

AN OPTIMAL PRIOR KNOWLEDGE-BASED DOA ESTIMATION METHOD

Guillaume Bouleux¹, Petre Stoica² and Rémy Boyer³

¹ LASPI

Saint Etienne University
IUT de Roanne, Roanne, France

guillaume.bouleux@univ-st-etienne.fr

² Systems and Control Division

Uppsala University
Uppsala, Sweden

peter.stoica@it.uu.se

³ LSS-Supelec

Paris-Sud University
Gif-Sur-Yvette, France

remy.boyer@lss.supelec.fr

ABSTRACT

In some applications involving line spectra or direction of arrival (DOA) estimation, we may have known a priori information. This information could consist of known frequencies in Magnetoencephalography (MEG) or mechanical signals, or of known DOA's in a RADAR urban scenario. With this fact in mind we propose an optimal method for array processing that exploits the information of the known DOA's for estimating the unknown DOA's as accurately as possible. This new **Prior-knowLEDGE (PLEDGE)** method is based on the method of direction estimation (MODE) approach. To show the benefits of incorporating prior-knowledge we also present the corresponding stochastic Cramér-Rao Bound (CRB). Finally, we apply PLEDGE for estimating frequencies in the current spectrum of an induction motor to perform the diagnosis on rotor bars.

1. INTRODUCTION

In some applications involving direction of arrival (DOA) or line spectra estimation, we may have known a priori information. We support on two observations to prove that fact. First, in a RADAR urban scenario, the presence of stationary sources such as buildings, brings about the reflection of the waves emitted. Those one impinge then on arrays with always the same direction. In addition and as an important remark, in such a multi-path problem, the sources are highly correlated and even coherent. Next, the sources of Magnetoencephalography (MEG) are synchronous synaptic currents in the cerebral cortex. Since some neural activities of human brain are repetitive we know frequencies present in the synaptic current spectrum. A more industrial application but still in the same context is the diagnosis of rotating machines. The kinematic of these systems is well known and particular frequencies are so well identified, that could be the on line signal frequency or gears frequencies. The point is that these prior knowledge might not carry any information and might mask what is of interest. It is then crucial to take advantage of the prior-information for estimating the parameters of interest as accurately as possible. Following this way, recent work has been proposed [4] in the context of MEG and [8] in the context of array processing. Each these methods uses the concept of reduced signal subspace by deflating the sample covariance matrix. The Cramér-Rao Bound (CRB) associated with this model has been given and studied in [2]. This bound, the Prior-CRB, shows that this kind of methods are limited and suffer of drastic assumptions as well. We could advance that the non-bijective transformation (the deflation) causes an information loss. From this it is obvious and easy to understand why these algorithms are limited.

So, the idea is to propose an optimal algorithm in which we could easily use and fix the known parameters. We have based our derivation on the popular method of direction estimation (MODE) [12] from which we have deduced the optimal **Prior-knowLEDGE (PLEDGE)** algorithm for the array processing. The PLEDGE is presented for the Uniform and Linear Array (ULA) case which allows us to easily transpose the algorithm to the line spectra estimation. To

show the benefice of incorporating prior-knowledge we also present the corresponding Cramér-Rao Bound (CRB). We give simulation results from which we propose a discussion on what prior-hypothesis are essential. We end this work by presenting a diagnosis problem on rotor bars in an induction motor. As we will show PLEDGE is in total adequation with this context and solve the problem.

2. MODEL STATEMENT

Consider n narrowband and Far-field plane waves impinging on an ULA composed by L sensors separated by a half wavelength. Let t be a sample ("snapshot") and assume that the total number of available samples is N , then $t = 1, \dots, N$. The one sample response, or equivalently the single-experiment time series model can be parametrized in the following manner

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\omega})\mathbf{x}(t) + \mathbf{n}(t) \quad (1)$$

where $\mathbf{A}(\boldsymbol{\omega}) = [\mathbf{a}(\omega_1) \dots \mathbf{a}(\omega_n)]$ and where $\mathbf{a}(\omega_i)$ is the i -th steering vector defined by

$$\mathbf{a}(\omega_i) = [1 \ e^{j\omega_i} \dots e^{j(L-1)\omega_i}]^T \quad (2)$$

with $\omega_i = -\pi \sin(\theta_i)$ the spatial pulsation for the direction of arrival (DOA) estimation problem (θ the DOA) and where ω_i could be the temporal pulsation, i.e $\omega_i = 2\pi \frac{f_i}{f_e}$ (f_e the sample frequency) for the line spectra estimation problem. Amplitude waves $\mathbf{x}(t)$ and noise signal $\mathbf{n}(t)$ are assumed to be jointly Gaussian with zero-mean, stationary and circular stochastic process of second order moments

$$\mathbb{E}[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{P} \quad \text{and} \quad \mathbb{E}[\mathbf{n}(t)\mathbf{n}^H(t)] = \sigma^2 \mathbf{I} \quad (3)$$

where \mathbb{E} stands for the mathematical expectation and H for the transpose and conjugate. For the latter use we introduce T as the transpose and * as the conjugate.

The aim of this work is to furnish an optimal method for the DOA estimation which integrates the prior-knowledge of some directions. The optimality has to be understood in the Maximum Likelihood (ML) sense and elaborating ML DOA estimation has been treated in abundance since the two last decades (see for instance [9]). MODE rose up particularly by its good performances and computational cost (among other things) [10, 6, 12]. We briefly recall its principle before using it to tackle the problem of prior-information integration.

3. MODE

The derivation of MODE can be seen by different points of view and we chose to give a pragmatic manner without going into deep, for more details you can refer to [12] and the references therein. Assume that the directions can be found by polynomial rooting. Consequently, we define the polynomial

$$b_0 z^n + b_1 z^{n-1} + \dots + b_n = b_0 \prod_{i=1}^n (z - e^{j\omega_i}) \quad (4)$$

in which the n angle roots are the n directions of interest ω . Since all the roots of (4) belongs to the unit circle, its coefficients must satisfy the conjugate symmetry constraint [12], and thus $b_i = b_{n-i}^*$, $i = 0, \dots, n$. Let us define now the sylvester matrix whose rows are formed by the polynomial coefficients $\{b_i\}$ such that

$$\mathbf{B}^H = \begin{bmatrix} b_n & b_{n-1} & \dots & b_0 & & \mathbf{0} \\ & \ddots & \ddots & & \ddots & \\ \mathbf{0} & & b_n & b_{n-1} & \dots & b_0 \end{bmatrix} \quad (5)$$

meaning that the columns of \mathbf{B} span the null space of \mathbf{A} . Consider now by deterministic derivation, the concentrated negative log-likelihood function [9] $F_{ML}(\omega) = \text{Trace}[\mathbf{\Pi}_A^\perp \hat{\mathbf{R}}]$ or equivalently

$$F_{ML}(\mathbf{b}) = \text{Trace}[\mathbf{B}(\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \hat{\mathbf{R}}] \quad (6)$$

with $\mathbf{b} = [b_0 \dots b_n]^T$, where $\mathbf{\Pi}_A^\perp = \mathbf{I} - \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H$, and where the sample covariance matrix is defined by

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{t=1}^N \mathbf{y}(t) \mathbf{y}^H(t). \quad (7)$$

Let besides

$$\hat{\mathbf{R}} = \hat{\mathbf{E}}_S \hat{\mathbf{\Sigma}}_S \hat{\mathbf{E}}_S^H + \hat{\mathbf{E}}_N \hat{\mathbf{\Sigma}}_N \hat{\mathbf{E}}_N^H \quad (8)$$

be an eigendecomposition of $\hat{\mathbf{R}}$. A consistent estimate of the noise variance is given by $\hat{\sigma}^2 = \frac{1}{L-n} \text{Trace}[\hat{\mathbf{\Sigma}}_N]$.

It has been shown that the ML estimate was computationally heavy and not efficient, i.e. the variance of the estimation errors ($\hat{\omega}_i - \omega_i$) do not achieve the CRB [12]. So, a better approach is given by a large sample realization of (6) given by

$$\text{Trace}[\mathbf{A}^H \hat{\mathbf{E}}_N \hat{\mathbf{E}}_N^H \mathbf{A} \mathbf{W}] \quad (9)$$

where \mathbf{W} is a positive definite weighting matrix. The derivation which comes up with the optimal (ML sense) \mathbf{W} can be found in [12] and after algebraic manipulations we obtain the MODE minimizer. The coefficients $\{b_i\}$ are therefore the solution of the minimizer [12]

$$F_{Mode}(\mathbf{b}) = \text{Trace}[\mathbf{B}(\hat{\mathbf{B}}^H \hat{\mathbf{B}})^{-1} \mathbf{B}^H \hat{\mathbf{E}}_S \hat{\mathbf{\Lambda}} \hat{\mathbf{E}}_S^H] \quad (10)$$

with

$$\hat{\mathbf{\Lambda}} = \hat{\mathbf{\Sigma}}_S (\hat{\mathbf{\Sigma}}_S - \hat{\sigma}^2 \mathbf{I})^{-2}. \quad (11)$$

We make use of the rank-1 equivalence of $\hat{\mathbf{E}}_S \hat{\mathbf{\Lambda}} \hat{\mathbf{E}}_S^H$ to observe that (10) can be viewed as a quadric minimizer. To this end, we use conjointly

$$\hat{\mathbf{E}}_S \hat{\mathbf{\Lambda}} \hat{\mathbf{E}}_S^H = \sum_{k=1}^n \hat{\lambda}_k \hat{\mathbf{e}}_k \hat{\mathbf{e}}_k^H \quad (12)$$

and property

$$\mathbf{B}^H \hat{\mathbf{e}}_k = \hat{\tilde{\mathbf{E}}}_k \mathbf{b} \quad (13)$$

introduced in [3] and where $\hat{\tilde{\mathbf{E}}}_k$ is a $(L-n+1) \times (n+1)$ Hankel matrix defined such that

$$\hat{\tilde{\mathbf{E}}}_k = \begin{bmatrix} \hat{e}_k(0) & \hat{e}_k(1) & \dots & \hat{e}_k(n) \\ \hat{e}_k(1) & \dots & \dots & \hat{e}_k(n+1) \\ \vdots & & & \vdots \\ \hat{e}_k(L-n) & \dots & \dots & \hat{e}_k(L) \end{bmatrix}, \quad (14)$$

to remark that (10) can effectively be written

$$\text{Trace}[\mathbf{B}(\hat{\mathbf{B}}^H \hat{\mathbf{B}})^{-1} \mathbf{B}^H \hat{\mathbf{E}}_S \hat{\mathbf{\Lambda}} \hat{\mathbf{E}}_S^H] = \mathbf{b}^H \hat{\mathbf{Q}} \mathbf{b} \quad (15)$$

where

$$\hat{\mathbf{Q}} = \sum_k^n \hat{\lambda}_k \hat{\tilde{\mathbf{E}}}_k^H (\hat{\mathbf{B}}^H \hat{\mathbf{B}})^{-1} \hat{\tilde{\mathbf{E}}}_k. \quad (16)$$

As (16) gives prominence, for solving (15) the $\hat{\mathbf{B}}$ must be estimated. It has been proved [12] that an initial consistent estimate $\tilde{\mathbf{b}}$ is obtained by taking $(\hat{\mathbf{B}}^H \hat{\mathbf{B}}) = \mathbf{I}$. Constraining the minimizer with $\|\mathbf{b}\|^2 = 1$, we obtained $\tilde{\mathbf{b}}$ as the eigenvector corresponding to the smallest eigenvalue of $\hat{\mathbf{Q}} = \sum_k^n \hat{\lambda}_k \hat{\tilde{\mathbf{E}}}_k^H \hat{\tilde{\mathbf{E}}}_k$. Making use of this consistent estimate, we form $\hat{\mathbf{B}}$ and (15) is solved with the constraint $\|\mathbf{b}\|^2 = 1$ by selecting the eigenvector associated with the smallest eigenvalue of (16). Lastly, the n DOA's are deduced from the n angle roots of (4).

4. PLEDGE

We deduce PLEDGE directly from MODE. Thanks to the polynomial rooting, we can easily write a new polynomial which integrates known zeros. In such a case, the polynomial defined in (4) can be factorized in the following manner

$$b_0 \prod_{i=1}^n (z - e^{j\omega_i}) = b_0 \prod_{i=1}^{n_k} (z - e^{j\omega_i}) \prod_{i=1}^{n_u} (z - e^{j\omega_i}) \quad (17)$$

where are introduced the two following polynomials

$$\mathcal{Q}_k(z) = q_0 \prod_{i=1}^{n_k} (z - e^{j\omega_i}) = q_0 z^{n_k} + \dots + q_{n_k} \quad (18)$$

$$\mathcal{Q}_u(z) = \bar{b}_0 \prod_{i=1}^{n_u} (z - e^{j\omega_i}) = \bar{b}_0 z^{n_u} + \dots + \bar{b}_{n_u}, \quad (19)$$

which are respectively the polynomials whose zeros are the known DOA's and the unknown DOA's. The \mathbf{b} vector can therefore be formulated in algebraic correspondence. It is enough for this to convolve the coefficients of the two polynomials $\mathcal{Q}_k(z)$ and $\mathcal{Q}_u(z)$ and it yields

$$[b_0 \ b_1 \ \dots \ b_n]^T = \mathbf{C}^T \bar{\mathbf{b}} \quad (20)$$

where

$$\bar{\mathbf{b}} = [\bar{b}_0 \ \bar{b}_1 \ \dots \ \bar{b}_{n_u}]^T$$

$$\mathbf{C} = \begin{bmatrix} q_0 & q_1 & \dots & q_{n_k} & & \mathbf{0} \\ & \ddots & \ddots & & \ddots & \\ \mathbf{0} & & q_0 & q_1 & \dots & q_{n_k} \end{bmatrix}.$$

PLEDGE estimates the n_u unknown DOA's from the polynomial of coefficients $\{\bar{b}_i\}$. To solve this problem, we substitute (20) into (15) and the following modified quadratic optimization problem is deduced

$$\text{Trace}[\mathbf{B}^H \hat{\mathbf{E}}_S \hat{\mathbf{\Lambda}} \hat{\mathbf{E}}_S^H \mathbf{B}] = \bar{\mathbf{b}}^H \bar{\mathbf{Q}} \bar{\mathbf{b}} \quad (21)$$

with $\bar{\mathbf{Q}} = \mathbf{C} \mathbf{Q} \mathbf{C}^H$. The first consistent estimate is the eigenvector associated with the smallest eigenvalue of $(\mathbf{C} \mathbf{C}^H)^{-1} \bar{\mathbf{Q}}$. We stress on the fact that as it might not be an evidence, the best performances are obtained when the constraint is equal to $\|\mathbf{b}\|^2 = 1$

and not $\|\bar{\mathbf{b}}\|^2 = 1$. Then, the estimation of the n_u DOA's is obtained by replacing the consistent estimates $\hat{\mathbf{b}}$ into (16) to once again form the $\hat{\mathbf{B}}$ matrix, find the PLEDGE estimate $\hat{\mathbf{b}}$ by selecting from the smallest eigenvalue of $(\mathbf{C}\mathbf{C}^H)^{-1}\mathbf{C}\mathbf{Q}_2\mathbf{C}^H$ its associated eigenvector. Finally the angles of the n_u polynomial roots achieve the unknown DOA's.

5. PLEDGE CRB

The CRB proposed in [2] is based on a subspace reduction by an orthogonal deflation method which guarantees to suppress the known informations present. This bound, named the Prior-CRB (P-CRB), has been derived under the deterministic assumption. The results and analysis which were given, pointed out that exploiting the prior-information by an orthogonal projection was not an optimal method. Actually the main observations are : (1) the $P\text{-CRB} = \text{CRB}$ in general conditions and (2) the $P\text{-CRB} < \text{CRB}$ when sources associated with the known and unknown directions are correlated. So, in such a procedure, the only way to improve the estimation of the unknown DOA's is to be in presence of full correlated sources. We can advance two reasons for that. One is surely the determinist derivation of the P-CRB, and the other could be felt by the fact we project the data leading to an irremediable information loss. Starting from this admission, we propose in this section the stochastic CRB based on PLEDGE model and thereby named PLEDGE CRB.

We drew our inspiration from [11] to formulate and derive the PLEDGE CRB. Accordingly, let

$$\boldsymbol{\alpha} = [\boldsymbol{\omega}_u^T \boldsymbol{\rho}^T \sigma^2]^T \quad (22)$$

be the unknown parameter vector, where $\boldsymbol{\omega}_u = [\omega_1 \dots \omega_u]^T$. Actually $\boldsymbol{\omega}_u$ is the set of unknown DOA's and indices here have no importance, they are arbitrary and help the mathematical formulation. The $n^2 \times 1$ vector $\boldsymbol{\rho}$ is made from $\{\mathbf{P}_{ii}\}$ and $\{\text{Re}(\mathbf{P}_{ij}), \text{Im}(\mathbf{P}_{ij}) \text{ for } j > i\}$. Under the previous assumptions and the Gaussian hypothesis, the Fisher Information Matrix (FIM) for the parameter vector $\boldsymbol{\alpha}$ is given by

$$\text{FIM}_{p,q} = N \text{Trace} \left[\frac{\partial \mathbf{R}}{\partial \alpha_p} \mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \alpha_q} \mathbf{R}^{-1} \right] \quad (23)$$

for $p, q = 1, \dots, n_u + n^2 + 1$

with the true covariance matrix $\mathbf{R} = \mathbf{A}\mathbf{P}\mathbf{A}^H + \sigma^2\mathbf{I}$. The further derivation steps require first the vectorization of (23)

$$\frac{1}{N} \text{FIM} = \left(\frac{\partial \mathbf{r}}{\partial \boldsymbol{\alpha}^T} \right)^H (\mathbf{R}^{-T} \otimes \mathbf{R}^{-1}) \left(\frac{\partial \mathbf{r}}{\partial \boldsymbol{\alpha}^T} \right) \quad (24)$$

with \otimes the kronecker product and

$$\mathbf{r} = \text{vec}(\mathbf{R}) = (\mathbf{A}^* \otimes \mathbf{A}) \text{vec}(\mathbf{P}) + \sigma^2 \text{vec}(\mathbf{I}), \quad (25)$$

and second the following partitioning

$$(\mathbf{R}^{-T/2} \otimes \mathbf{R}^{-1/2}) \left[\frac{\partial \mathbf{r}}{\partial \boldsymbol{\omega}_u^T} \middle| \frac{\partial \mathbf{r}}{\partial \boldsymbol{\rho}^T} \frac{\partial \mathbf{r}}{\partial \sigma^2} \right] = [\mathbf{G}_u | \boldsymbol{\Delta}] \quad (26)$$

which leads to

$$\begin{aligned} \frac{1}{N} \text{FIM} &= \begin{bmatrix} \mathbf{G}_u^H \\ \boldsymbol{\Delta}^H \end{bmatrix} [\mathbf{G}_u \boldsymbol{\Delta}] \\ &= \begin{bmatrix} \mathbf{G}_u^H \mathbf{G}_u & \mathbf{G}_u^H \boldsymbol{\Delta} \\ \boldsymbol{\Delta}^H \mathbf{G}_u & \boldsymbol{\Delta}^H \boldsymbol{\Delta} \end{bmatrix}. \end{aligned} \quad (27)$$

We conduct the derivation for the upper left submatrix $\mathbf{G}_u^H \mathbf{G}_u$ along with the methodology of [11]. With no surprise, the final result, agreeing with an intuitive approach, gives us an expression of the PLEDGE CRB

$$\text{PLEDGE CRB}(\boldsymbol{\omega}_u) = \frac{\sigma^2}{2N} \left[\text{Re} \left\{ \left(\mathbf{D}_u^H \boldsymbol{\Pi}_A^\perp \mathbf{D}_u \right) \odot \left(\mathbf{P}_u^H \mathbf{A}^H \mathbf{R}^{-1} \mathbf{A} \mathbf{P}_u \right)^T \right\} \right]^{-1} \quad (28)$$

with \odot the Hadamard product and where the partitioning $\mathbf{P} = [\mathbf{P}_u | \mathbf{P}_k]$ is considered. Once again this is an arbitrary choice and the \mathbf{P}_u matrix is composed by the columns of \mathbf{P} associated with the sources whose directions are unknown. Lastly, $\mathbf{D}_u = [d_1 \dots d_{n_u}]$, $d_i = (da(\omega_i)/d\omega_i)$.

6. SIMULATIONS

In all the plots attached, we have two sources impinging on a ULA with a half-wavelength distance between the sensors. The sources have the directions θ_1 and θ_2 respectively. We define the power of each source with respect to the correlation matrix as

$$\frac{\mathbf{P}}{\sigma^2} = \begin{bmatrix} 10 \frac{\text{SNR}_1}{10} & \\ \rho^* & 10 \frac{\text{SNR}_2}{10} \end{bmatrix}$$

where σ^2 is kept equal to one for each simulation and ρ is the correlation. The DOA of interest is the second one, namely θ_2 . The number of sensors, snapshots, the power of the sources are varying along with the figures but the two DOA's are located at $\theta_1 = 10^\circ$ and $\theta_2 = 12^\circ$ for all experiments concerning Fig.1 and lastly each results are the mean of 1000 independent trials. We compare PLEDGE to MODE and P-MUSIC [2] except we have improved the performance of the latter by using the noise-free sample covariance matrix instead of directly the sample one (i.e. $\hat{\mathbf{R}} - \hat{\sigma}^2 \mathbf{I}$ instead of $\hat{\mathbf{R}}$). We have plotted the stochastic CRB for n parameters, the PLEDGE CRB, the CRB for uncorrelated sources of Jansson et al [7] and the CRB for n_u unknown DOA's, $n_u = 1$ in the context of these simulations.

6.1. On PLEDGE performances

First, PLEDGE achieves the PLEDGE CRB whatever be the scenario. This fact concludes that PLEDGE is an optimal solution to the prior knowledge-based DOA estimation. Next, the set of Fig.1 and Fig.2's plots show the benefit of using PLEDGE instead of MODE. For example, one would wonder what could be the gain brought by PLEDGE when the sources of interest are less or much less powerful than the ones which are known. An hint can be found on Fig.1-(b) which reveals that the θ_2 's estimation has been improved. To be sure, one can observe on this figure that when the source of interest is half powerful than the known one, having then $\text{SNR}_2/\text{SNR}_1 = 0.5$, the gain is greatly significant. For equipower sources the advantage could be used at low SNR, less than 10dB, where the gain can nearly reach 5dB. PLEDGE is thus a possible solution to improved the precision into the threshold. We also show that PLEDGE could be much less sensitive to the correlation than MODE and naturally than P-MUSIC which is suboptimal, to be convinced see Fig.2. Lastly, we can see on Fig.1-(c) that the P-MUSIC is better than PLEDGE at low SNR. For this SNR range, P-MUSIC follows a CRB that we named the PLEDGE uncorrelated CRB. That CRB takes into account the prior knowledge of uncorrelated sources and the knowledge of θ_1 (or generally n_k known DOA's), which are the implicit hypothesis of the P-MUSIC algorithm. Due to the lack of space and since this bound is not the key contribution of this work, no closed-form expression of this bound was brought in this paper. That, will be done in a future work. These additionally assumptions on prior-informations introduce the next reflexion.

6.2. On prior-knowledge

We develop herein a study on using prior-knowledge. We base our reflexion on the CRBs plotted. Compared to the CRB for n parameters of interest, the PLEDGE CRB which is of reduced dimension, shows that the prior-information is useful especially at low SNR. In addition, increasing the number of known parameters should increase that difference. Pay now attention to the CRB for uncorrelated sources. Make use of this prior hypothesis leads to derive the FIM with $2n + 1$ parameters instead of $n^2 + n + 1$ for the CRB. The numbers of FIM elements for the PLEDGE is $n_u + n^2 + 1$. Then we could have thought that knowing the sources are uncorrelated could have improved more significantly the variance. We can verify this with the help of Fig.1-(c). However for the 2 DOA's case, we can have a scenario from which knowing some directions is better than knowing the uncorrelation state of the sources. That fact is illustrated by Fig.1-(d). We conclude by saying that the minimum variance is obtained when we couple together both the assumption of uncorrelated sources and the assumption of known directions. The corresponding derivation leads to the PLEDGE uncorrelated CRB. This bound has by far the best gain, whatever be the scenario. It is then necessary to give the PLEDGE for uncorrelated sources. This future work will be the optimal prior-knowledge algorithm taking into account both the correlation and the direction. We can imagine furthermore that this new algorithm could be a new tool to test the assumption of correlated/uncorrelated sources.

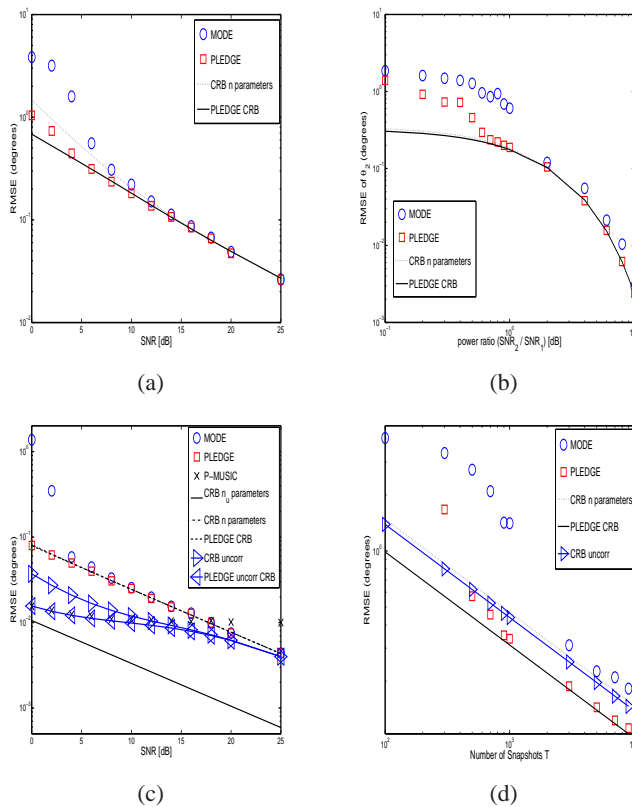


Fig. 1. (a) 5 sensors, 1000 snapshots and equipower sources, (b) $SNR_1 = 4$ dB, 10 sensors and 100 snapshots, (c) 10 sensors, 100 snapshots $SNR_1 = 4$ dB and $SNR_2 = 14$ dB, (d) $SNR=2$ dB, 6 sensors and equipower sources.

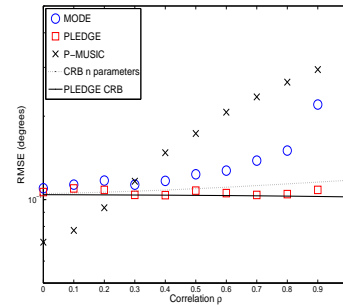


Fig. 2. 10 sensors, 100 snapshots $SNR = 2$ dB, $\theta_1 = 10^\circ$, $\theta_2 = 14^\circ$ with equipower.

7. UTILITY FOR MECHANICAL DIAGNOSIS

The mechanical diagnosis seems to be an attractive application by essence. Indeed, many rotating systems generates frequencies linked to the rotation of gears or rotor and stator in induction motors. Thus, acquiring a vibratory signal or a current signal brings out informations on the mechanical structure and healthiness of the system. In addition, thanks to the kinematic, some frequencies are known and either do not bring out useful information or prevent the frequencies of interest from being clearly estimated. The application concerned corresponds to the second case mentioned and we will depict the utility of PLEDGE on the estimation of two sideband components masked by a much powerful one. Those frequencies are besides of major importance in rotor bars diagnosis [5].

The following explanations are strongly inspired from <http://www.ieee-kc.org/library/motors/motorslip.htm>. An induction motor consists of two basic assemblies : a stator and a rotor. The name "induction motor" comes from the alternating current induced into the rotor via the rotating magnetic flux produced in the stator. Motor torque is developed from the interaction of currents flowing in the rotor bars and the stators' rotating magnetic field. In actual operation, rotor speed always lags the magnetic field's speed, allowing the rotor bars to cut magnetic lines of force and produce useful torque. This speed difference is called slip speed. The slip is therefore defined as $s = \frac{v_s - v}{v_s}$ expressed in percent where v is the actual speed and v_s the synchronous speed (stator). For industrial induction motors, the slip is of 1% or 2%. As it is developed in [5], a useful indicator of broken rotor bars is the sideband components around the on line frequency. These sidebands are located in the current spectrum at frequencies $f_s = (1 \pm 2s)f_{line}$ where f_{line} is the network frequency, say the 50 Hz. Accordingly, the frequencies of interest are closed from the on line frequency and since the acquisition signal is the current one, the dynamic of the 50 Hz is very high. There is another drawback caused by the stability of the network frequency. In France, the electrical producer company ensures the stability of the network frequency on a day, meaning that for a while this frequency moves. To estimate the sideband components, the Fourier transform needs a long observation time to increase the precision but the effect expected is exactly the opposite because of the fluctuation of the 50 Hz. The more data you acquire, the less accurate is the estimation. That is illustrated by Fig.3. From this figure the sideband frequencies would have been visible on each side of the 50 Hz lobe but due to instability of the network frequency the Fourier analysis is ineffective. The problem is then to acquire a weak number of data and to have the best precision with possibly get rid of the 50 Hz which does not bring out information and is awkward for the estimation. So, PLEDGE seems to be exactly what we need and even if this algorithm has been derived for the array

processing we will show that it is suitable in the spectral analysis case.

7.1. Experimental conditions

We have used the current signal acquired from an induction motor (<http://www.laspi.fr>). The machine as a weak broken bars and thus two sideband components should appear at frequencies 51,17 Hz and 48,83 Hz. The experimental conditions are the following : we have acquired 60000 samples, say 100 revolutions, at the sample frequency $f_e = 25,6$ KHz of the current signal and we have decimated of a factor 200 leading to 3 samples per revolution. PLEDGE needs an estimation of the signal subspace by eigendecomposition of the sample covariance matrix. In the context of the spectral line search, we use an Hankel matrix formed by the data samples instead and the MDL technic to estimate the dimension of the signal subspace, see [1] and the references therein. The MDL estimator has given 30 parameters of interest. The 50 Hz network frequency is first estimated by MODE and we use this estimation as the known frequency in PLEDGE.

7.2. Results

The frequencies estimated by PLEDGE are reported inside Table 1. We focus on the area around the 50 Hz since the key information on the rotor bars healthiness is present in this area. first, PLEDGE has totally suppress the known frequency, that is to say the 50 Hz and secondly the grey cells give prominence to the diagnosis of the broken bars since the frequencies estimated are 48.87 Hz and 50.98 Hz. So, we have shown that PLEDGE was able to estimate 2 frequencies with much less power than the one which does not carry any information. Consequently, PLEDGE seems to be adapted for the diagnosis of mechanical defects, its high resolution property allows to acquire a weak number of samples, struggling then the network frequency deviation and improving the computational cost of the processing.

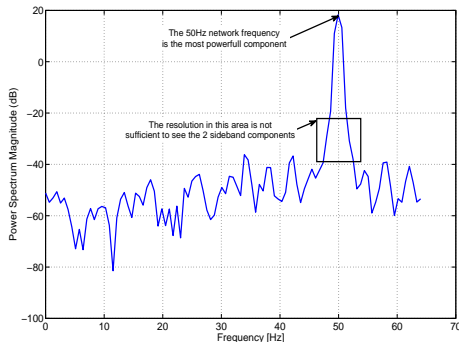


Fig. 3. Power Spectral Density (PSD) of current signal with 300 samples and a sampling frequency of 128 Hz.

Estimated frequencies [Hz] by PLEDGE					
0.15	1.24	6.13	9.76	14.74	15.64
18.05	22.61	23.71	25.17	25.68	26.76
30.04	31.70	34.19	35.49	37.86	39.12
41.20	42.09	45.45	48.87	50.98	52.26
54.69	57.83	58.93	61.92	62.67	63.49

Table 1. The gray cells correspond to the 2 sideband frequencies (slip frequencies)

8. CONCLUSION

In this paper we have proposed an optimal **Prior-knowLEDGE (PLEDGE)** method based on the method of direction estimation (MODE) approach. PLEDGE exploits the information of known DOA's to better estimate the unknown ones. To show the benefic of incorporating known information we have also proposed the corresponding stochastic CRB. The simulation results showed that PLEDGE could significantly help the estimation of unknown DOA's especially when the sources corresponding to the unknown DOA's are much less powerful than those which are known. We saw that PLEDGE has good performances into the threshold and is robust to the correlation between the sources as well, even at low SNR. Finally, we have presented an induction motor diagnosis problem. The diagnosis consists of estimating to slip frequencies masked by a close and much powerful one. PLEDGE was totally in adequation and we showed that PLEDGE solved this problem.

9. ACKNOWLEDGMENT

G. Bouleux thanks the Rhone-Alpes local government for having financially supported his stay at the university of Uppsala, Sweden and R. Boyer thanks both the Ile-de-France local government and Digiteo Research Park.

10. REFERENCES

- [1] G. Bouleux, A. Ibrahim, F. Guillet, and R. Boyer. A Subspace-Based Rejection Method for Detecting Bearing Fault in Asynchronous Motor. In *IEEE International Conference on Condition Monitoring and Diagnosis (CMD 08)*, Beijing, 2008.
- [2] R. Boyer and G. Bouleux. Oblique Projections for Direction-Of-Arrival Estimation with Prior Knowledge. *IEEE Trans. on Signal Processing*, 56(4):1374–1387, April 2008.
- [3] Y. Bresler and A. Macovski. Exact Maximum Likelihood Parameter Estimation of Superimposed Exponential Signals in Noise. *IEEE ASSP Magazine*, 34:1081–1089, 1986.
- [4] H. Chen, S. V. Huffel, D. V. Ormond, and R. de Beer. Parameter Estimation with Prior Knowledge of Known Signal Poles for the Quantification of NMR Spectroscopy Data in the Time Domain. *Journal of Magnetic Resonance*, 119(2):225–234, 1996.
- [5] C. Concari, G. Franceschini, and C. Tassoni. Differential Diagnosis Based on Multivariable Monitoring to Assess Induction Machine Rotor Conditions. *IEEE Trans. on Industrial Electronics*, 55(12):4156–4166, Dec. 2008.
- [6] A. B. Gershman and P. Stoica. On Unitary and Forward-Backward MODE. *Digital Signal Processing*, 9:67–75, 1999.
- [7] M. Jansson, B. Göransson, and B. Otersten. A Subspace Method for Direction of Arrival Estimation of uncorrelated Emitter Signals. *IEEE Trans. on Signal Processing*, 47(4):945–956, April 1999.
- [8] D. Linebarger, R. DeGroat, E. Dowling, P. Stoica, and G. Fudge. Incorporating a Priori Information Into MUSIC-Algorithms and Analysis. *Signal Processing*, 46(1), 1995.
- [9] B. Ottersten, M. Viberg, P. Stoica, and A. Nehorai. *Exact and Large Sample ML techniques for Parameter Estimation and Detection in Array Processing*. Radar Array Processing (second ed.). Springer-Verlag, New-York, 1999.
- [10] P. Stoica and M. Jansson. On Forward-Backward MODE for Array Signal Processing. *Digital Signal Processing*, 7:239–252, 1997.
- [11] P. Stoica, E. G. Larsson, and A. B. Gershman. The Stochastic CRB for Array Processing : A Textbook derivation. *IEEE Signal Processing Letters*, 8(5):148–150, May 2001.
- [12] P. Stoica and K. Sharman. Maximum Likelihood Methods for Direction Of Arrival Estimation. *IEEE Trans. on Acoustics Speech and Signal Processing*, 38:1132–1143, 1990.