DIVERSITY TECHNIQUES FOR RF-BEAMFORMING IN MIMO-OFDM SYSTEMS: DESIGN AND PERFORMANCE EVALUATION

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ABSTRACT

In this paper we present a low-complexity MIMO-OFDM system based on adaptive signal combining in radio-frequency (RF). The limitations of the proposed architecture require the development of specific transmission and reception schemes. Specifically, we consider the problem of selecting the transmit and receive RF weights (beamformers), as well as the time and frequency linear precoders, under the assumption of i.i.d. Rayleigh channels perfectly known at the receiver side. The transmission scheme is based on orthogonal beam division multiplexing (OBDM) and minimum MSE beamforming, i.e., the data is sequentially transmitted by means of different transmit beamformers, whereas the receive beamformers are selected to minimize the MSE of the linear MMSE receiver. Finally, the performance of the proposed transmission scheme is illustrated by means of Monte Carlo simulations.

1. INTRODUCTION

In conventional baseband architectures of multiple-input multiple-output (MIMO) systems, the signals in all the branches have to be processed simultaneously, which requires the replication of the hardware for each antenna. Thus, the system cost and power consumption associated to this paradigm have been in part responsible for the delay in the comercial deployment of MIMO systems.

These problems can be solved by means of a novel RF-MIMO transceiver architecture, which moves some of the baseband processing to the RF front-end. As an example, Fig. 1 shows the receiver for this architecture, which applies a set of complex weights (gain factor and phase shift) to the received signals. Thus, after combining the RF signals, a single stream of data is acquired and processed in baseband, reducing the hardware cost and the associated power consumption compared to a full baseband MIMO architecture. Finally, we must point out that the interest in this kind of architectures is also propelled by recent advances in RF integrated circuits designed in SiGe-BiCMOS technology [1].

From a signal processing point of view, the new architecture imposes the following challenges: Firstly, since only one data stream is acquired and processed, the multiplexing gain of a MIMO system is always limited to one [2]. However, it is easy to prove [3] that other important benefits of the MIMO channel, such as diversity or array gain, are kept by the new architecture. Secondly, in the case of OFDM-based transmissions, the novel architecture is restricted to the application of a common pair of Tx-Rx beamformers for all the subcarriers, which contrasts with the ability of conventional MIMO systems for processing the subcarriers independently. This coupling makes the design of the Tx-Rx beamformers a challenging task [4].

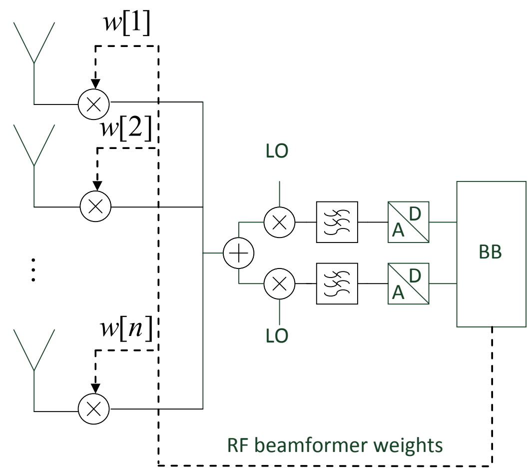

Figure 1: Analog antenna combining in the RF path for MIMO communications systems. Exemplarily shown for a direct-conversion receiver.

In this paper we consider OFDM-based transmissions over frequency selective and spatially white Rayleigh channels, which are assumed to be perfectly known at the receiver side, but unknown at the transmitter. Specifically, we show that under the assumption of linear receivers, the optimal transmission scheme is based on the combination of two previously proposed techniques: OBDM [3] and MinMSE beamforming [4]. On one hand, OBDM was proposed for single-carrier systems without channel state information (CSI) at the transmitter side, and the general idea is based on the sequential transmission, through orthogonal directions defined by several transmit beamformers, of a linearly precoded version of the data.[1] On the other hand, MinMSE beamforming was proposed for OFDM-based systems with perfect CSI at both the transmitter and the receiver. In this case, the main idea consists of minimizing the MSE associated to the equivalent SISO channel after Tx-Rx beamform-

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement n° 213952 (MIMAX), and by the Spanish Government (MICINN) under projects TEC2007-68020-C04-02 (MultiMIMO) and TEC2007-30929-E.

[1]See [3] for the details and [5] for an extension to spatially correlated channels.

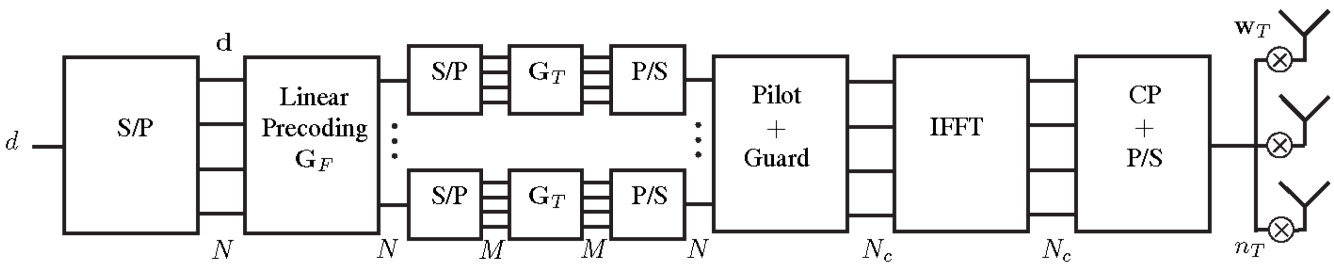

Figure 2: Block diagram of the OFDM-based transmitter with analog antenna combining. The receiver diagram is equivalent.

ing. Finally, the performance of the RF-combining architecture is evaluated and compared, by means of Monte Carlo simulations, with that of conventional SISO and MIMO systems. The results, which illustrate the tradeoff between performance and complexity, show that the proposed architecture provides an intermediate solution between conventional SISO and MIMO systems, but with a hardware complexity and power consumption close to that of the SISO system.

Throughout this paper we will use bold-faced upper case letters to denote matrices, bold-faced lower case letters for column vector, and light-faced lower case letters for scalar quantities. $\mathbf{X}^H$, $\|\mathbf{X}\|$ and $\text{vec}(\mathbf{X})$ denote the Hermitian, Frobenius norm, and columnwise vectorized version of matrix $\mathbf{X}$. Finally, $\otimes$ denotes the Kronecker product and $\mathbf{I}$ is the identity matrix of the required dimensions.

2. MULTICARRIER BEAMFORMING WITH ANALOG COMBINING SCHEMES

In this paper we consider OFDM-based transmissions over a frequency selective MIMO channel, which is assumed to be perfectly known at the receiver but unknown at the transmitter side. We consider a general OFDM-based $n_T \times n_R$ MIMO system, whose transmitter is shown in Fig. 2, and where N out of N_c subcarriers contain linearly precoded information symbols. The precoding scheme operates both in frequency and time. On one hand, frequency precoding is commonly used in OFDM-based systems in order to exploit the frequency diversity of the channel (roughly speaking we could say that each information symbol is transmitted through different subcarriers), and it is also closely related to single-carrier systems with frequency domain equalization (SCFDE) [6]. On the other hand, in order to exploit the spatial diversity in analog combining systems, the information symbols have to be time-precoded and transmitted using different beamformers (RF weights), which can be interpreted as the replacement of the spatial diversity by a virtual time-diversity [3]. Finally, after insertion of pilot and null carriers, inverse fast Fourier transform (IFFT), and addition of the cyclic prefix, the OFDM symbol is transmitted through the n_T transmit antennas using the analog combining architecture shown in Fig. 1.

2.1 Tx-Rx data model

Let us start by defining a transmission block as a set of M OFDM symbols.[2] These symbols will be transmitted and received using M different pairs of beamformers, which are assumed to change synchronously.[3] Although the implementation details of the proposed RF-MIMO architecture are beyond the scope of this paper, we must point out that, due to technological reasons, the beamformer weights could have to remain fixed during the transmission of several OFDM symbols. However, in that case the transmission blocks could be distributed among several beamformers as shown in Fig. 3.

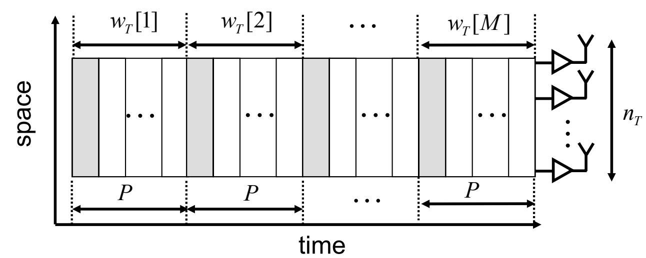

Figure 3: Distribution of a transmission block. The grey columns denote the M time and frequency precoded OFDM symbols of one block, which are transmitted using M different beamformers. In this example the beamformers remain fixed during the transmission of several (P) OFDM symbols.

Now, considering a data matrix $\mathbf{D} \in \mathbb{C}^{N\times M}$ containing NM information symbols, we can define the transmission matrix

$$\mathbf{S} = \mathbf{G}_F \mathbf{D} \mathbf{G}_T,$$

where $\mathbf{G}_F \in \mathbb{C}^{N\times N}$ and $\mathbf{G}_T \in \mathbb{C}^{M\times M}$ are, respectively, the frequency and time precoding matrices. Here, we must note that the total energy associated to a transmission block is

$$\|\mathbf{S}\|^2 = \|\mathbf{s}\|^2 = \|\mathbf{G}\mathbf{d}\|^2,$$

where $\mathbf{s} = \text{vec}(\mathbf{S})$, $\mathbf{d} = \text{vec}(\mathbf{D})$ and $\mathbf{G} = \mathbf{G}_T \otimes \mathbf{G}_F$. Therefore, in order to preserve the transmission energy, the precoding matrices should have a unitary Kronecker product, i.e., $\mathbf{G}^H\mathbf{G} = \mathbf{I}$.

After linear precoding, each row of $\mathbf{S}$ is associated to a subcarrier, whereas each column represents the linearly precoded data in one OFDM symbol. Thus, the n-th column of $\mathbf{S}$ will be transmitted using the transmit and receive beamformers $\mathbf{w}_T[n] \in \mathbb{C}^{n_T\times 1}$ and $\mathbf{w}_R[n] \in \mathbb{C}^{n_R\times 1}$, whose elements are given by the RF weights shown in Fig. 1. Finally, we will assume, without loss of generality, unit energy beamformers, i.e., $\|\mathbf{w}_T[n]\| = \|\mathbf{w}_R[n]\| = 1$.

[2] A sensible solution consists in choosing $M \geq n_T$.

[3] Practical issues such as the effect of channel estimation errors or lack of synchronism between the beamformers will be addressed in a future work.

With the above definitions, the received signal for the n-th pair of beamformers ($n = 1, \ldots, M$) can be written as

$$y_k[n] = h_k[n]s_k[n] + n_k[n], \qquad k = 1, \ldots, N,$$

where $y_k[n]$ denotes the observed signal at the k-th subcarrier, $n_k[n]$ denotes the i.i.d. Gaussian noise with variance σ^2, $s_k[n]$ is the element in the k-th row and n-th column of $\mathbf{S}$, and the equivalent channel after Tx and Rx beamforming is

$$h_k[n] = \mathbf{w}_R^H[n]\mathbf{H}_k\mathbf{w}_T[n],$$

where $\mathbf{H}_k \in \mathbb{C}^{n_R \times n_T}$ represents the response of the MIMO channel at the k-th subcarrier.

3. TRANSCEIVER DESIGN

In this section, we design the system parameters $\mathbf{G}_F$, $\mathbf{G}_T$, $\mathbf{w}_T[n]$ and $\mathbf{w}_R[n]$ to minimize the averaged bit error rate (BER) associated to the linear minimum mean square (MMSE) receiver, i.e., our optimization problem is

$$\begin{aligned} \underset{\mathbf{G}_T, \mathbf{G}_F, \mathbf{w}_T[n], \mathbf{w}_R[n]}{\text{Minimize}} \quad & \text{BER}\left(\mathbf{G}_T, \mathbf{G}_F, \mathbf{w}_T[n], \mathbf{w}_R[n]\right), \\ \text{subject to} \quad & \|\mathbf{w}_T[n]\| = \|\mathbf{w}_R[n]\| = 1, \\ & \mathbf{G}^H\mathbf{G} = \mathbf{I}, \\ & \mathbf{G} = \mathbf{G}_T \otimes \mathbf{G}_F. \end{aligned}$$

3.1 Design of the Frequency and Time Precoders

The design of linear precoding schemes for OFDM systems has been addressed, under different criteria, in [7, 8, 9]. Here, we follow the same principles applied on the matrix $\mathbf{G}$. In the case of linear receivers and QAM constellations, the basic idea consists of writing the averaged BER as a function of the MSE associated to the information symbols

$$\text{BER} = \frac{1}{NM}\sum_{k=1}^{N}\sum_{n=1}^{M}\text{BER}_k[n] = \sum_{k=1}^{N}\sum_{n=1}^{M} g(\text{MSE}_k[n]),$$

where $\text{BER}_k[n]$ and $\text{MSE}_k[n]$ represent, respectively, the BER and MSE associated to the information symbol in the k-th row and n-th column of $\mathbf{D}$, and $g(\cdot)$ is an increasing convex function.[4]

Interestingly, due to the unitarity of the precoding matrix $\mathbf{G}$, the total MSE is[5]

$$\overline{\text{MSE}} = \sum_{k=1}^{N}\sum_{n=1}^{M}\text{MSE}_k[n] = \sum_{k=1}^{N}\sum_{n=1}^{M}\text{MSE}_{s_k}[n],$$

where

$$\text{MSE}_{s_k}[n] = \frac{1}{1 + \gamma|h_k[n]|^2}, \tag{1}$$

denotes the MSE in the estimate of $s_k[n]$, and $\gamma = 1/\sigma^2$ is the SNR. Thus, noting that $\overline{\text{MSE}}$ does not depend on the specific unitary precoding matrix $\mathbf{G}$, and taking into account that the averaged BER is a Schur-convex function [10], we have

$$\text{BER} = \sum_{k=1}^{N}\sum_{n=1}^{M} g(\text{MSE}_k[n]) \geq g\left(\overline{\text{MSE}}\right),$$

and the lower bound is achieved when all the $\text{MSE}_k[n]$ are equal [7, 10].

Finally, in order to ensure a uniform distribution of the total MSE among the information symbols, the optimal precoding matrix $\mathbf{G}$ must be unitary with constant modulus entries, such as the Fourier or Walsh-Hadamard matrices [7, 10]. However, we must note that, in our particular problem, $\mathbf{G}$ must also satisfy the Kronecker structure $\mathbf{G} = \mathbf{G}_T \otimes \mathbf{G}_F$. Fortunately, the Kronecker product preserves the unitarity and constant modulus properties, i.e., given two unitary matrices $\mathbf{G}_T$ and $\mathbf{G}_F$ with constant modulus entries, the product $\mathbf{G}_T \otimes \mathbf{G}_F$ is unitary with constant modulus elements. Thanks to this property, we can conclude that the separated time and frequency precoding structure proposed in this paper is optimal. To summarize, we propose to independently chose the precoding matrices $\mathbf{G}_T$ and $\mathbf{G}_F$ as any Fourier or Walsh-Hadamard matrices, which reduces the averaged BER to

$$\text{BER} = g\left(\overline{\text{MSE}}\right) = g\left(\sum_{k=1}^{N}\sum_{n=1}^{M}\frac{1}{1 + \gamma|h_k[n]|^2}\right). \tag{2}$$

3.2 Design of the Beamformers

In this subsection, the transmit and receive beamformers are designed in order to minimize the BER of the analog combining system. The proposed scheme assumes spatially uncorrelated Rayleigh channels, and it is based on a set of $M \geq n_T$ transmit beamformers distributing the transmit power isotropically. At the receiver side the channel and transmit beamformers are known, which reduces the design problem to the minimization of the MSE.

3.2.1 Receive Beamformers

As we have shown in the previous subsection, under the optimal precoding matrices $\mathbf{G}_T$ and $\mathbf{G}_F$, the problem of minimizing the BER reduces to the minimization of the total MSE. Thus, taking (2) into account, the criterion for the design of the receive beamformers can be rewritten as the following uncoupled optimization problems

$$\underset{\mathbf{w}_R[n]}{\text{Minimize}} \quad \sum_{k=1}^{N}\frac{1}{1 + \gamma|h_k[n]|^2}, \qquad \text{s.t.} \qquad \|\mathbf{w}_R[n]\| = 1,$$

for $n = 1, \ldots, M$. Since the receiver knows the MIMO channel and the transmit beamformers, the above problems are equivalent to that of designing the minimum MSE (MinMSE) receive beamformer in an analog combining SIMO system under OFDM transmissions. This problem, which in the flat fading case reduces to the well known maximum ratio combining (MRC) receiver, has been studied in [4]. Specifically, in [4] we have proposed a gradient search algorithm, which converges very fast to the optimal solution. Here, we summarize the updating rule for the MinMSE beamforming problem

$$\mathbf{w}_R[n] \leftarrow \mathbf{w}_R[n] + \mu\mathbf{R}[n]\mathbf{w}_R[n], \qquad n = 1, \ldots, M,$$

[4]The convexity of $g(\cdot)$ can be easily proven [10].

[5]We assume unit power transmissions, i.e., $E[|s_k[n]|^2] = 1$.

where μ is the step-size,

$$\mathbf{R}[n] = \sum_{k=1}^{N} \mathrm{MSE}_{s_k}^2[n]\mathbf{h}_k[n]\mathbf{h}_k^H[n], \qquad n = 1, \ldots, M,$$

can be seen as a weighted correlation matrix, $\mathrm{MSE}_{s_k}[n]$ is obtained from (1), and

$$\mathbf{h}_k[n] = \mathbf{H}_k\mathbf{w}_T[n], \qquad n = 1, \ldots, M,$$

define the equivalent frequency selective SIMO channels after fixing the transmit beamformers.

3.2.2 Transmit Beamformers

The design of the transmit beamformers is more complicated due to the fact that both the receive beamformers and the MIMO channel are unknown at the transmitter side. A simpler alternative consists in the minimization of the pairwise error probability (PEP). In the case of flat-fading and spatially uncorrelated Rayleigh channels, this problem has been addressed in [3] (see also [5] for an extension to spatially correlated channels), and the results can be easily extended to the multicarrier case. Specifically, defining the $n_T \times M$ matrix

$$\mathbf{W}_T = [\mathbf{w}_T[1] \quad \cdots \quad \mathbf{w}_T[M]],$$

the optimal solution is

$$\mathbf{W}_T^H\mathbf{W}_T = \mathbf{I} \qquad \text{for} \quad n_T \geq M,$$
$$\mathbf{W}_T\mathbf{W}_T^H = \mathbf{I} \qquad \text{for} \quad n_T \leq M,$$

i.e., as expected, the available power has to be isotropically distributed, which is the idea after the OBDM scheme proposed in [3]. Furthermore, it is easy to prove that for $M \geq n_T$, the beamformers in $\mathbf{W}_T$ extract the spatial diversity at the transmitter side (n_T) and also maximize the coding gain.

4. SIMULATION RESULTS

In this section we evaluate the performance of the RF-combining architecture using the proposed techniques. All the results are based on Monte Carlo simulations. Specifically, we consider a block-fading model in which the propagation coefficients remain constant for a coherence interval of PM symbols (i.e., the frame duration in Fig. 3). A 4×4 Rayleigh channel with exponential tap delay profile has been assumed [11]. In particular, the power associated to the l-th tap is

$$n_T n_R \rho^l (1 - \rho),$$

and we have selected $\rho = 0.7$, which provides channels with approximately 16 significative taps.[6] We have used $N_c = N = 64$ data subcarriers, $M = n_T = 4$ pairs of beamformers, and QPSK information symbols, which can be uncoded or linearly precoded in frequency and time with the Fourier matrices. The proposed scheme, which we refer to as OBDM+MSE, has been compared with the following systems:

- Full-MIMO: We consider a scheme performing maximum ratio transmission (MRT) and maximum ratio combining (MRC) in each subcarrier. Obviously, this implies perfect channel knowledge at both sides of the link, and therefore it can be seen as a non-tight upper bound for the performance of the proposed system.
- SISO: This can be seen as the natural competitor of the proposed system, which provides better performance at the expense of a slight increase of complexity.
- SIMO+MRC: A SIMO system with MRC in each subcarrier at the receiver.
- SIMO+MSE: A SIMO system with MinMSE beamforming at the receiver. In comparison with the previous system, it illustrates the loss due to the use of a common receive beamformer for all the subcarriers.
- OBDM+MRC: OBDM scheme at the transmitter and MRC in each subcarrier at the receiver. In comparison with SIMO+MRC, it illustrates the gain provided by the OBDM scheme.

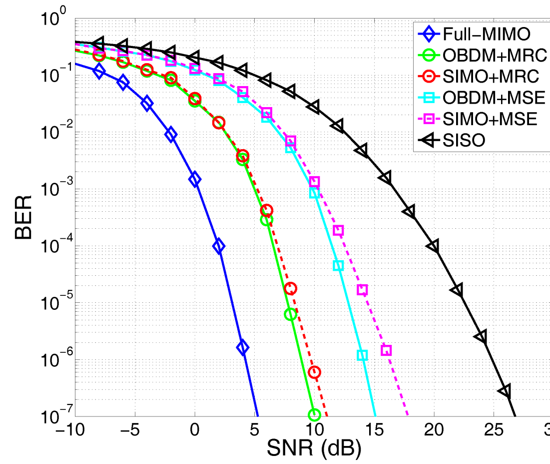

Figure 4: BER performance of the evaluated schemes with time and frequency precoding.

In the first experiment we have evaluated the BER performance using both time and frequency precoding. As can be seen in Fig. 4, the performance of each OBDM-based system is always better than the analogous system with single antenna in transmission. The noticeable decrease of BER in OBDM in comparison to the single antenna system is due to the spreading of the information symbols along the n_T channel uses and beamformers. Furthermore, we can see that the gap between MinMSE schemes and those with MRC in each subcarrier is less than 5 dB.

The second experiment, illustrated in Fig. 5, shows the system performance when linear frequency precoding is not applied. As we can see, OBDM-based systems outperform those with single antenna in transmission by more than 3 dB in medium SNRs. In this case the improvement due to the time precoding in OBDM is more noticeable.

In the final example we have evaluated the effect of the frequency diversity, controlled by the exponential parameter ρ, on the proposed architecture. On the one hand, conventional SISO and MIMO systems suffer a performance degradation when the diversity decreases. However, we must remember that the proposed scheme applies a common pair of beamformers to all the subcarriers, which is only optimal in

[6] This can be seen as the limit to avoid inter-carrier-interference in 802.11a systems.

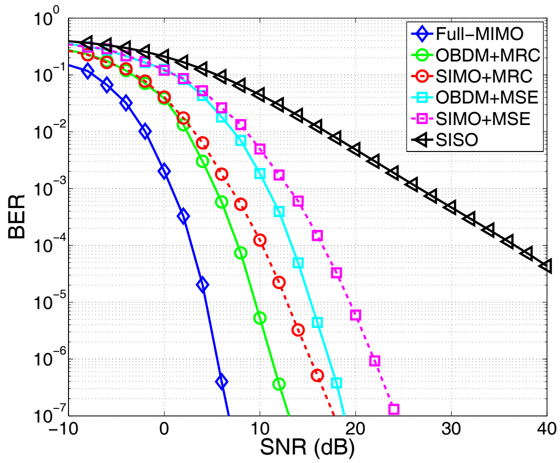

Figure 5: BER performance of the evaluated schemes with time precoding.

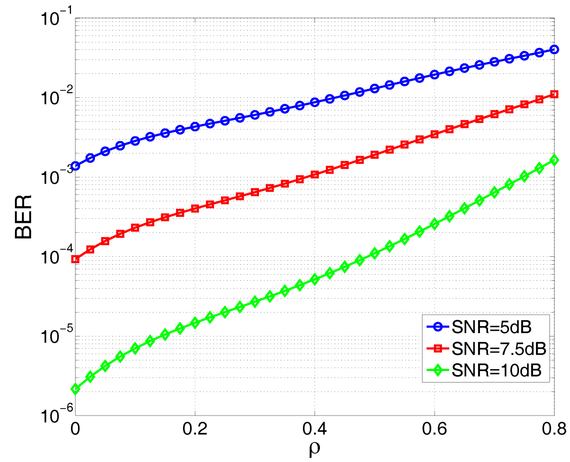

Figure 6: BER of the proposed scheme as a function of the exponential channel parameter.

the case of flat fading channels. In other words, the gap between the proposed scheme and a Full-MIMO approach increases with the frequency selectivity. The obtained results are shown in Fig. 6, where we can see the BER as a function of ρ for three different SNRs. As can be seen, the effect of the frequency selectivity on the proposed RF-combining architecture is just opposite to that in SISO and Full-MIMO schemes. Thus, since $\rho = 0.7$ provides very dispersive channels, the results in Figs. 4 and 5 can be seen as a worst-case performance for the proposed scheme.

5. CONCLUSIONS

In this paper we have proposed a transmission scheme for a novel RF-combining architecture. Specifically, we have considered OFDM-based transmissions with perfect channel state information (CSI) at the receiver side, and statistical CSI at the transmitter. The main design challenges are due to the limitations imposed by the novel architecture, i.e., transmission/reception of a single data stream and application of a common pair of beamformers to all the subcarriers. Under the assumption of i.i.d Rayleigh channels, the optimal transmission scheme consists in the combination of two techniques: OBDM and MinMSE beamforming. At the transmitter side, a linearly precoded version of the information symbols is transmitted in several directions, defined by different transmit beamformers. At the receiver side, the beamformers are selected in order to minimize the overall MSE of the linear MMSE receiver. Finally, simulation results show that the proposed architecture clearly outperforms a conventional SISO system at the expense of a slight increase on the system complexity. On the other hand, the gap between the proposed architecture and a conventional full baseband MIMO system is justified by the important reduction on hardware cost and power consumption.

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