A NOVEL DATA-FUSION-BASED IMPROVEMENT TO DEBIASED CMKF TRACKING

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ABSTRACT

A novel modification to the debiased converted-measurement Kalman filter (CMKF-D) is proposed and implemented. The resulting CMKF-D evaluates the average true measurement-error bias and covariance with polar target-position estimates obtained by nonlinearly transforming Cartesian estimates formed by traditional weighted-least-squares fusion of the debiased converted measurements and the predicted Cartesian estimates of the CMKF-D. A tracking-performance comparison is made between the resulting CMKF-D and the previously presented CMKF-D which demonstrates the improvements obtained from the new technique when bearing measurement errors are large.

1. INTRODUCTION

The problems of debiasing the converted measurements and approximating the converted-measurement error covariance have been examined by Lerro and Bar-Shalom [1] for the CMKF-D which tracks a target in Cartesian coordinates given polar measurements. This present work demonstrates a technique for improving the tracking performance of the previously published method by calculating better polar estimates for use in the average true bias and measurement-error covariance. The better polar estimates are calculated by transforming to polar coordinates the Cartesian estimates obtained by weighted-least-squares fusion of the CMKF-D's predicted target-position estimates and the debiased converted measurements.

2. TECHNICAL BACKGROUND

2.1 Polar-Measurement Model

A sensor, located at the origin of the plane in which the target moves, produces measurements of the target's range and bearing. This investigation assumes a traditional polar-measurement model in which the *k*th measurements consist of the true range and bearing quantities, $r[k]$ and $\beta[k]$, corrupted by additive, uncorrelated, white, zero-mean normal measurement noises [2]. Thus, the measurements are mathematically described by

$$r_m[k] = r[k] + \tilde{r}[k]$$

and

$$\beta_m[k] = \beta[k] + \tilde{\beta}[k],$$

where $\tilde{r}[k] \sim N\left(0, \sigma_r[k]\right)$, $\tilde{\beta}[k] \sim N\left(0, \sigma_\beta[k]\right)$, and

$$\operatorname{cov}\left(\begin{bmatrix}\tilde{r}[k] & \tilde{\beta}[k]\end{bmatrix}', \begin{bmatrix}\tilde{r}[l] & \tilde{\beta}[l]\end{bmatrix}'\right) = \begin{bmatrix}\sigma_r^2[k] & 0 \\ 0 & \sigma_\beta^2[k]\end{bmatrix}\delta_K[k-l],$$

with $\delta_K[\bullet]$ indicating the Kronecker delta function.

2.2 The Debiased CMKF (CMKF-D)

To be used in the considered CMKF-D, the polar measurements are first transformed to Cartesian converted measurements with

$$x_m[k] = r_m[k]\cos\left(\beta_m[k]\right) = x[k] + \tilde{x}[k] \tag{1}$$

and

$$y_m[k] = r_m[k]\sin\left(\beta_m[k]\right) = y[k] + \tilde{y}[k] \tag{2}$$

where

$$x[k] = r[k]\cos\left(\beta[k]\right)$$

and

$$y[k] = r[k]\sin\left(\beta[k]\right).$$

Lerro and Bar-Shalom demonstrated [1] that the nonlinear transformations (1) and (2) introduce biases in the converted measurements even though the original polar measurements were unbiased. These biases can become unacceptably severe when the bearing-measurement error variance is large.

The true vector bias is given by

$$\boldsymbol{\mu}_t[k] = E\left\{\begin{bmatrix}\tilde{x}[k] \\ \tilde{y}[k]\end{bmatrix} \middle| r[k], \beta[k]\right\} = \begin{bmatrix}\mu_t^{(x)}[k] \\ \mu_t^{(y)}[k]\end{bmatrix}$$

where $E(\bullet)$ represents mathematical expectation. For the assumed normal range- and bearing-measurement noises, the elements of $\boldsymbol{\mu}_t[k]$ are explicitly given by (6) of [1]; note that both elements require the target's true range and bearing for evaluation. Similarly, the true measurement-error covariance is given by

$$\mathbf{R}_t[k] = \mathrm{cov}\left\{ \begin{bmatrix} \tilde{x}[k] \\ \tilde{y}[k] \end{bmatrix} \middle| r[k], \beta[k] \right\} = \begin{bmatrix} R_t^{11}[k] & R_t^{12}[k] \\ R_t^{21}[k] & R_t^{22}[k] \end{bmatrix}.$$

For the assumed normal range- and bearing-measurement noises, the elements of $\mathbf{R}_t[k]$ are explicitly given by (7) of [1]; note that all four elements require the target's true range and bearing for evaluation.

Since the true polar coordinates of the target are unavailable in practice, Lerro and Bar-Shalom proposed conditioning $\boldsymbol{\mu}_t[k]$ and $\mathbf{R}_t[k]$ on $r_m[k]$ and $\beta_m[k]$ to obtain the average true measurement-error bias

$$\boldsymbol{\mu}_a[k] = E\left\{ \boldsymbol{\mu}_t[k] \middle| r_m[k], \beta_m[k] \right\} = \begin{bmatrix} \mu_a^{(x)}[k] & \mu_a^{(y)}[k] \end{bmatrix}'$$

and the average true measurement-error covariance

$$\mathbf{R}_a[k] = E\left\{ \mathbf{R}_t[k] \middle| r_m[k], \beta_m[k] \right\}.$$

Both $\boldsymbol{\mu}_a[k]$ and $\mathbf{R}_a[k]$ depend only on the polar measurements. The debiased converted measurements are thus given by

$$x_m^{(d)}[k] = r_m[k]\cos\left(\beta_m[k]\right) - \mu_a^{(x)}[k]$$

and

$$y_m^{(d)}[k] = r_m[k]\sin\left(\beta_m[k]\right) - \mu_a^{(y)}[k].$$

In order to improve dynamic tracking performance, Lerro and Bar-Shalom specified the additional requirement of evaluating $\boldsymbol{\mu}_a[k]$ [3] and $\mathbf{R}_a[k]$ [1, 3] using the best available polar estimates rather than exclusively using the polar measurements on which $\boldsymbol{\mu}_a[k]$ and $\mathbf{R}_a[k]$ had been conditioned. To provide a practical means of meeting this additional requirement, Lerro and Bar-Shalom presented a simple test, (35) of [1], which chose the more accurate of the measured and predicted polar quantities (obtained via nonlinear transformation of the CMKF-D's predicted Cartesian measurements) based on the sizes (computed with determinants) of the respective error covariances in Cartesian coordinates.

2.3 Proposed Improvement to the Tracking Algorithm

Whereas the additional requirement given in [1, 3] and correctly explained in [4] calls for using the *best* available *polar* quantities to evaluate $\boldsymbol{\mu}_a[k]$ and $\mathbf{R}_a[k]$, it should be noted that the given rule actually determines the *better* (i.e., the less uncertain) of the measured and predicted *Cartesian* estimates. Although using true range and bearing quantities to evaluate $\boldsymbol{\mu}_a[k]$ and $\mathbf{R}_a[k]$ is obviously impractical, it does motivate a search for polar estimates superior to both the measured and predicted quantities to use in the evaluation of $\boldsymbol{\mu}_a[k]$ and $\mathbf{R}_a[k]$.

The polar measurements and Cartesian predictions comprise the only practically and immediately available estimates of the target's position. Since the errors in the Cartesian predictions depend upon measurement noise *prior to the current measurement only*, and since the measurement noises are white processes, the errors in the polar measurements and the errors in the Cartesian predictions are statistically independent. We therefore propose producing polar estimates for the evaluation of $\boldsymbol{\mu}_a[k]$ and $\mathbf{R}_a[k]$ by first fusing the (converted) measured and predicted target-position estimates in Cartesian coordinates and then nonlinearly transforming the fused Cartesian estimates to polar coordinates. Section III provides details for the proposed data-fusion and coordinate-transformation method.

3. PROPOSED METHOD OF ESTIMATING THE POLAR TARGET-POSITION VALUES

3.1 Weighted-Least-Squares Fusion of Measured and Predicted Target-Position Estimates

The traditional method of weighted-least-squares estimation [5] assumes a set of l measurements $\mathbf{z}$ that is a linear combination of the elements of a constant vector $\mathbf{x}$ corrupted by a zero-mean random additive measurement noise vector $\mathbf{v}$. Mathematically,

$$\mathbf{z} = \mathbf{M}\mathbf{x} + \mathbf{v}.$$

The estimate $\hat{\mathbf{x}}$ of $\mathbf{x}$ that minimizes the scalar cost function

$$J = \left(\mathbf{z} - \mathbf{M}\hat{\mathbf{x}}\right)' \mathbf{C}_{\mathbf{v}}^{-1} \left(\mathbf{z} - \mathbf{M}\hat{\mathbf{x}}\right),$$

where $\mathbf{C}_{\mathbf{v}} = \mathrm{cov}(\mathbf{v})$, is

$$\hat{\mathbf{x}} = \left(\mathbf{M}'\mathbf{C}_{\mathbf{v}}^{-1}\mathbf{M}\right)^{-1} \mathbf{M}'\mathbf{C}_{\mathbf{v}}^{-1}\mathbf{z}. \tag{3}$$

The measured target-position estimate and associated error covariance are in the polar coordinate system, but the predicted target-position estimate and associated error covariance are in the Cartesian coordinate system. As a result, the two target-position estimates cannot be directly fused using the weighted-least-squares approach of [5, eq. (4.0-5)] (in which the weighting matrix is conventionally formed from the inverses of the two estimates' error covariances), since this technique requires the two estimates and their respective error covariances to be in identical coordinate systems. Thus, some means of converting one estimate and its error covariance to the coordinate system of the other estimate is required.

Since converted-measurement debiasing with Lerro and Bar-Shalom's technique was shown to produce statistically consistent converted measurements for even reasonably large measurement-error variances, the kth debiased *converted* measurements $\begin{bmatrix} x_m^{(d)}[k] & y_m^{(d)}[k] \end{bmatrix}'$, when conditioned on the true target coordinates, approximately represent a randomly drawn sample from a joint distribution with mean $\begin{bmatrix} x[k] & y[k] \end{bmatrix}'$ and covariance $\mathbf{C}_{Cart}^{meas}[k] \cong \mathbf{R}_a[k]$. Thus,

$$\begin{bmatrix} x_m^{(d)}[k] & y_m^{(d)}[k] \end{bmatrix}' = \begin{bmatrix} x[k] & y[k] \end{bmatrix}' + \begin{bmatrix} \tilde{x}^{(d)}[k] & \tilde{y}^{(d)}[k] \end{bmatrix}'$$

where

$$\mathrm{cov}\left(\begin{bmatrix} \tilde{x}^{(d)}[k] & \tilde{y}^{(d)}[k] \end{bmatrix}' \right) = \mathbf{C}_{Cart}^{meas}[k] \cong \mathbf{R}_a[k].$$

In basic (linear) Kalman-filter theory, the predicted state at index k is a linear transformation of the state estimate at index $k-1$ which is itself a linear combination of the measurements taken at indices up to and including $k-1$ [6]. Therefore, the kth predicted target-position estimates, $\left[x_p[k] \quad y_p[k]\right]'$, when conditioned on the true target coordinates, represent a randomly drawn sample from a joint distribution with mean $\left[x[k] \quad y[k]\right]'$ and error covariance

$$\mathbf{C}_{Cart}^{pred}[k] = \mathbf{HP}[k\,|\,k-1]\mathbf{H}'. \tag{4}$$

In (4),

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

is the observation or measurement matrix and $\mathbf{P}[k\,|\,k-1]$ is the predicted state-estimate-error covariance matrix at measurement index k. $\mathbf{P}[k\,|\,k-1]$ is in turn given by

$$\mathbf{P}[k\,|\,k-1] = \mathbf{FP}[k-1\,|\,k-1]\mathbf{F}' + \mathbf{Q}[k]$$

where

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is the state-transition matrix (with T representing the measurement interval), $\mathbf{P}[k-1\,|\,k-1]$ is the state-estimate-error covariance at measurement index $k-1$, and $\mathbf{Q}[k]$ is the process-noise covariance. Thus,

$$\left[x_p[k] \quad y_p[k]\right]' = \left[x[k] \quad y[k]\right]' + \left[\tilde{x}_p[k] \quad \tilde{y}_p[k]\right]'$$

where

$$\operatorname{cov}\left(\left[\tilde{x}_p[k] \quad \tilde{y}_p[k]\right]'\right) = \mathbf{C}_{Cart}^{pred}[k].$$

We may now apply (3) to the problem of fusing the measured and predicted Cartesian quantities:

$$\begin{bmatrix} \hat{x}[k] \\ \hat{y}[k] \end{bmatrix} = \left(\mathbf{M}'\mathbf{C}^{-1}[k]\mathbf{M}\right)^{-1} \mathbf{M}'\mathbf{C}^{-1}[k] \begin{bmatrix} x_m^{(d)}[k] \\ y_m^{(d)}[k] \\ x_p[k] \\ y_p[k] \end{bmatrix},$$

where

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$\mathbf{C}[k] = \begin{bmatrix} \mathbf{C}_{Cart}^{meas}[k] & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & \mathbf{C}_{Cart}^{pred}[k] \end{bmatrix}.$$

3.2 Transformation of the Fused Cartesian Estimates to Polar Coordinates

The estimates $\hat{x}[k]$ and $\hat{y}[k]$ are fused Cartesian estimates, which are expected to be more accurate than either $x_m^{(d)}[k]$ and $y_m^{(d)}[k]$ or $x_p[k]$ and $y_p[k]$. However, *polar* estimates are required for the evaluation of $\boldsymbol{\mu}_a[k]$ and $\mathbf{R}_a[k]$, so some method of coordinate transformation from Cartesian to polar is required. The conventional method adopted here converts an estimate in one coordinate system to an estimate in another coordinate system via the nonlinear transformation relating the two coordinate systems:

$$\hat{r}[k] = \sqrt{\hat{x}^2[k] + \hat{y}^2[k]}$$

and

$$\hat{\beta}[k] = \tan^{-1}\left(\hat{y}[k]/\hat{x}[k]\right).$$

4. TRACKING PERFORMANCE WITH THE PROPOSED IMPROVEMENT

4.1 Debiased Converted-Measurement Kalman Filters

A performance comparison of two CMKF-D configurations is conducted. All CMKF-D configurations have the general structure given in Section IV.A of [1]. The two considered CMKF-D configurations differ in the method by which the polar values used to evaluate $\boldsymbol{\mu}_a[k]$ and $\mathbf{R}_a[k]$ are chosen. The first CMKF-D is that described explicitly by [1]. The second CMKF-D uses the polar estimates obtained as described in Section III.

4.2 Simulation Results

The two considered CMKF-D implementations are tested using three standard-deviation values of bearing-measurement error (10° and 15°) and one standard-deviation value of range-measurement error (50 m). The initial target state and kinematics model of Section IV.C of [1] are replicated. Specifically, the initial target location is at a range of 70 km and a bearing of 45°, and the initial target velocity is 15 m/s in the positive y direction. Additionally, the target-trajectory model is the second-order kinematics model of [7, eq. (2-297)], and the process-noise covariance, given by [7, eq. (2-312)], has a standard deviation of 0.01 m/s^2 in each Cartesian dimension.

The CMKF-D implementations are initialized using a slightly modified version of the two-point differencing method of [7, eqs. (2-287), (2-288), and (2-289)] which accounts for the fact that, in general, $\mathbf{R}_a[k] \neq \mathbf{R}_a[l]$, $k \neq l$. Specifically, designating the indices of the first two measurements as $k=0$ and $k=1$, respectively, the Cartesian state estimate is initialized with

$$\hat{\mathbf{x}}[1] = \begin{bmatrix} x_m^{(d)}[1] \\ \left(x_m^{(d)}[1] - x_m^{(d)}[0]\right)/T \\ y_m^{(d)}[1] \\ \left(y_m^{(d)}[1] - y_m^{(d)}[0]\right)/T \end{bmatrix},$$

and the elements of the initial Cartesian state-estimate-error covariance $\mathbf{P}[1]$ are set to

$$P^{11}[1] = R_a^{11}[1],$$

$$P^{12}[1] = P^{21}[1] = R_a^{11}[1]/T,$$
$$P^{13}[1] = P^{31}[1] = R_a^{12}[1],$$
$$P^{14}[1] = P^{41}[1] = R_a^{12}[1]/T,$$
$$P^{22}[1] = \left(R_a^{11}[1] + R_a^{11}[0]\right)/T^2,$$
$$P^{23}[1] = P^{32}[1] = R_a^{12}[1]/T,$$
$$P^{24}[1] = P^{42}[1] = \left(R_a^{12}[1] + R_a^{12}[0]\right)/T^2,$$
$$P^{33}[1] = R_a^{22}[1],$$
$$P^{34}[1] = P^{43}[1] = R_a^{22}[1]/T, \text{ and}$$
$$P^{44}[1] = \left(R_a^{22}[1] + R_a^{22}[0]\right)/T^2.$$

Fifty measurements are taken with an interval of $T = 60$ s. The presented results are those averaged over 1000000 Monte-Carlo runs.

The RMS position errors of the original and proposed CMKF-D implementations are shown in Figures 1 and 2 for the cases of $\sigma_\beta[k] = 10°$ and $\sigma_\beta[k] = 15°$, respectively.

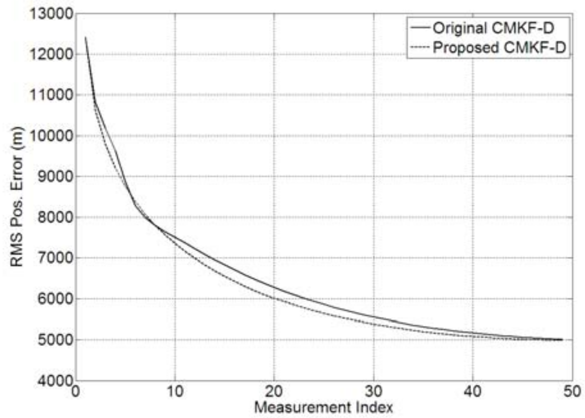

Figure 1 – RMS position errors for $\sigma_\beta[k] = 10°$

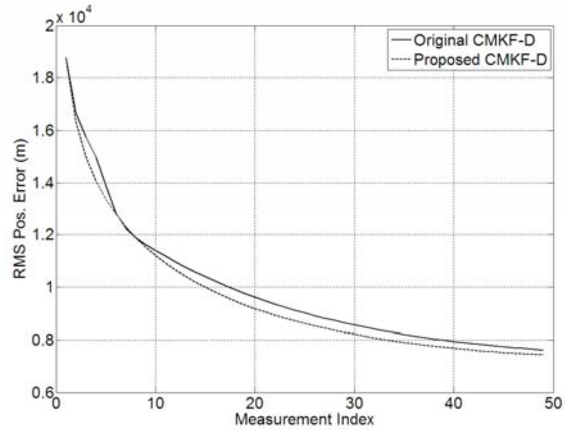

Figure 2 – RMS position errors for $\sigma_\beta[k] = 15°$

For both cases, the proposed CMKF-D provides noticeably better position-tracking performance than the original CMKF-D. Note that qualitatively similar performance results are obtained when the target's initial position and velocity are randomly selected.

The percent improvement of the proposed CMKF-D algorithm over the original CMKF-D algorithm, defined as

$$I = \frac{\text{Orig. alg. RMS Error - Prop. alg. RMS Error}}{\text{Orig. alg. RMS Error}}\left(100\%\right),$$

is shown in Figure 3 for the cases of $\sigma_\beta[k] = 10°$ and $\sigma_\beta[k] = 15°$.

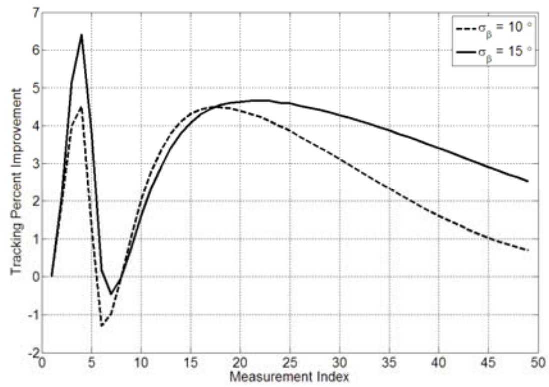

Figure 3 – Percent Improvement of the Proposed CMKF-D over the Original CMKF-D

The average time required to execute a single iteration of the CMKF-D using MATLAB version 7.4 on an Intel® Core™ 2 Duo CPU T7300 running at 1.99 GHz with 2 GB of RAM was 0.10554 ms. The time required to execute a single iteration of the proposed CMKF-D using the same hardware and software was 0.62692 ms, so the tracking improvement comes at the cost of significantly increased computational time.

5. CONCLUSIONS

A novel CMKF-D which fuses measured and predicted target-position information to evaluate the measurement-error covariance approximation is proposed and implemented. The new CMKF-D generally outperformed the original CMKF-D technique for both considered scenarios. The proposed CMKF-D achieves its improved performance, however, at the cost of additional computations to perform (1) weighted-least-squares estimation with the predicted Cartesian coordinates and the converted Cartesian measurements and (2) the nonlinear transformation of the fused Cartesian values.

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