BLIND ESTIMATION OF TIMING AND CARRIER FREQUENCY OFFSETS IN OFDM SYSTEMS

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ABSTRACT

A novel approach is proposed for blind synchronization of Orthogonal Frequency Division Multiplexing (OFDM) systems. The power spectrum and spectral correlation, computed using frequency domain signals, are exploited to independently recover the timing and carrier frequency offsets in closed form. The proposed estimators performance is evaluated for the Additive White Gaussian Noise (AWGN) and the Rayleigh fading channels. They show enhanced performance in terms of Mean Square Error (MSE) when compared to estimators using the temporal autocorrelation. Moreover, the proposed estimators do not require any channel knowledge.

1. INTRODUCTION

The OFDM modulation is being adopted in several present and future high rate wireless communications standards. These standards include the European digital audio and video broadcasting, various versions of the wireless local area networking IEEE 802.11 and the wireless local loop IEEE 802.16. OFDM modulation is especially interesting for such systems because it copes with the frequency selective channels met in high data rate transmissions.

For systems employing OFDM, Inter-Symbol Interference (ISI) can be suppressed if a temporal guard interval of convenient length is inserted. In this way, equalization is drastically simplified.

The main drawback of OFDM modulation, is however, its increased sensitivity to synchronization errors [1]. A timing error prevents to correctly discard the guard interval affected by ISI. A frequency offset results in Inter-Carrier Interference (ICI). OFDM modulations including a Cyclic Prefix (CP) are here considered where a copy of the last part of the OFDM symbol is inserted within the guard interval.

To recover the synchronization parameters, data aided methods can be employed [2]-[5]. Nevertheless, for better spectral efficiency, blind solutions are preferable.

This paper addresses the problem of blind estimation of the timing error and the carrier frequency offset in OFDM systems employing CP.

Classical approaches for blind estimation of the timing error in cyclic prefixed OFDM exploit the redundancy introduced by the CP [4, 6]. Several blind synchronization methods use the cyclostationarity of OFDM signals, such as quoted in [7, 8] where it is shown that for non circular constellations, the frequency offset can be recovered from the cyclic frequencies of the received signal via the maximization of a sum of cyclic autocorrelation coefficients.

The cyclostationarity of OFDM signals is also exploited in [9, 10] where the frequency and timing offsets are estimated either sequentially, by using the cyclic correlation coefficients or independently, by maximizing the cyclic spectrum. The main constraint of the algorithm cited above is that the channel needs to be known. This constraint is avoided in the algorithm proposed in [11] where the channel response is modeled by a random process.

Other more computationally efficient techniques exist for carrier frequency synchronization, assuming a correct timing acquisition (i.e. the OFDM symbol start is correctly detected). For instance, [12] uses oversampling and [13]-[15] take advantage of virtual (not loaded) subcarriers.

In this paper, a frequency domain processing is proposed, through power spectrum and spectral correlation computation, to blindly recover the carrier frequency and the timing offsets. The remaining of this paper is organized as follows. After introducing the data model in section 2, the proposed closed form estimators are derived in section 3. Finally, some simulations results are provided to assess the proposed estimators relevance.

2. DATA MODEL

Consider the following continuous time OFDM signal which employs a CP

$$s_0(t) = e^{j2\pi f_c t} \times \left(\sum_p \left(\sum_{k=k_{min}}^{k_{max}} C_{k,p} e^{j2\pi \frac{k}{T_u}(t-T_g-pT_s)} \right) rect_{[0,T_s]}(t-pT_s) \right), \quad (1)$$

where the occupied frequency band W is partitioned into $N = k_{max} - k_{min} + 1$ subchannels or subcarriers. The subcarriers are equally spaced by $\frac{W}{N} = \frac{1}{T_u}$ which corresponds to the symbol rate on each subcarrier. T_s denotes the OFDM symbol duration including the CP of length T_g such that $T_s = T_u + T_g$. $C_{k,p}$ denotes the data symbol transmitted over the k^{th} subcarrier within the p^{th} OFDM block. $rect_{[0,T_s]}(t)$ is a rectangular pulse shaping filter of length T_s and f_c is the carrier frequency.

We are here concerned with systems with known symbol rate and subcarriers spacing, *i.e.* T_s and T_u are known or previously estimated (see [16, 17]). We aim to recover the synchronization parameters, namely the carrier frequency offset Δf_c and the timing error t_0. The channel coherence band is assumed to be larger than the subcarriers spacing such that the channel effect, on each subchannel, reduces to a phase rotation and an amplitude attenuation.
Let $\hat{f}_c$ denote the estimate of the carrier frequency f_c obtained by roughly recentring the received signal spectrum around its continuous component. As t_0 denotes the time offset with respect to the OFDM symbol start, the received baseband signal is given by

$$\begin{aligned}&\tilde{s}(t) = s_0(t-t_0)e^{-j2\pi\hat{f}_c t} + n(t) = \\ &\sum_{p,k} H_k C_{k,p} e^{j2\pi\frac{k}{T_u}(t-t_0-T_g-pT_s)} rect_{[0,T_s]}(t-t_0-pT_s) \\ &\times e^{j2\pi\Delta f_c t} e^{-j2\pi f_c t_0} + n(t), \qquad (2)\end{aligned}$$

where $s_0(t)$ is modified to account for the channel gains $\{H_k\}$ over the different subcarriers. $n(t)$ is an Additive White Gaussian Noise (AWGN) with zero mean and variance σ^2. $\Delta f_c = f_c - \hat{f}_c$ denotes the carrier frequency offset.

3. ESTIMATION METHODS USING A FREQUENCY DOMAIN FORMULATION

In the following, two closed form estimators are derived respectively for Δf_c and t_0.

3.1 Frequency synchronization : estimation of Δf_c

Let

$$y(t) \overset{def}{=} E\left((\tilde{s}(t) \otimes \tilde{s}^*(-t)\right), \qquad (3)$$

where $\otimes$ corresponds to the convolution operator. The Fourier Transform (FT) of $y(t)$ is given by

$$\begin{aligned} Y(f) &= FT_\delta\left(E\left(\int \tilde{s}(u)\tilde{s}^*(u-\delta)du\right)\right) & (4)\\ &= E\left(\int\int \tilde{s}(u)\tilde{s}^*(u-\delta)e^{-j2\pi f\delta}dud\delta\right) & (5)\\ &= E\left(\left|\int \tilde{s}(u)e^{-j2\pi fu}du\right|^2\right), & (6)\end{aligned}$$

where FT_δ denotes FT with respect to δ. The expression (6) can be identified to the power spectrum of $\tilde{s}(t)$. It is assumed that the transmitted symbols $\{C_{k,p}\}$ are uncorrelated for different OFDM symbols and subcarriers, i.e. $E(C_{k,p}C^*_{k',p'}) = \sigma_c^2 \delta_{p,p'}\delta_{k,k'}$, where $\delta_{x,y}$ is the kronecker symbol. Then, applying (6) on the data model (2) yields

$$Y(f) = \lambda \sum_{k=k_{min}}^{k=k_{max}} |H_k|^2 \frac{sin(\pi T_s(f-\Delta f_c - \frac{k}{T_u}))^2}{(\pi T_s(f-\Delta f_c-\frac{k}{T_u}))^2} + \sigma^2 \qquad (7)$$

where $\lambda = P\sigma_c^2 T_s^2$ and P denotes the number of observed OFDM blocks.
Computing $y(t)$ of Eq. (3) as the Inverse Fourier Transform (IFT) of $Y(f)$, we obtain

$$\begin{aligned} y(\tau) &= FT^{-1}(Y(f))|_{t=\tau} = \int Y(f)e^{j2\pi\tau f}df & (8)\\ &= \lambda e^{j2\pi\tau\Delta f_c}\left(\sum_k |H_k|^2 e^{j2\pi\tau\frac{k}{T_u}}\right)g(\tau) + \sigma^2\delta_{\tau,0}, & (9)\end{aligned}$$

where

$$g(\tau) = \int \frac{\sin(\pi T_s f)^2}{(\pi T_s f)^2}\cos(2\pi\tau f)df. \qquad (10)$$

Some computations lead to

$$\begin{aligned} g(\tau) &= \frac{T_s-\tau}{T_s^2} \ for \ 0 \le \tau \le T_s, & (11)\\ &= 0 \ for \ \tau \ge T_s. & (12)\end{aligned}$$

For arbitrary channel gain coefficients $\{H_k\}$, the expression (8) presents two amplitude maxima for $\tau = 0$ and $\tau = T_u$. In particular, for $\tau = T_u$,

$$y(T_u) = \beta e^{j2\pi T_u \Delta f_c}, \qquad (13)$$

where $\beta = P\sigma_c^2 T_g \sum_k |H_k|^2$ is a real strictly positive. The phase of $y(T_u)$ is thus given by

$$\Phi^1_{f_c} = 2\pi T_u \Delta f_c. \qquad (14)$$

The closed form estimator of Δf_c, derived from $Y(f)$, is thus

$$\widehat{\Delta f_c} = \frac{Arg(FT^{-1}(Y(f))|_{t=T_u})}{2\pi T_u}. \qquad (15)$$

It allows to estimate frequency offsets smaller than the spacing between adjacent subcarriers $|\Delta f_c| \le \frac{1}{2T_u}$ without ambiguity.

• ***Comparison to Bolcskei algorithm [9, 10]:*** It is worth noting that $y(\tau)$ defined by (3) can be expressed otherwise involving the temporal autocorrelation function

$$R_{\tilde{s}}(t,\tau) = E\left(\tilde{s}(t)\tilde{s}^*(t-\tau)\right) \qquad (16)$$

as follows

$$y(\tau) = \int E(\tilde{s}(t)\tilde{s}^*(t-\tau))dt = \int R_{\tilde{s}}(t,\tau)dt. \qquad (17)$$

In this way, Eq. (17) identifies $y(\tau)$ to the cyclic autocorrelation coefficient of order 0 denoted $r^0(\tau)$. In [9, 10], the cyclic autocorrelation of OFDM signals, computed as the FT of $R_{\tilde{s}}(t,\tau)$ with respect to t, is used to derive the synchronization parameters.
In practice, $R_{\tilde{s}}(t,\tau)$ is estimated from $\{\tilde{s}(t), \tilde{s}(t-\tau)\}$ by $\widehat{R_{\tilde{s}}}(t,\tau) = \tilde{s}(t)\tilde{s}(t-\tau)^*$. The method proposed above involves an averaging of frequency domain transformed signals to compute $Y(f)$, which is further exploited to recover the frequency offset.

3.2 Timing synchronization : estimation of t_0

We now define

$$\begin{aligned}&Z(\Delta f, f-\Delta f) \overset{def}{=} \\ &\int\int R_{\tilde{s}}(t,\tau)e^{-j2\pi\Delta f t}e^{-j2\pi(f-\Delta f)\tau}dtd\tau = \\ &E\left(\left(\int \tilde{s}(u)e^{-j2\pi fu}du\right)\left(\int \tilde{s}(v)e^{-j2\pi(f-\Delta f)v}dv\right)^*\right) \qquad (18)\end{aligned}$$

which can be identified to the spectral correlation evaluated for a frequency shift Δf. Applying (18) on the data model (2) leads to

$$Z(\Delta f, f-\Delta f) = \sigma_c^2 T_s^2 \sum_p e^{-j2\pi\Delta f(t_0+pT_s+\frac{T_s}{2})} \times$$
$$\sum_k |H_k|^2 \frac{sin(\pi T_s(f-\Delta f_c - \frac{k}{T_u}))}{\pi T_s(f-\Delta f_c-\frac{k}{T_u})} \frac{sin(\pi T_s(f-\Delta f-\Delta f_c-\frac{k}{T_u}))}{\pi T_s(f-\Delta f-\Delta f_c-\frac{k}{T_u})}$$
$$+\sigma^2\delta_{\Delta f,0}. \quad (19)$$

For $\Delta f = \frac{n}{T_s}$, ($n \in \mathbf{Z}^*$), one obtains

$$Z(\frac{n}{T_s}, f-\frac{n}{T_s}) = \lambda e^{-j2\pi\frac{n}{T_s}t_0} \times$$
$$\sum_k |H_k|^2 \frac{sin^2(\pi T_s(f-\Delta f_c-\frac{k}{T_u}))}{\pi T_s(f-\Delta f_c-\frac{k}{T_u})\pi T_s(f-\frac{n}{T_s}-\Delta f_c-\frac{k}{T_u})}. \quad (20)$$

Over frequencies f that verify $(f-\Delta f_c-\frac{k}{T_u})(f-\Delta f-\Delta f_c-\frac{k}{T_u}) > 0$, (i.e. $f < \Delta f_c + \frac{k_{min}}{T_u}$ and $f > \Delta f_c + \frac{k_{max}}{T_u} + \Delta f$), the phase of $Z(\frac{n}{T_s}, f-\frac{n}{T_s})$ is given by

$$\Phi_{t_0}^n = -2\pi\frac{n}{T_s}t_0, \quad (21)$$

and the timing error can be recovered in closed form by

$$\widehat{t_0} = -\frac{Arg(Z(\frac{n}{T_s}, f-\frac{n}{T_s}))}{2\pi\frac{n}{T_s}}. \quad (22)$$

The so obtained $\widehat{t_0}$ accuracy is enhanced if (22) is averaged over the considered frequencies interval. For $n = 1$ (*i.e.* $\Delta f = \frac{1}{T_s}$), timing errors up to half the OFDM symbol length $\frac{T_s}{2}$ can be recovered without ambiguity.

• ***Comparison to Bolcskei algorithm [9, 10]:*** Expression (18) can be expressed otherwise as

$$Z(\Delta f, f-\Delta f) = \int r^{\Delta f}(\tau) e^{-j2\pi(f-\Delta f)\tau} d\tau \quad (23)$$

where $r^{\Delta f}(\tau)$ denotes the cyclic autocorrelation coefficient for the cyclic frequency Δf. In this way, Eq. (23) identifies $Z(\Delta f, f-\Delta f)$ to the cyclic spectrum. In [9, 10], the cyclic spectrum, computed as the FT of the cyclic autocorrelation $r^{\Delta f}(\tau)$, is exploited for synchronization purpose. In this paper, rather than using the temporal autocorrelation, the spectral correlation is computed based on a frequency domain signals averaging.

4. NUMERICAL EXAMPLES

This section aims to assess the performance of the algorithms proposed herein for blind synchronization of OFDM signals including a CP.

4.1 Conditions of simulations

The considered OFDM signal has the following characteristics $T_s = 1.25T_u$ ($T_g = 0.2T_s$) with $T_u = 13.3\mu s$. The number of subcarriers is $N = 20$. The sampling frequency used to generate the signal is $F_e = 2\frac{N}{T_u}$. Therefore, one OFDM symbol contains $T_sF_e = 50$ temporal samples. Each subcarrier is randomly modulated from a 16-QAM.

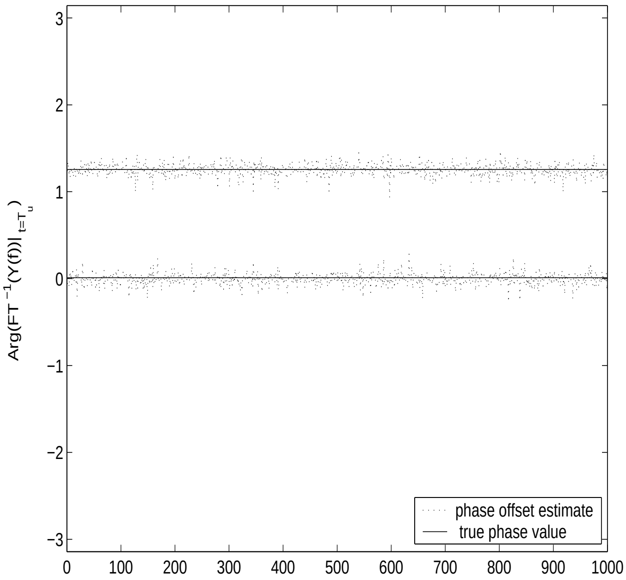

Figure 1: $\Phi_{f_c}^1$ estimation for $\Delta f_c = 0$ and $\Delta f_c = 1/5T_u$. AWGN channel with $SNR = 10dB$.

The data used to derive one estimate of each of the parameters of interest corresponds to samples over 100 OFDM symbols (5.10^3 samples).
To compute expressions (6) and (18), an averaging over frequency domain signals is operated. Each of these signals is computed using one temporal data block. To this aim, the data are cut into blocks of 1024 samples and adjacent blocks are shifted by 124 samples. Thus, 33 blocks are considered, each of which consists of about 20.48 OFDM symbols.
The performance is evaluated in terms of normalized Mean Square Error (MSE) and bias (with respect to true values of the estimated parameter) for a Signal to Noise Ratio (SNR) ranging from $-5dB$ to $15dB$ over 10^4 Monte Carlo trials. In each trial, the channel and the transmitted data are randomly generated. The OFDM signal incorporates both a frequency offset and the symbol start is not correctly detected ($t_0 \neq 0$). Unless specifically mentioned otherwise, $\Delta f_c = \frac{1}{10T_u} = 7.5188KHz$ and $t_0 = \frac{T_s}{6} = 2.7708\mu s$.

4.2 Results

Figure 1 exhibits $\Phi_{f_c}^1$ estimates, recovered as the phase of $FT^{-1}(Y(f))|_{t=T_u}$, and the true phase of $y(T_u)$ for two distinct values of the frequency offset Δf_c over 10^2 Monte Carlo trials. It shows the good accuracy of the proposed estimator. Figure 2 displays the phase of $Z(1/T_s, f-1/T_s)$, which provides an estimate of $\Phi_{t_0}^1$ given by Eq. (21) over frequencies that verify $f < \Delta f_c|_{min} + \frac{k_{min}}{T_u} \simeq -751KHz$ or $f > \Delta f_c|_{max} + \frac{k_{max}}{T_u} + \frac{1}{T_s} \simeq 812KHz$, where $\Delta f_c|_{min} = -\frac{1}{2T_u}$ (resp. $\Delta f_c|_{max} = \frac{1}{2T_u}$) is the minimum (resp. maximum) carrier offset value that can be estimated without ambiguity. This discards the central discontinuities of $Arg(Z(1/T_s, f-1/T_s))$ observed on Figure 2. This phase estimate is used to evaluate t_0 by Eq. (22) for $n = 1$.

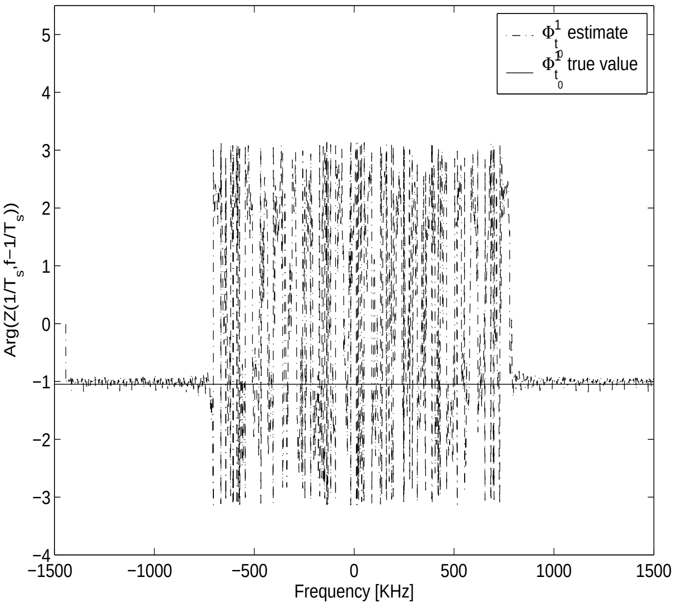

Figure 2: Phase of $Z(1/T_s, f-1/T_s)$. AWGN channel with $SNR = 10dB$. (one realization of Monte Carlo trials)

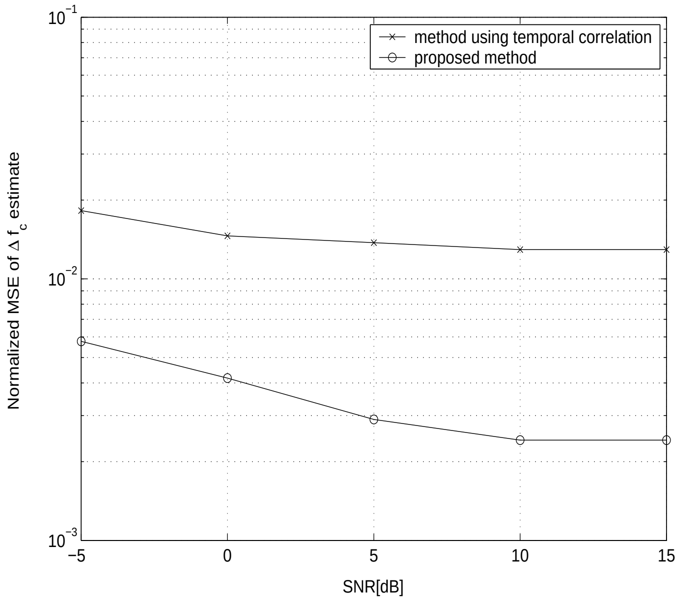

Figure 3: Normalized Mean Square Error of $\widehat{\Delta f_c}$. Case of AWGN channel.

	AWGN proposed	AWGN temporal	Rayleigh proposed	Rayleigh temporal
MSE	$2.4\ 10^{-3}$	$1.33\ 10^{-2}$	$3.47\ 10^{-2}$	0.50
bias	$1.75\ 10^{-5}$	0.1	$6.2\ 10^{-3}$	0.32

Table 1: Performance of Δf_c estimation. Normalized MSE and bias over the AWGN and the Rayleigh channels. $SNR = 15dB$.

	AWGN proposed	AWGN temporal	Rayleigh proposed	Rayleigh temporal
MSE	$1.7\ 10^{-2}$	$7.59\ 10^{-2}$	$2.45\ 10^{-2}$	$7.7\ 10^{-2}$
bias	$5.7\ 10^{-3}$	$1.24\ 10^{-2}$	$3.42\ 10^{-2}$	$7.08\ 10^{-2}$

Table 2: Performance of t_0 estimation. Normalized MSE and bias over the AWGN and the Rayleigh channels. $SNR = 10dB$.

Figures 3 and 4 compare the normalized MSE (over 10^4 Monte Carlo trials) obtained by the method proposed in this paper to that using the cyclic autocorrelation in [9, 10], and referred to by method using temporal correlation. The normalized MSE is plotted versus the SNR, respectively for Δf_c and t_0 estimates. Both figures show that an enhanced performance is obtained by the proposed estimators. This enhancement can be related to the better accuracy of $Y(f)$ and $Z(\Delta f, f-\Delta f)$ estimation (by averaging different realizations over different temporal signal blocks) compared to that of $R_{\tilde{s}}(t, \tau)$.

Tables 1 and 2 display the normalized bias and MSE of the compared algorithms in the cases of the AWGN and Rayleigh channels and respectively for Δf_c and t_0 estimates. The results show again the better accuracy of the proposed approach for both kind of channels. For the carrier frequency recovery, a degradation is observed in the case of the Rayleigh channel with respect to the AWGN case. However, for the timing error estimation, the accuracy is only slightly affected by the channel selectivity.

5. CONCLUSION

In this paper, the power spectrum and spectral correlation of OFDM signals are evaluated based on frequency domain transformed signals. They are further used to estimate the timing and carrier frequency offsets. The proposed estimators are independent and computationally efficient. They show good performance for both the AWGN and the Rayleigh fading channels. Enhanced accuracy is obtained with respect to the approach exploiting the temporal autocorrelation and cyclic autocorrelation. Also, no channel knowledge is required to apply the proposed estimators.

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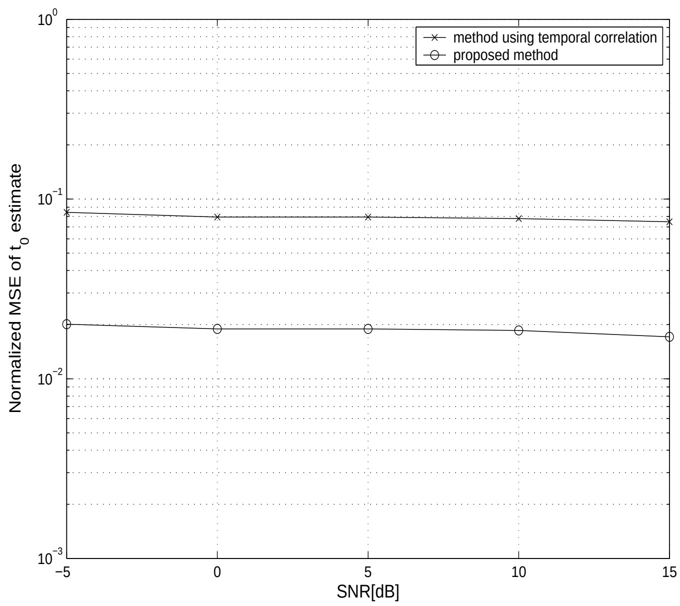

Figure 4: Normalized Mean Square Error of $\widehat{t}_0$. Case of AWGN channel.

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