INTERFERENCE MITIGATION USING WIDELY LINEAR ARRAYS

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ABSTRACT

Widely Linear Processing (WLP), proposed by Brown and Crane in 1969, has recently received a great deal of attention, in particular due to its potential of application in some important problems in array processing. The present work investigates the implementation of the linearly constrained minimum variance (LCMV) algorithm proposed by Frost in the context of widely and strictly linear processing. The results are compared with those obtained via MVDR widely and strictly linear processing.

1. INTRODUCTION

The use of signal processing tools for interference mitigation in the context of RF telecommunications has been investigated along many decades. One important issue concerns the application of array processing in a multiuser scenario, where the classical techniques of linear (optimal and/or adaptive) filtering have found a large range of application [1]. On the other hand, Widely Linear Processing (WLP), proposed by Brown and Crane in 1969 [2], has recently received a great deal of attention, in particular due to its potential of application in some important problems in array processing. Indeed, in many cases, the involved signals are non-circular, so that the use of widely linear processing should represent an advantage in terms of performance when compared with Strictly Linear Processing (SLP). This raises promising possibilities of performance enhancement in interference mitigation in wireless communications, where the propagation and multipath effects introduce non-circular effects [3, 4].

For instance, Chevalier et al. [5] proposed a time-invariant widely linear minimum variance distortionless response (MVDR) beamformer for the optimal reception of an unknown signal. The properties and the performance of the approach were analysed and results of implementation in a noncircular context were presented. In another work, Chevalier et al. [6] proposed a new array-based receiver associated with several unknown signal parameters. The detection of a known BPSK signal corrupted by noncircular interference was considered and it was shown that the performance of the receivers was enhanced. Additionally, the capability of the new detectors of performing single-antenna cancellation of rectilinear interferences was verified.

The mentioned works open interesting perspectives, like studying the performance of an adaptive antenna array by analysing the process of interference mitigation under different situations involving time-variant BPSK signals corrupted by noncircular interferences. In this sense, the present work investigates the implementation of the linearly constrained minimum variance (LCMV) algorithm proposed by Frost [7] in the context of widely and strictly linear processing. The results are compared with those obtained via MVDR widely and strictly linear processing.

In this sense, the present work introduces the linearly constrained minimum variance (LCMV) algorithm proposed by Frost [7] in the context of widely linear processing (FROST-WL) and compares its performance with the strictly linear implementation of the LCMV (FROST-SL). Moreover, the results obtained with the FROST-WL are compared with those obtained via MVDR widely (MVDR-WL) and strictly (MVDR-SL) linear processing. Another novelty presented in this article is the use of the link error rate performance instead of theoretical SINR in the comparisons.

The paper is organized as follows: Section 2 presents the constrained spatial filtering problem, including the linearly constrained minimum variance (LCMV) algorithm and the MVDR technique. The mathematical foundations of widely linear processing are introduced in section 3. Section 4 is devoted to the simulations results and discussions. Some concluding remarks in section 5 close the work.

2. CONSTRAINED SPATIAL FILTERING

In digital communications, special attention must be paid to the eventual presence of interfering signals due to the pervasive requirement that several users share limited resources in an orderly way, i.e., without degenerating into a noxious interference process. A widespread solution to achieve this aim is the use of an *adaptive antenna array*, a device formed by a set of antennas ordered in a chosen geometry (usually linear or planar) and endowed with adjustable gains, as shown in Figure 1.

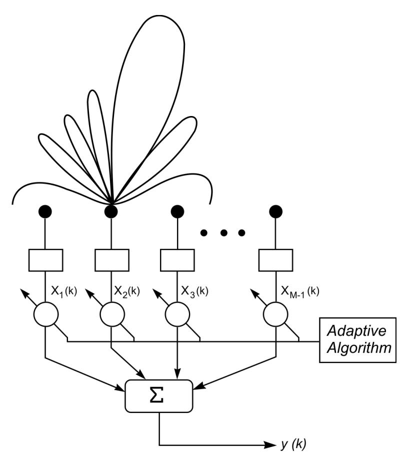

Figure 1 – Adaptive antenna array.

The array works as a *spatial filter*, because an adequate choice of its parameters can either amplify or cancel signals coming from distinct directions. Consequently, the structure is able to separate signals of interest from undesirable interferers, exactly in the spirit of the problem we posed. A crucial issue concerns the determination of the parameters of the array in order to engender a given desired response. Two possibilities are studied in this work: the classical MVDR (Minimum Variance Distortionless Response) technique and the Frost algorithm, which is based on the knowledge of the directions of arrival (DOAs) of the desired signals. For both possibilities, strictly and widely linear processing will be investigated. First the model to be employed along the present work must be posed.

2.1 Signal Model

We consider a uniform linear array (ULA), in which sensors are equally spaced by a distance d equal to one half of the wavelength along a straight line. The array is illuminated by isotropic sources located in the far field, such that a plane waves impinge on the array. These signals are digitally modulated, which means that their samples belong to a finite alphabet.

Consider that N_S narrowband signals $s_1(k), s_2(k), \ldots, s_{Ns}(k)$ impinge on the M-element ULA with DOAs $\theta_1, \theta_2, \ldots, \theta_{Ns}$. The signals are uncorrelated and composed of i.i.d.. The array input vector, which contains the signals at all antennas, sampled at the instant k, can be written as:

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{v}(k), \quad (1)$$

where $\mathbf{s}(k)$ denotes the vector containing the samples of the signals, $\mathbf{v}(k)$ is the sensor noise vector assumed to be formed by zero mean complex white Gaussian noise samples, and the matrix:

$$\mathbf{A} = [\mathbf{a}(\phi_1) \quad \mathbf{a}(\phi_2) \quad \cdots \quad \mathbf{a}(\phi_{Ns})] \quad (2)$$

contains the *steering vectors* $\mathbf{a}(\phi_k) = [1 \; e^{-j\phi_k} \; e^{-j2\phi_k} \; \cdots \; e^{-j(M-1)\phi_k}]^T$, where $\phi_k = (2\pi d/\lambda)\sin(\theta_k) + \beta$ and T is the transposition operator. The electrical angle ϕ is related to the incidence angle θ of a plane wave, measured with respect to the normal to the linear array. The incidence angle θ lies within the range $-\pi/2$ to $\pi/2$ and the angle ϕ may be assumed to lie within the range $-\pi < \phi \leq \pi$. The phase offset β changes from 0° to 360°. Given the filter output:

$$y(k) = \mathbf{w}^H(k)\mathbf{x}(k), \quad (3)$$

the objective is to minimize the mean square error (MSE):

$$\varepsilon(k) = E\left[|s_D(k) - y(k)|^2\right], \quad (4)$$

with respect to the parameters of the spatial filter, $\mathbf{w}(k)$, where $s_D(k)$ is the value of a desired signal at the sample k.

2.2 MVDR Technique

The Linearly Constrained Minimum Variance (LCMV) beamformer consists in minimizing the variance of the output error subject to a set of constraints that can be represented by

$$\mathbf{C}^T\mathbf{w}(k) = \mathbf{f} \quad (5)$$

where $\mathbf{C}$ and $\mathbf{f}$ are pre-established.

The MVDR technique is a special case of the LCMV beamformer, reached when the constraint is restricted to ensuring unit gain in a given direction. In such case, the optimal solution is given by [8]:

$$\mathbf{w}_o = \mathbf{R}_{xx}^{-1}\mathbf{a}(\phi_D)[\mathbf{a}^H(\phi_D)\mathbf{R}_{xx}^{-1}\mathbf{a}(\phi_D)]^{-1} \quad (6)$$

where $\mathbf{R_{xx}} = E[\mathbf{xx}^H]$. In simple terms, this beamformer seeks the condition of minimum output variance under the constraint, producing a distortionless response along the direction of the desired electrical angle ϕ_D. Based upon this result, it is possible to express the output variance as a function of the DOA, which leads to a MVDR spatial power spectrum [8]:

$$S_{MVDR}(\phi_D) = [\mathbf{a}^H(\phi_D)\mathbf{R}_{xx}^{-1}\mathbf{a}(\phi_D)]^{-1} \quad (7)$$

2.3 Frost Algorithm

The Frost (FROST) algorithm is a constrained LMS-based adaptive technique to attain the optimal parameters in (5). The procedure is given by the following updating expressions [7]:

$$\mathbf{w}(k+1) = \mathbf{P}[\mathbf{w}(k) - \mu y(k)\mathbf{x}(k)] + \mathbf{F}$$

$$\mathbf{w}(0) = \mathbf{F} \quad (8)$$

where μ is the step size, $\mathbf{F} \triangleq \mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{f}$ and $\mathbf{P} \triangleq \mathbf{I} - \mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T$.

3. WIDELY LINEAR PROCESSING

Pincibono and Chevalier [9] proposed in 1995 the utilization of the received signal and its complex conjugate for the optimal filter determination. In this way, the filter output becomes:

$$y(k) = \mathbf{u}^H(k)\mathbf{x}(k) + \mathbf{v}^H(k)\mathbf{x}^*(k) \quad (9)$$

where $(.)^*$ denotes the complex conjugate. From the orthogonality principle, in order to reach the minimum of cost function expressed by (4), it is necessary and sufficient that the Wiener filter coefficients are such that the error ε is orthogonal to the samples of the filter input vector, that is, orthogonal to all vector elements of $\mathbf{x}$ and $\mathbf{x}^*$. In this way, $E[y^*\mathbf{x}] = E[s_D^*\mathbf{x}]$ and $E[y^*\mathbf{x}^*] = E[s_D^*\mathbf{x}^*]$. After some algebraic manipulations, we can write:

$$\mathbf{R_{xx}u} + \mathbf{C_{xx}v} = \mathbf{r} \quad (10)$$

$$\mathbf{C_{xx}u} + \mathbf{R_{xx}}^*\mathbf{v} = \mathbf{z} \quad (11)$$

where $\mathbf{C_{xx}} = E[\mathbf{xx}^T]$, $\mathbf{r} = E[y^*\mathbf{x}]$ and $\mathbf{z} = E[s_D\mathbf{x}]$. The system solution is:

$$\mathbf{u} = [\mathbf{R_{xx}} - \mathbf{C_{xx}}(\mathbf{R_{xx}}^*)^{-1}\mathbf{C_{xx}}^*][\mathbf{r} - \mathbf{C_{xx}}(\mathbf{R_{xx}}^{-1})^*\mathbf{z}^*]$$
$$\mathbf{v} = [\mathbf{R_{xx}}^* - \mathbf{C_{xx}}^*\mathbf{R_{xx}}^{-1}\mathbf{C_{xx}}^*]^{-1}[\mathbf{z}^* - \mathbf{C_{xx}}^*\mathbf{R_{xx}}^{-1}\mathbf{r}] \quad (12)$$

System solution analysis allows concluding that, in the worst case, when both $\mathbf{C_{xx}}$ and $\mathbf{z}$ are equal to zero, the WLP leads to a performance equal to that of the SLP. In other cases, as that of QPSK modulated signals corrupted by rectilinear interference (e.g. BPSK), the WLP performs better than the SLP.

3.1 Widely Linear MVDR Technique

In the WLP MVDR beamforming formulation, the received data $\mathbf{x}(k)$ and its complex conjugate $\mathbf{x}^*(k)$ are taken into account to determine the optimal filter. For this case, $\tilde{\mathbf{x}}(k) \triangleq [\mathbf{x}(k)^T \, \mathbf{x}(k)^H]^T$, and the constraints become $\tilde{\mathbf{w}}^H\mathbf{a}_1(\phi_D) = 1$ and $\tilde{\mathbf{w}}^H\mathbf{a}_2(\phi_D) = 0$, where $\mathbf{a}_1(\phi_D) \triangleq [\mathbf{a}(\phi_D)^T, \mathbf{0}_M^T]^T$, $\mathbf{a}_2(\phi_D) \triangleq [\mathbf{0}_M^T, \mathbf{a}(\phi_D)^H]^T$ and $\mathbf{0}_M$ is the zero vector. The WLP solutions is given by

$$\tilde{\mathbf{w}}_{MVDR} \triangleq \mathbf{R}_{\tilde{x}\tilde{x}}^{-1}\mathbf{a}(\phi_D)[\mathbf{a}(\phi_D)^H\mathbf{R}_{\tilde{x}\tilde{x}}^{-1}\mathbf{a}(\phi_D)]^{-1}\mathbf{q} \quad (13)$$

where $\mathbf{q} \triangleq [1,0]^T$.

3.2 WLP Frost Algorithm

To formulate the Frost algorithm within the WLP framework, the received data are considered to be formed by $\mathbf{x}(k)$ and $\mathbf{x}^*(k)$. The solution is given by:

$$\tilde{\mathbf{w}}(k+1) = \tilde{\mathbf{P}}[\tilde{\mathbf{w}}(k) - \mu y(k)\tilde{\mathbf{x}}(k)] + \tilde{\mathbf{F}} \qquad (14)$$
$$\tilde{\mathbf{w}}(0) = \tilde{\mathbf{F}}$$

with $\tilde{\mathbf{C}} \triangleq [\mathbf{C}^T \mathbf{C}^H]^T$, $\tilde{\mathbf{F}} \triangleq \tilde{\mathbf{C}}(\tilde{\mathbf{C}}^T\tilde{\mathbf{C}})^{-1}\mathbf{f}$ and $\tilde{\mathbf{P}} \triangleq \mathbf{I} - \tilde{\mathbf{C}}(\tilde{\mathbf{C}}^T\tilde{\mathbf{C}})^{-1}\tilde{\mathbf{C}}^T$.

4. SIMULATION RESULTS

In order to study the performance of the array in the task of adaptive and static interference mitigation, the SER originated by SLP and WLP approaches were compared for different representative scenarios. The employed ULA is composed of a set of M = 2 omnidirectional antennas spaced by d. It is considered that a desired BPSK signal s_D impinges on the array and suffers the interference of P = 2 signals with interference-to-noise ratio (INR) of 20dB, being also subject to additive noise such that the signal-to-noise ratio (SNR) is 10dB. The DOA of the desired signal θ_D was changed from -90° to +90° in steps of 1° and, for each DOA, the resulting SER is calculated.

At the reception, the contributions of the interferers and the noise are added as described in (1), resulting in a complex stream of vectors $\mathbf{x}(k)$ that are applied to a spatial filter designed to mitigate the interference. The output filter signal $y(k)$ feeds a decision device that convert the received values into symbols belonging to the original alphabet. Then, the recovered symbols were compared with the transmitted symbols and the SER is calculated by dividing the number of errors by the number of transmitted symbols.

Two kinds of beamforming were considered in the simulations, MVDR and FROST, both in its SLP and WLP forms. For all the simulations the SER was evaluated over one million symbols.

4.1 Definitions

Three signals were considered in the simulations. The first one, named s_D, were considered as a desired signal and the others two, s_1 and s_2, as interferers. Each signal was formed according to:

$$s_i = \frac{A_i}{\left|\sqrt{(C_i - 1)^2 + 1}\right|}\left(S_{Ii} + j(C_i - 1)S_{Qi}\right) \qquad (15)$$

where i can assume values *1* or *2* for the interferers signals, or D for the desired signal, C_i is the circularity coefficient with real value such that $0 \leq C_i \leq 1$, S_{Ii} and S_{Qi} are i.i.d. BPSK symbols -1 or +1, and A_i represent the amplitude of the signals being A_D = 1 and $A_1 = A_2 = \sqrt{10}$. According to equation (15), circularity coefficient defines the relation between the real and imaginary parts of the signal. So, if C_i = 0 the signal s_i is totally circular as its real and imaginary parts have the same amplitude. If C_i = 1, the signal s_i is said to be rectilinear, as its imaginary part is null.

The signals defined by (15) impinge the array under DOAs θ_D, θ_1 and θ_2, related to the desired signal, interferer s_1 and interferer s_2, respectively. The electrical phase β_D of the signal s_D is considered zero for the whole set of simulations, while β_1 and β_2, related to the interferers s_1 and s_2 respectively, could be changed. For the whole set of simulations, C_D was considered to be 1.

4.1 Simulation Results for MVDR

In this section, simulation results for SLP and WLP MVDR are presented and compared for several situations.

4.1.1 Interferers Circularity Impact

In order to investigate de performance of WLP related to SLP when one of the interferers has its circularity coefficient changed, SER results were converted to the equivalent SNR related to an AWGN channel using the inverse of the probability distribution function of standardized Gaussian random variable, $Q(x)^{-1}$, given by:

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{(-t^2/2)}dt \qquad (16)$$

knowing the fact that $Q(x) = \sqrt{SNR}$, where x = SER is the probability of error of a BPSK modulation over an AWGN channel. For this set of simulations, θ_1 = 90°, θ_2 = 0°, $\beta_1 = \beta_2$ = 0°, C_2 = 1 and C_1 assumed several values. The SNR results for this situation when a WLP MVDR filter is used are illustrated in Figure 2.

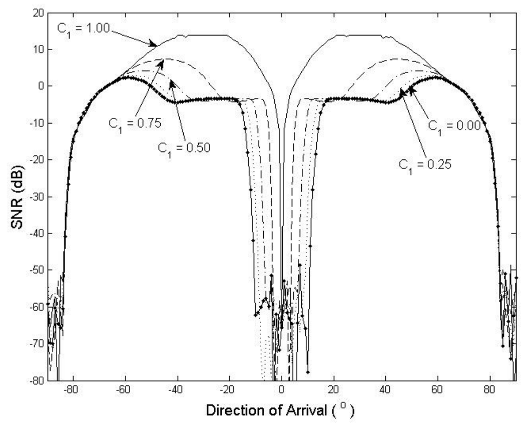

Figure 2 – Equivalent SNR for MVDR-WL

As expected, for $|\theta_D|$ = 90° and θ_D = 0°, the MVDR-WL beamformer cannot mitigate the interference, as those DOAs are coincident with the interferers DOA. As a result, due to a constellation superposition the SER is maximized, independently of the C_1 value. In the figure, the SER maximization is represented by very low values of the equivalent SNR. Conversely, when $|\theta_D|$ lies around 20° and 40° and C_1 = 1, the WLP MVDR completely nulls the interferers, and the resultant equivalent SNR tends to around 10dB. As C_1 changes towards to 0, the equivalent SNR is reduced.

Figure 3 shows the gain G of the MVDR-WL over the MVDR-SL using the equivalent SNR. In this case,

$$G = 10\log\left(\frac{SNR_{WL}}{SNR_{SL}}\right) \qquad (17)$$

where SNR_{WL} is the equivalent SNR related to the MVDR-WL beamformer and SNR_{SL} is the equivalent SNR related to the MVDR-SL. As can be seen, θ_D around 0° and ±90° results in no gain, indicating that, in those cases, WLP presents the same behavior of SLP. However, when s_1 is rectilinear WLP has performance considerable better than SLP, especially when for $|\theta_D|$ ranges from 20° and 40°. As C_1 decreases to zero, i. e., s_1 becomes circular, the gain decreases as well. Those results confirm the theoretical assumptions presented mainly in [4] and [5].

4.1.2 Interferers Phase Offset Impact

In the next two simulation sets, interferer s_1 phase offset β_1 was changed in order to SLP and WLP performances be compared.

Figures 4 and 5 show SER results when MVDR-SL and MVDR-WL were applied, in a scenario where θ_1 = 90° and θ_2 = 0°, all the impinging signals were BPSK, i. e., C_S, C_1 and C_2 were unitary, and β_2 = 0°.

It can be observed that, in the case of SLP, independently of the value of β_1, the maximum SER lies in the interval -20° ≤ θ_D ≤ +20°. Moreover, when β_1 = 0° the BPSK constellation of the interferer s_1

completely overlaps the BPSK constellation of the desired signal when $|\theta_D| = 90^o$, resulting in the worst case of SER. Only for $|\theta_D|$ around 30^o the maximum SER is not achieved.

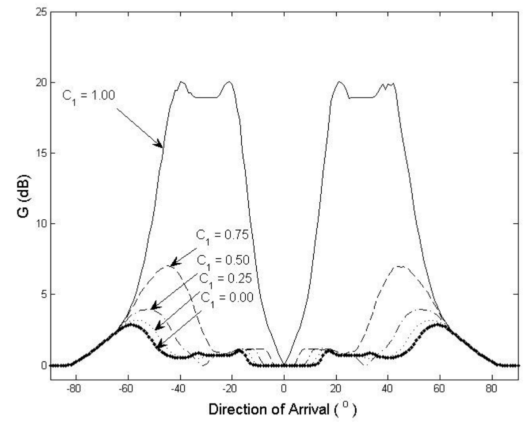

Figure 3 – WLP gain over SLP for MVDR

As the electrical phase offset grows, the SER starts to show lower values and an abrupt change in the SER behaviour happens for β_1 changing from 60^o to 80^o and 90^o. For this case, SER is null for $|\theta_D|$ larger than 60^o, however, for θ_D around 0^o, the SER shows the same behaviour presented for lower values of β_1.

The MVDR-WL beamformer, as shown in Figure 5, presents a better performance than the MVDR-SL beamformer. As expected, for $|\theta_D| = 90^o$ and $|\theta_D| = 0^o$, the SER is maximal, but, for $50^o \leq |\theta_D| \leq 15^o$, the SER is null for $\beta_1 = 0^o$. As β_1 changes towards 90^o, the SER grows, until it approaches the MVDR-SL beamformer. Another common characteristic is the abrupt change of behaviour for β_1 larger than 80^o.

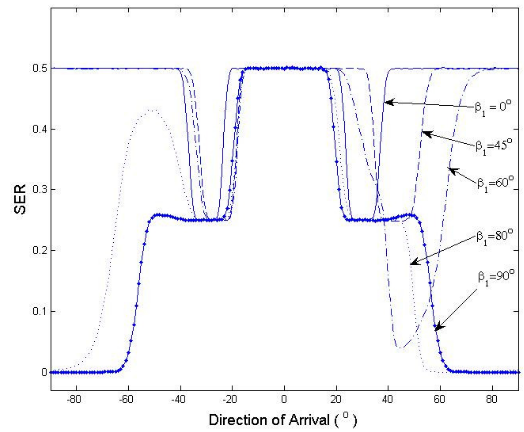

Figure 4 – MVDR-SL SER

Although presented in the literature [5], $\theta_1 = 90^o$ is, in a certain sense, an unusual DOA since, in practical scenarios, antenna arrays tend to cancel signals that are collinear with the array. In order to present WLP and SLP comparison in a more realistic scenario, Figures 6 and 7 present SER behaviors considering $\theta_1 = -45^o$ and $\theta_2 = +45^o$, keeping the other parameters (C_S, C_1 and C_2 unitary, and $\beta_2 = 0^o$).

For the MVDR-SL beamformer shown in Figure 6, as β_1 changes from 0^o towards 90^o, few modifications are noted in SER behavior, but, for β_1=80^o and $\beta_1 = 90^o$, SER becomes null for $|\theta_D|$ around 45^o. Significant changes can be noted when the MVDR-WL is used for this case. SER is null for almost all values of θ_D and achieves high values only around ±45^o, which are the values of interferers DOA.

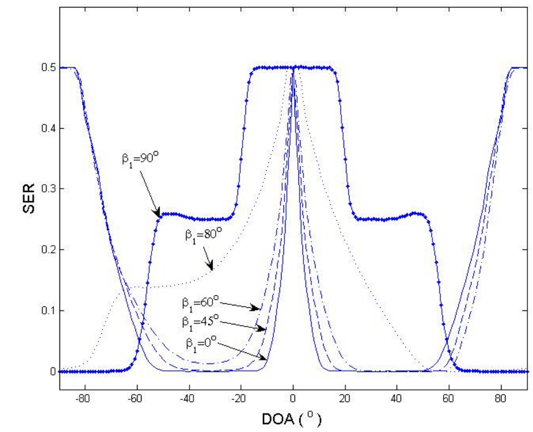

Figure 5 – MVDR-WL SER

However, as β_1 changes from 0^o to 90^o, SER around $\theta_D = -45^o$ decreases, reaching null values for $\beta_1 = 90^o$. These confirm the WLP superior performance in the task of interference mitigation when the interferers have their constellations in quadrature in relation to the desired signal constellation, as predicted theoretically in [3], [4] and [5].

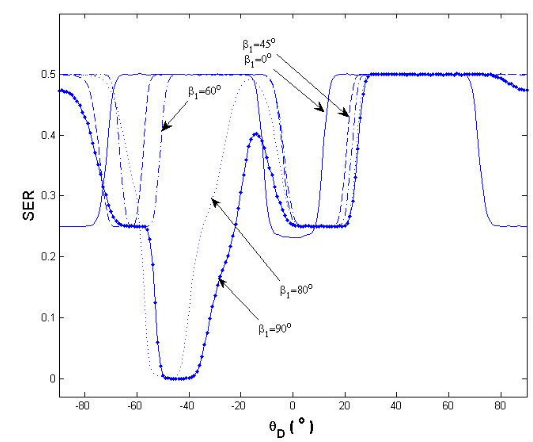

Figure 6 –MVRD-SL SER

4.2 Simulations Results for FROST

In order to evaluate FROST-SL and FROST-WL performances in terms of SER, the last scenario proposed in section 4.1 was used in the simulations. As mentioned, for this scenario, $\theta_1 = -45^o$, $\theta_2 = +45^o$, C_S, C_1 and C_2 are unitary, $\beta_2 = 0^o$ and β_1 is changed.

Figure 8 shows the SER behavior for a FROST-SL algorithm. A comparison with Figure 6, which represents the MVDR-SL SER results for the same situation, allows concluding that the FROST-SL algorithm has a similar performance. However, when using the WLP approach, the SER decreases more quickly as β_1 increases, as shown in Figure 9. In this case, for β_1 larger than 25^o, the SER around $\theta_D = -45^o$ is almost null.

In this case, it is important to remark that, in a real system, the interferers s_1 and s_2 probably would be modulated over different frequency carriers in relation to s_D. So, β_1 and β_2 would be changing continuously from 0 to 2π and the SER peak showed in Figures 7 and 9 presents a behavior of successive changes between $0 \leq SER \leq 0.5$. Considering this behavior in time, it is possible to establish the expected value for SER in a total cycle of β_1 for $\theta_D = -45^o$. In doing so, the MVDR-WL gives an average SER = 0.38 and the FROST-WL gives an average SER of 0.07, which means around 7 errors in a stream of 100 symbols. In this scenario, the SER could be mitigated by the use of some well known block error correcting code like the Hamming code.

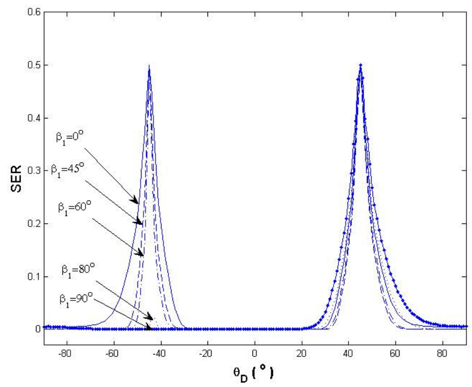

Figure **7 -** MVDR-WL SER

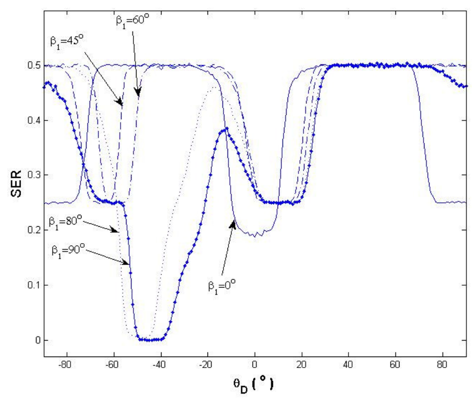

Figure 8 – FROST-SL SER

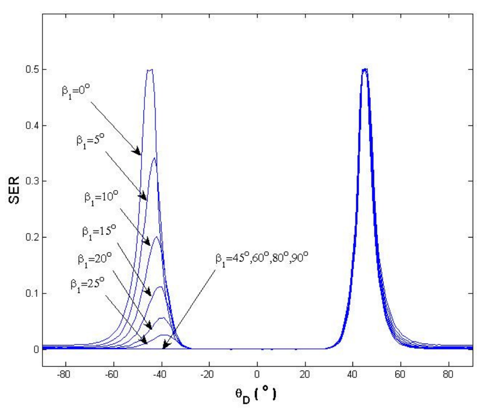

Figure 9 – FROST-WL SER

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5. CONCLUSION

In this work, the WLP was compared with SLP in a scenario of underparameterized array processing in terms of SER performance. The main contributions of this paper, supported by computer simulations, are the use of the link error rate performance instead of theoretical SINR in the comparisons evaluation and to have confirmed the ability of the linearly constrained minimum variance (LCMV) algorithm, proposed by Frost, to provide interference mitigation in the context of widely linear processing. The results were compared with those obtained via MVDR widely and strictly linear processing and also with the original (SLP) Frost algorithm. Several situations were considered, involving changing in DOA, circularity coefficient and electrical offset. For all proposed situations, it was shown that WLP has superior performance over SLP when using MVDR technique or when an adaptive processing is employed, like in the Frost algorithm. In the same way, it was shown that the adaptive approach outperforms the non-adaptive one. As an important remark, it was shown that when an interferer impinges on the array under the same DOA of a desired signal and its electrical offset is changing in time, the SER behaves in time in a cyclic way. In that case, it is possible to establish the SER mean value for an entire cycle. Using the LCMV algorithm, this value is low enough that allows to conclude that if an error correcting block code is used in the desired signal, the SER could be improved.

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