2]$ value and therefore it reduced the complexity of the SE, simulation results contradict this initial hypothesis. Despite the fact that the iterative ordering results in a smaller amount of nodes visited per level in the SE in almost all the levels of the tree search, the non-iterative method performs better under a fixed tree structure. This might be due to the fact that the non-iterative approach requires a smaller amount of points in the two lower levels, namely $i = 1$ and $i = 2$, for all the antenna configurations that have been studied in this research (e.g. see Table 2 for the results on the $K = 6$ antenna case). Regarding the BER curves for the iterative ordering approach, they are not displayed for sake of clarity. However, some numerical results on the difference in performance between the two ordering approaches will be provided. The non-iterative approach outperforms the iterative method by 0.06 dB at a BER of 10^{-3} for the $K = 4$ system configuration. Nevertheless, the difference between the two ordering approaches narrows as the number of antennas is increased.

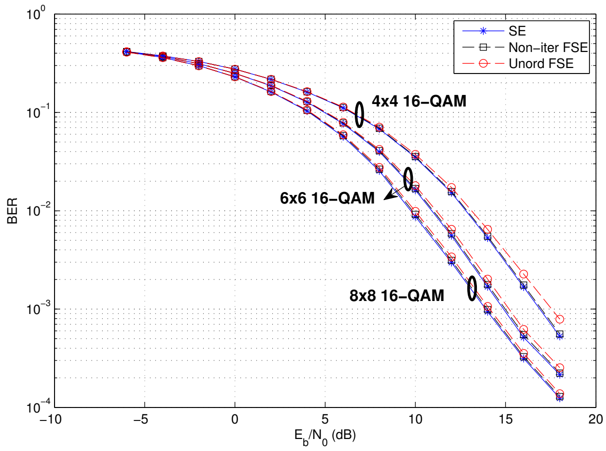

Figure 4: BER performance of the SE, the ordered FSE and the unordered FSE under several system configurations.

Finally, Figure 5 shows the number of visited nodes per level for several antenna configurations. Note that the markers for the SE case stand for mean values only, as the complexity of the SE is variable. The number of nodes visited during the FSE tree search is calculated as $\prod_{j=i}^{K} n_j$ for level i. The data displayed in this plot show that the number of visited nodes by the proposed algorithm is considerably smaller for most of the levels. It is worth to mention as well the great reduction in searched paths that is achieved with the VP-FSE compared to the exhaustive search method. If the number of candidate values for the elements of the perturbing vector is set to $B = 25$, the total number of paths searched by the exhaustive search approach for a 4 antenna system would be $25^4 = 3.90 \cdot 10^5$, as opposed to the 10 paths required for the FSE. This huge gap in complexity is increased if more antennas are added to the system. In a 6×6 system the complexity of the exhaustive search would be of $25^6 = 2.44 \cdot 10^8$ paths, and this value could be increased to $25^8 = 1.52 \cdot 10^{11}$ if 8 antennas are used. On the other hand, the FSE simulations carried out in this research only required the computation of 24 and 48 paths for the 6 and 8 antenna cases, respectively. These results verify the vast complexity reduction achieved by the proposed VP-FSE scheme.

6. CONCLUSIONS

The FSE applied to the search of the perturbing signal in a vector precoding system has proved to be a fully valid and applicable approach. The BER performance of the proposed algorithm is very close to the optimum solution whereas its complexity is far more reduced in comparison to that of the exhaustive search method or even the SE algorithm. Moreover, due to its fixed complexity, the proposed algorithm can be effectively implemented in FPGA devices, where the pipelining and parallelization of resources enhance the throughput of the overall communication system.

As for the best ordering approach, the results on BER performance depicted in Figure 4 have shown that the non-iterative matrix ordering performs slightly better than the V-BLAST matrix ordering, while not requiring any of the $K-1$ matrix inverse operations required by the latter.

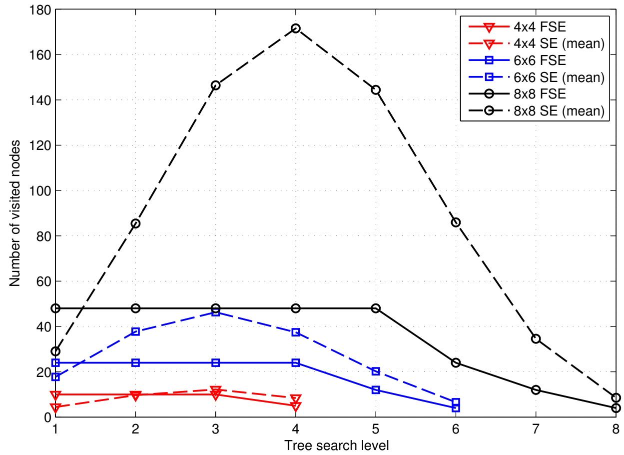

Figure 5: Number of visited nodes per level by the FSE and SE algorithms for $K = 4$, $K = 6$ and $K = 8$ systems at 20 dB.

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