EFFECTS OF FREQUENCY-DEPENDENT ATTENUATION ON THE PERFORMANCE OF TIME DELAY ESTIMATION TECHNIQUES USING GROUND PENETRATING RADAR

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ABSTRACT

In this paper, we investigate the effects of frequency-dependent attenuation on the performance of some time delay estimation techniques applied to ground penetrating radar (GPR) data. The signal model is based on a complex power law of frequency for dielectric permittivity which describes wave propagation using two parameters, the quality factor Q and the phase velocity at an arbitrary reference frequency V_r. Hence, the adopted model deviates from the damped exponential model and it is this mismatch that is likely to deteriorate the performance of the employed techniques. At first, we carry out a sensitivity study by determining the variations of the relative root mean square error of the time delay estimates as a function of the SNR, Q and the product $B\tau$ for three algorithms, namely, root-MUSIC, ESPRIT and the matrix pencil method (MPM). These variations reveal a systematic error which is quantified by means of a first-order approximation and is found to be the ratio of the phase delay to the group delay. The bandwidth over which this approximation is reasonably accurate depends on the reference frequency and the quality factor of the medium. Then, we use this error to compensate for the bias introduced by the model mismatch with the aim of improving the estimates.

1. INTRODUCTION

Time delay estimation is a well-known problem encountered in many fields such as medical imaging, sonar, radar, and geophysics. For civil engineering applications in which the ground penetrating radar (GPR) is used as one of the nondestructive testing techniques, time delay is an important parameter for quantitative interpretation of data [1]. In the case of non-dispersive media where the received signal consists of delayed and scaled replicas of the transmitted signal, the literature provides a variety of parameter estimation techniques capable of accurately estimating the different time delays. These techniques are classified into two broad categories, namely, parametric and non-parametric approaches. The former category comprises subspace-based methods such as root-MUSIC and ESPRIT and linear prediction methods such as MPM which have been extensively studied in various domains including GPR [2, 5]. However, if the medium is lossy and dispersive, the premise of detecting replicas is no longer valid and the assumed signal model as well as the processing techniques should be modified.

Among the disparate physical phenomena contributing to signal attenuation, this paper tackles the effects of frequency-dependent dielectric permittivity. The attenuation of GPR energy decreases and shifts the amplitude spectrum of the radar pulse to lower frequencies (absorption) with increasing time delay and causes also a distortion of the wavelet (dispersion). To account for these effects, we consider a complex power function of frequency for the dielectric permittivity which yields a well-known constant-Q model. Not only does this model describe the wave propagation using only two parameters (Q and V_r), but it also honours the constraints of causality [6].

In an attempt to estimate the parameters of a data model which includes electromagnetic scattering components with frequency dependent amplitudes, [7] proposed a modification of the MUSIC algorithm. It is based on premultiplying the scattering data by the inverse of an assumed frequency dependence. In [8], another approach was adopted using MPM by carrying out a first-order approximation of the frequency dependence with the aim of retrieving the damped exponential (DE) model. Similarly, in this paper, we adapt a DE model to the constant-Q model via a first-order approximation. This adaptation offers a better understanding of the biased estimates provided by the algorithms and paves the way for partially counterbalancing the effects of model mismatch.

The rest of the paper is organized as follows. In section 2, the signal model introducing absorption and dispersion along with its linear approximation are given. A brief review of the used methods is presented in section 3. In section 4, computer simulations highlight the performance degradation and quantify the improvement introduced by compensation. Finally, our conclusions for this work are given in section 5.

2. FORMULATION

In this section, we present the accurate signal model derived from the complex power law of frequency for dielectric permittivity, and an approximate model which is a superposition of damped exponentials. Both models are defined within the scope of determining the geometry of a stratified medium.

2.1 The Constant-Q Model

In order to model radar wave propagation in a constant-Q medium, [6] used a complex power law of frequency for the effective dielectric permittivity of the form:

$$\varepsilon(f) = \varepsilon'(f) + j\varepsilon''(f) = \varepsilon^0 \left|\frac{f}{f_r}\right|^{n-1} \Big[-j\mathrm{sgn}(f)\Big]^{n-1} \quad (1)$$

in which f_r and ε^0 are constants, f_r is an arbitrary reference frequency, and ε^0 will have the dimension of permittivity and is equal to ε for $n = 1$. The index n ($0 < n < 1$) is related to the quality factor Q as follows:

$$n = \frac{2}{\pi} \arctan(Q) \tag{2}$$

where $\frac{1}{Q} = \frac{\varepsilon''}{\varepsilon'} = \tan\left[\frac{\pi}{2}(1-n)\right]$.

In addition, considering only the positive frequencies and substituting equation (1) in the expression of the complex wavenumber k, we obtain

$$k(f) = \beta(f) - j\alpha(f) = \beta(f)\left\{1 - j\tan\left[\frac{\pi}{4}(1-n)\right]\right\} \tag{3}$$

with

$$\beta(f) = \frac{2\pi f}{V(f)}. \tag{4}$$

The dispersion and absorption terms are introduced by the real and imaginary parts of the wavenumber k, with $V(f)$ and α, the phase velocity and the absorption coefficient, respectively, given by

$$V(f) = V_r \left(\frac{f}{f_r}\right)^{\frac{(1-n)}{2}} \tag{5}$$

with

$$V_r = \frac{1}{\sqrt{\mu\varepsilon^0}\cos\left[\frac{\pi}{4}(1-n)\right]}$$

and

$$\alpha(f) = \beta(f)\tan\left[\frac{\pi}{4}(1-n)\right]. \tag{6}$$

μ is the magnetic permeability. The case $n = 1$ corresponds to loss-free propagation ($Q = \infty$, i.e. no attenuation or $k = \frac{2\pi f}{V_r}$ is real). From equation (5), it can be seen that V_r is simply the phase velocity at the arbitrary reference frequency f_r, and since V is slightly dependent on frequency, the absorption coefficient α in equation (6) obeys a frequency power law. The result in equation (5) is the dispersion relationship proposed for the first time by Kjartansson [9] for the case of mechanical losses in solids.

The complex power law for the dielectric constant equation (1) is easy to use in the frequency domain and is valid for positive values of Q. The wave propagation properties of materials can be described completely by only two parameters, Q and the phase velocity at an arbitrary reference frequency V_r. This simplicity makes it practical to use in the inverse Q imaging technique and in any inversion schema. This approach has been used successfully in 3D forward modeling of GPR data [10], in the estimation of water content of saturated rocks [11], and more recently for the characterization of the dielectric permittivity of concrete [12].

Assuming a horizontally stratified medium, the backscattered complex signal can be modeled as a linear combination of d echoes each of which emanates from the interface between two horizontally superposed layers under normal incidence. Each layer is considered to be homogeneous and characterized by a thickness e, a constant quality factor Q, and a dielectric constant ε^0. Upon substituting for each layer the expression of the corresponding wavenumber k in the equation of a plane wave as propagating along the $z-$axis, $e^{j(\omega t - kz)}$, the following signal model is obtained:

$$s(f) = \sum_{m=1}^{d} A_m(f) \prod_{l=1}^{m} e^{2\pi f \tau_l \left(\frac{f}{f_r}\right)^{\frac{n_l-1}{2}} \left\{-j-\tan\left[\frac{\pi}{4}(1-n_l)\right]\right\}} + b(f) \tag{7}$$

where $A_m(f)$ is a function gathering the different reflections and transmissions undergone by the m^{th} echo, $\tau_l = 2e_l/V_{rl}$ is the time delay corresponding to the l^{th} layer, $b(f)$ is additive white Gaussian noise with zero mean and variance σ^2. The product in the signal model shows the cumulative effect of the $(m-1)$ traversed layers on the m^{th} echo. The function $A_m(f)$ depends on the inter-layer dielectric contrast via the Fresnel coefficients that are generally frequency dependent. For example, the reflection coefficient emanating from the interface of the first and second layers is given as:

$$R_{12}(f) = \frac{\sqrt{\varepsilon_{r1}^0}\left(-j\frac{f}{f_r}\right)^{\frac{n_1-1}{2}} - \sqrt{\varepsilon_{r2}^0}\left(-j\frac{f}{f_r}\right)^{\frac{n_2-1}{2}}}{\sqrt{\varepsilon_{r1}^0}\left(-j\frac{f}{f_r}\right)^{\frac{n_1-1}{2}} + \sqrt{\varepsilon_{r2}^0}\left(-j\frac{f}{f_r}\right)^{\frac{n_2-1}{2}}} \tag{8}$$

where ε_r^0 is the relative dielectric constant. In such a case, we have $A_1(f) = R_{12}(f)$. The frequency variations of this term are slight and in the opposite manner to those resulting from propagation. According to [13], signal distortion is still mainly due to propagation and not to interface transmission. In what follows, the frequency variations of $A_m(f)$ are not taken into consideration.

2.2 Approximate Damped Exponential Model

As seen in the previous subsection, the constant-Q model deviates from the DE model. This subsection presents the first-order approximation enabling the retrieval of the DE model. It also discusses in terms of Q and f_r the bandwidth limits within which the approximation is considered sufficiently accurate. In [5], the author formulates a DE model from the first-order approximation of the complex wavenumber k. However, he considers no specific model for the frequency variations of the dielectric permittivity.

2.2.1 *First-order Approximation*

Applying the Taylor series expansion of the phase function $\phi(f) = -2\pi f\tau\left(\frac{f}{f_r}\right)^{\frac{n-1}{2}}$ at $f = f_r$ and considering only the first two terms, we obtain:

$$\phi(f) \approx -2\pi\tau f_r - 2\pi\tau\left(\frac{n+1}{2}\right)(f - f_r) \tag{9}$$

Using the above approximation, the data model becomes:

$$s_a(f) = \sum_{m=1}^{d} \prod_{l=1}^{m} A_l'(f) e^{2\pi f \tau_l \left(\frac{n_l+1}{2}\right)\left\{-j-\tan\left[\frac{\pi}{4}(1-n_l)\right]\right\}} + b(f) \tag{10}$$

where $s_a(f)$ denotes the approximated model, and $A_m(f)$ changes to $A_l'(f)$ to accommodate the constant complex terms resulting from the approximation. Therefore, replacing the nonlinear phase function by its first-order approximation results in the classical damped exponential model where the poles are given by:

$$z_m = \prod_{l=1}^{m} e^{2\pi f_s \tau_l \left(\frac{n_l+1}{2}\right)\left\{-j-\tan\left[\frac{\pi}{4}(1-n_l)\right]\right\}} \tag{11}$$

where $m = 1, 2, \cdots, d$ and f_s is the frequency shift. From equation (11), any method capable of estimating the parameters of the DE model when applied to the model in equation (7) would reveal, to a first-order approximation, a bias due to the term $(n+1)/2$ since they consider the time delay estimate to be:

$$\tau_m = -\left[\frac{\Im(\log z_m) - \Im(\log z_{m-1})}{2\pi f_s}\right] \tag{12}$$

where $\Im$ is the imaginary part. To gain insight into the physical significance of the bias term, we determine the phase and group delays:

$$\tau_\phi(f) = -\frac{1}{2\pi}\frac{\phi(f)}{f} = \tau\left(\frac{f}{f_r}\right)^{\frac{n-1}{2}} \tag{13}$$

$$\tau_g(f) = -\frac{1}{2\pi}\frac{\mathrm{d}\phi(f)}{\mathrm{d}f} = \tau\left(\frac{f}{f_r}\right)^{\frac{n-1}{2}}\left(\frac{n+1}{2}\right) \tag{14}$$

From the above two equations, expressing $\tau_g(f)$ in function of $\tau_\phi(f)$ gives:

$$\tau_g(f) = \left(\frac{n+1}{2}\right)\tau_\phi(f) \tag{15}$$

So, we deduce that the first-order approximation estimate gives access to the value of $\tau_g(f)$ at $f = f_r$ and not that of $\tau_\phi(f)$. Consequently, we expect the algorithms to reveal a systematic error of $(1-n)/2$, which may rise to 25% for the most dispersive medium (i.e. for $Q = 1$). Noting that both the modulus and argument of the pole depend on n and τ, the approximated model allows us to define an unbiased time delay estimate as follows:

$$\tau_m = -\left(\frac{2}{n_m+1}\right)\left[\frac{\Im(\log z_m) - \Im(\log z_{m-1})}{2\pi f_s}\right] \tag{16}$$

where n_m is estimated as:

$$n_m = 1 - \frac{4}{\pi}\arctan\left(\frac{\Re(\log z_m) - \Re(\log z_{m-1})}{\Im(\log z_m) - \Im(\log z_{m-1})}\right) \tag{17}$$

and $\Re$ is the real part. It is worth mentioning that the error made on the estimate of τ_m does not "propagate to" or affect the estimate of τ_{m+1}. This is because the estimation of a certain time delay involves the subtraction of all the previously estimated time delays. The subtraction removes the bias introduced by each. In other words, the bias of an estimate of a certain echo stems from its propagation in the last traversed layer, only. In what follows, this motivates us to carry out simulations on a one-layered medium.

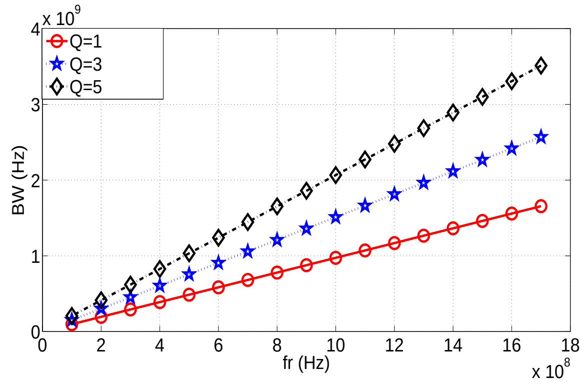

Figure 1: *The bandwidth variation in terms of Q and f_r.*

2.2.2 Approximation Adequacy

In order to define bandwidth limits within which the approximation yields an error inferior to 2%, it is required to solve the following transcendental equation which represents the 2% relative difference between $\phi(f)$ and its approximation in equation (9):

$$\left(\frac{n+1}{2}\right) f - \left(\frac{1.02}{f_r^{\frac{n-1}{2}}}\right) f^{\frac{n+1}{2}} + \left(\frac{1-n}{2}\right) f_r = 0 \tag{18}$$

We proceed by transforming the above equation into a polynomial. This requires rationalizing the exponents through approximating n by a/b ($a, b \in \mathbf{N}^*$ and $a < b$) and then changing f to Ω^{2b} leading to a polynomial of degree $2b$:

$$\left(\frac{a+b}{2b}\right)\Omega^{2b} - \left(\frac{1.02}{f_r^{\frac{a-b}{2b}}}\right)\Omega^{a+b} + \left(\frac{b-a}{2b}\right) f_r = 0 \tag{19}$$

Two of the $2b$ roots are real and correspond to the lower and upper limits of the bandwidth. Fig. 1 shows, for a given value of Q, an almost-linear increase in the absolute bandwidth BW as f_r increases which indicates that the nonlinear phase $\phi(f)$ develops a broader linear behavior at higher frequencies. Moreover, the relative bandwidth $RBW = BW/f_r$ is almost constant for a given Q and increases with increasing values of Q. For example, at $Q = 1$ $RBW = 97\%$ and attains 200% at $Q = 5$. Therefore, for GPR applications in which the bandwidth is generally equal to the central frequency of the radar wavelet, the first-order approximation seems adequate.

3. TDE TECHNIQUES

This section presents briefly the principles of MUSIC, ESPRIT and MPM. They all afford the estimation of the pole z_m in equation (11) through either a root-finding technique or a singular value decomposition. The details of these algorithms are well documented elsewhere [14, 15].

3.1 MUSIC

The MUSIC algorithm is based on the eigendecomposition of the correlation matrix which serves to partition the eigenvectors into noise and signal subspaces. Estimates of the time delays are obtained by computing the projection of a mode vector onto the noise subspace. For an echo at τ_m, its corresponding mode vector is orthogonal to the noise subspace. This property is used to estimate τ_m. Root-MUSIC, the polynomial version of MUSIC, computes the time delays by finding the roots of a polynomial called the root-MUSIC polynomial. Such an approach is less computationally demanding and better suited for the estimation of complex poles.

3.2 ESPRIT

The ESPRIT algorithm exploits an underlying rotational invariance between two adjacent data sub-bands. In comparison with MUSIC, ESPRIT provides a direct estimation of the time delays along with a reduced computational burden.

3.3 Matrix Pencil Method

The MPM is a linear prediction technique that exploits the structure of the matrix pencil of the underlying noiseless signals for the estimation of the time delays. MPM neither needs to estimate the correlation matrix nor employs a rooting procedure which dramatically reduces the computational burden. In contrast to MUSIC and ESPRIT, MPM carries out estimation from one snapshot and so can operate on correlated echoes without using any spatial smoothing process.

4. COMPUTER SIMULATIONS

In this section, the variations of the relative root mean square error (RRMSE) of the time delay estimates, defined as

$$\mathrm{RRMSE}(\%) = 100 \times \frac{\sqrt{\frac{1}{U}\sum_{i=1}^{U}(\hat{\tau}_i - \tau)^2}}{\tau}$$

($\hat{\tau}_i$ and τ denote the estimated and true time delays, respectively), are given as a function of Q, SNR, and the product $B\tau$, where B is the bandwidth of the GPR and τ is the smallest discernible time delay between two backscattered echoes. For the simulations, an air-coupled scenario is considered where the antennas are 30 cm above a one-layered medium placed on a perfect electric conductor. The antennas have a central frequency of $f_c = 700$ MHz which is taken to be the reference frequency, i.e., $f_r = f_c$ and the processed bandwidth is 1 GHz. The data set contains 100 equispaced frequency samples generated from equation (7) which becomes:

$$s(f) = R_{12}(f)e^{-j2\pi f\tau_1} - \left[1 - R_{12}^2(f)\right]e^{-j2\pi f\tau_1} e^{2\pi f\tau_2\left(\frac{f}{f_r}\right)^{\frac{n_2-1}{2}}\left\{-j-\tan\left[\frac{\pi}{4}(1-n_2)\right]\right\}} + b(f) \quad (20)$$

where $n_1 = 1$ since it characterizes air, a dispersionless medium. In all simulations and for the sake of clarity, only the estimates of MPM are compensated for the bias term. The spatial smoothing process (SSP) is employed for root-MUSIC and ESPRIT to decorrelate the echoes.

4.1 RRMSE versus Q

Q is varied between 1 and 30 which correspond to the limit values reported in the literature for civil engineering materials [16]. Each estimate is the result of $U = 100$ Monte Carlo runs at $SNR = 20$ dB. The horizontal line represents the RRMSE in the absence of dispersion at $SNR = 20$ dB and serves as an asymptote or a lower bound for the errors produced by the algorithms. From Fig. 2, we observe that

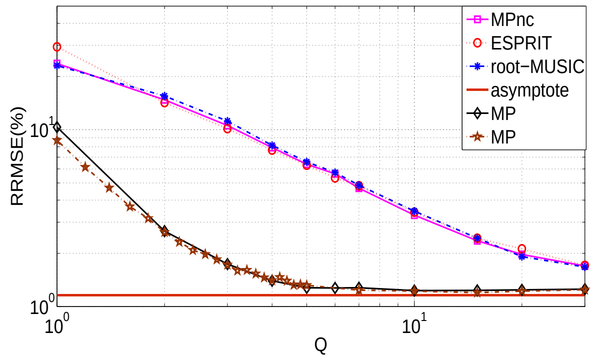

Figure 2: *The RRMSE variations of τ_2 versus Q at $SNR = 20$ dB where $\star$ and $\diamond$ designate compensated MP applied on the approximate and accurate models, respectively. MPnc stands for the non-compensated MP.*

the RRMSE of all algorithms decreases as Q increases. This is because a higher Q leads to less absorption and dispersion and thus a higher SNR for the second echo and a smaller discrepancy between the accurate and approximated models. However, the uncompensated algorithms do not attain the asymptote due to the bias term $(n_2+1)/2$ whereas the compensated MPM does for $Q > 10$. In the case of compensation, a major percentage of the error on the time delay τ_2, especially for low values of Q, is due to the error on n_2 which is known to have a higher noise threshold [8], and not due to model mismatch. To verify this conclusion, the RRMSE variation for a data set generated from the approximated model of equation (10) is added to Fig. 2 where it appears to be almost confounded with the one of the accurate model. Consequently, increasing the SNR would result in a significant improvement in the RRMSE for the compensated MPM and only a slight one for the uncompensated algorithms.

4.2 RRMSE versus $B\tau_2$

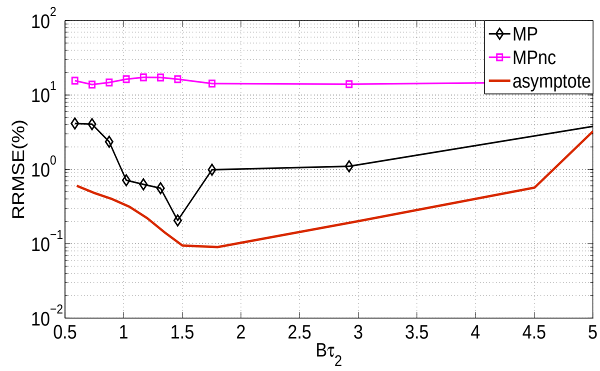

Figure 3: *The RRMSE variations of τ_2 versus $B\tau_2$ at $SNR = 40$ dB and $Q = 2$.*

Fig. 3 shows the variation of the RRMSE as a function of the product $B\tau_2$ at $SNR = 40$ dB and $Q = 2$. The uncompensated MPM produces a systematic error represented by a nearly constant RRMSE. This error is predicted by the term $(1-n_2)/2$ which for $Q = 2$ gives 14.75%. Compensating

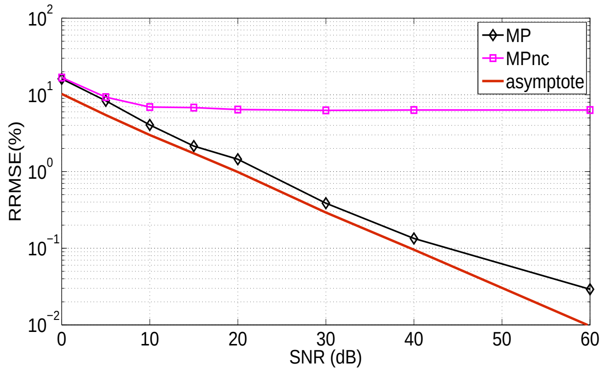

Figure 4: *The RRMSE variations of τ_2 versus SNR at $Q = 5$ and $e = 10$ cm.*

the results of MP has the effect of reducing the error until a threshold value of $B\tau_2 = 1.5$ is reached. Once again, this behavior is not to be attributed to model mismatch and the dispersionless curve supports this interpretation which is as follows. Starting from the threshold value, as the layer thickness increases, the *SNR* of the second echo decreases dramatically and overwhelms or dominates the well-established effects of dispersion which renders parameter estimation inaccurate, and so the RRMSE increases. Whereas for values less than the threshold, the effects of attenuation become more prominent with thickness but within reasonable decreasing *SNR* values enabling more and more accurate estimates. Consequently, the RRMSE decreases.

4.3 RRMSE versus *SNR*

The RRMSE versus *SNR* is shown in Fig. 4 for $Q = 5$ and $e = 10$ cm. As in the previous subsection, the uncompensated MPM presents a systematic error predicted by the term $(1 - n_2)/2$ which for $Q = 5$ gives 6.3%. Compensation improves the accuracy of the estimates and offers a performance comparable to that of the dispersionless case except for a deviation at high *SNR*. A deviation which we attribute to model mismatch since the influence of noise perturbation becomes negligible at high *SNR*.

5. CONCLUSION

In this paper, the effects of attenuation on the performance of some time delay estimation techniques were investigated in terms of Q, *SNR*, and $B\tau$. An approximate model was derived with the aim of retrieving the damped exponential model and its validity was discussed. Finally, the attenuation-induced systematic error was remedied by a compensation procedure which proved to provide better estimates over the considered bandwidth.

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