ON THE DESIGN OF GCF COMPENSATION FILTER BASED ON MINIMAX OPTIMIZATION

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ABSTRACT

This paper presents the design of the compensation filter of a generalized comb filter (GCF) based on minimax optimization method. The coefficients of the proposed filter are obtained by solving two simple linear equations. The filter operates at a low rate and considerably reduces the passband droop of the GCF filter.

1. INTRODUCTION

The simplest decimation filter, proposed by Hogenauer [1], is the cascaded-integrator-comb (CIC) filter. However, this filter has a high passband droop and a low stopband attenuation. Different methods have been proposed to improve the passband and the stopband characteristics of a CIC filter [2, 3, 4, 5, 6]. Recently, a generalized CIC decimation filter (GCF) has been proposed in [6]. As a result an increased stopband attenuation as well as extended bands around the zeros of the magnitude characteristic of the CIC filter are obtained. The bands around zeros, i.e., frequency points $2\pi k/D$ where D is the decimation factor, and $k = 1,\ldots,D-1$ are called folding bands [6, 7].

The transfer function of the GCF filter is expressed as [6]

$$H_{\mathrm{GCF}_N}(z) = \prod_{n=1}^{N} \frac{\sin(\alpha_n/2)}{\sin(\alpha_n D/2)} \prod_{n=1}^{N} \frac{1 - z^{-D} \mathrm{e}^{-\mathrm{j}\alpha_n D}}{1 - z^{-1} \mathrm{e}^{-\mathrm{j}\alpha_n}}, \quad (1)$$

where D stands for the decimation factor and α_n, $n = 1,\ldots,N$, are rotation parameters optimized such that the minimum attenuation within folding bands is maximized [6].

The discrete-time Fourier transform (DTFT) of $H_{\mathrm{GCF}_N}(z)$ is

$$H_{\mathrm{GCF}_N}(\mathrm{e}^{\mathrm{j}\omega}) = H(\omega) \exp\left(-\mathrm{j}\frac{(D-1)}{2}\left(\omega N + \sum_{n=1}^{N} \alpha_n\right)\right), \quad (2)$$

where

$$H(\omega) = \prod_{n=1}^{N} \frac{\sin(\alpha_n/2)}{\sin(\alpha_n D/2)} \prod_{n=1}^{N} \frac{\sin((\omega+\alpha_n)D/2)}{\sin((\omega+\alpha_n)/2)}. \quad (3)$$

In general case $H_{\mathrm{GCF}_N}(z)$ has linear-phase characteristics and complex-valued coefficients (see (2)). The real-valued filter coefficients of $H_{\mathrm{GCF}_N}(z)$ are obtained satisfying $\alpha_n = -\alpha_{N-n}$ [6]. A useful choice for α_n is $\alpha_n = q_n \pi/\nu D$, where ν is a positive integer and q_n is a real value in the range $[-1,1]$ [6]. Traditional CIC filter is obtained by setting $\alpha_n = 0$, $n = 1,\ldots,N$.

As one example, consider the design of a GCF filter using the following parameters: $N = 5$, $D = 7$, $\nu = 4$, and $q_n = [-0.55,\ -0.93,\ 0,\ 0.93,\ 0.55]$ [6]. Figure 1(a) shows the magnitude response of the resulting GCF filter, while the passband detail is illustrated in Fig. 1(b). Notice the increased width and attenuations at the folding bands. Unfortunately, the GCF filter exhibits a high passband droop (see Fig. 1(b)).

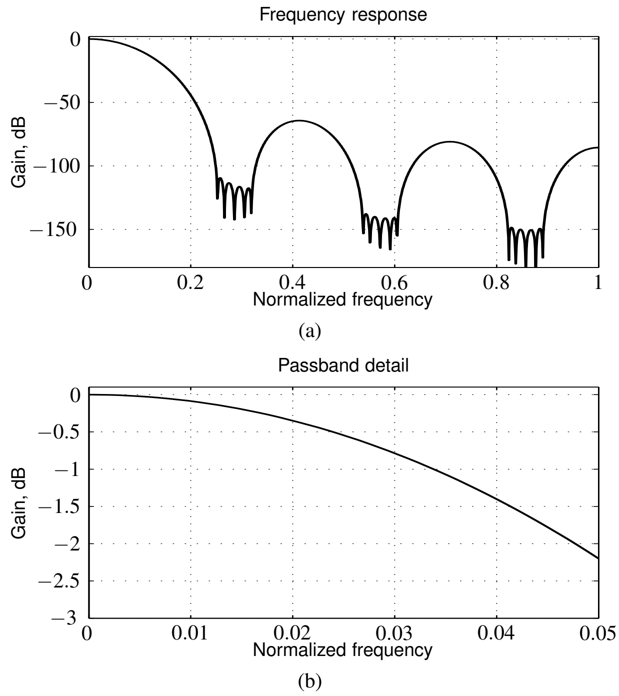

Figure 1: Magnitude response of the GCF filter for $N = 5$, $D = 7$, and $\nu = 4$.

The review of existing compensation methods shows that compensation filters are mainly presented for traditional CIC filters and not for GCF filters. See for example [2, 3, 4]. To this end, in this paper we introduce a design method for the GCF passband compensation, based on minimax optimization. More general approach that includes different design

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constraints, i.e. maximally flat, least square, and minimax is elaborated in [8].

The presented design also includes the CIC passband compensation as a special case.

The paper is organized as follows. Section 2 introduces the proposed second order compensation filter. Discussions and results are presented in Section 3.

2. PROPOSED GCF COMPENSATION FILTER

The transfer function of the proposed GCF compensation filter is given as

$$P(z^D) = a + bz^{-D} + az^{-2D}, \tag{4}$$

where a and b are real valued constants.

The compensation filter is cascaded with the GCF filter as shown in Fig. 2(a). Using the multirate identity [9] the filter $P(z^D)$ can be moved to lower rate resulting in more efficient structure shown in Fig. 2(b).

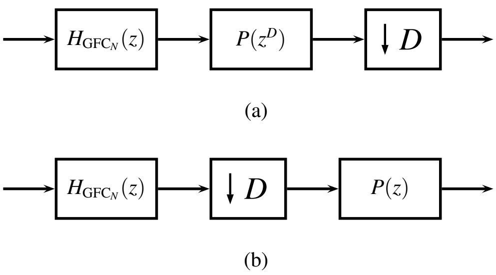

Figure 2: Decimation block diagrams. a) Generalized CIC filter $H_{\mathrm{GCF}_N}(z)$ and compensation filter. b) Efficient structure for decimation.

The cascade of the compensation filter $P(z^D)$ and the GCF filter yields the following overall transfer function:

$$G(z) = H_{\mathrm{GCF}}(z)P(z^D). \tag{5}$$

By performing the DTFT, equation (5) becomes

$$G(\mathrm{e}^{\mathrm{j}\omega}) = \mathrm{e}^{-\mathrm{j}\omega((D-1)N+2D)/2} H(\omega) P_{\mathrm{R}}(D\omega), \tag{6}$$

where $P_{\mathrm{R}}(D\omega)$ is the amplitude response of $P(\mathrm{e}^{\mathrm{j}\omega D})$, which is given by

$$P_{\mathrm{R}}(D\omega) = b + 2a\cos(D\omega). \tag{7}$$

We define the error function

$$E(\omega) = H(\omega)P_{\mathrm{R}}(D\omega) - 1. \tag{8}$$

In order to find the coefficients a and b, we impose the condition that the error function should be zero at frequencies $\omega = \omega_1$ and $\omega = \omega_2$, in the passband $[0, \omega_{\mathrm{p}}]$.

For $\omega = \omega_1$, from (3), (6)–(8), it follows that

$$H(\omega_1)(2a\cos(D\omega_1) + b) = 1. \tag{9}$$

Similarly, for $\omega = \omega_2$ (see (6)–(8)), the imposed condition results in

$$H(\omega_2)(2a\cos(D\omega_2) + b) = 1. \tag{10}$$

Solving equations (9) and (10), the values of a and b are, respectively,

$$a = \frac{1}{2}\frac{1/H(\omega_1) - 1/H(\omega_2)}{\cos(D\omega_1) - \cos(D\omega_2)}, \tag{11}$$

$$b = \frac{\cos(D\omega_1)/H(\omega_2) - \cos(D\omega_2)/H(\omega_1)}{\cos(D\omega_1) - \cos(D\omega_2)}. \tag{12}$$

Substituting (11) and (12) into (7), the error function becomes

$$E(\omega) = H(\omega)\left(\frac{1/H(\omega_1) - 1/H(\omega_2)}{\cos(D\omega_1) - \cos(D\omega_2)}\cos(D\omega) + \frac{\cos(D\omega_1)/H(\omega_2) - \cos(D\omega_2)/H(\omega_1)}{\cos(D\omega_1) - \cos(D\omega_2)}\right) - 1. \tag{13}$$

We apply the minimax optimization of the error function $E(\omega)$, i.e.,

$$\delta = \min_{\omega_1, \omega_2} \max_{[0, \omega_{\mathrm{p}}]} |E(\omega)|. \tag{14}$$

Figure 3 shows the error function, $E(\omega)$ in (13), obtained by minimax optimization (14).

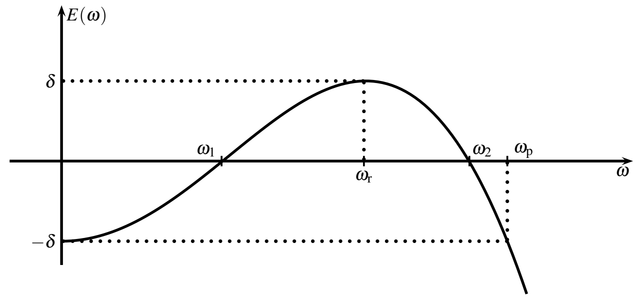

Figure 3: Error function $E(\omega)$.

From Fig. 3, at the frequency point $\omega = 0$, we have

$$E(0) = -\delta. \tag{15}$$

Similarly, for ω_{r} and ω_{p}, we have

$$E(\omega_{\mathrm{r}}) = \delta, \tag{16}$$

$$E(\omega_{\mathrm{p}}) = -\delta. \tag{17}$$

Additionally, the derivative of the error function evaluated at $\omega = \omega_{\mathrm{r}}$ equals zero, i.e.,

$$\left.\frac{\mathrm{d}E(\omega)}{\mathrm{d}\omega}\right|_{\omega=\omega_{\mathrm{r}}} = 0. \tag{18}$$

Next issue is to relate the frequencies ω_1 and ω_2 with the passband frequency ω_{p} for different values of N and D. To this end we present in Figs. 4(a) and 5(a) the frequencies ω_1 and ω_2 as a function of ω_{p}, respectively, for $N = 3, 4, 5, 6$, $D = 3, 5, 7, 11$, and $\nu = 4$. Similarly, Figs. 4(b) and 5(b) show the zoom of the upper part of Figs. 4(a) and 5(a). Note that

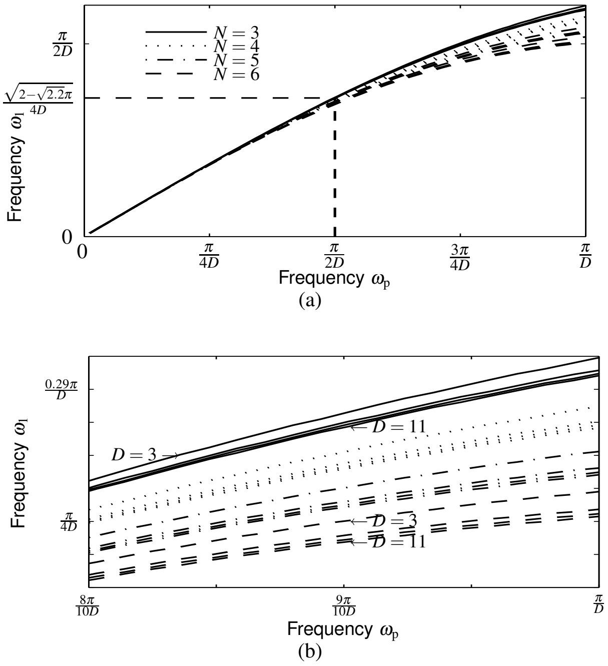

Figure 4: Frequency ω_1 as a function of the frequency ω_p for $N = 3,4,5,6$, $D = 3,5,7,11$, and $\nu = 4$.

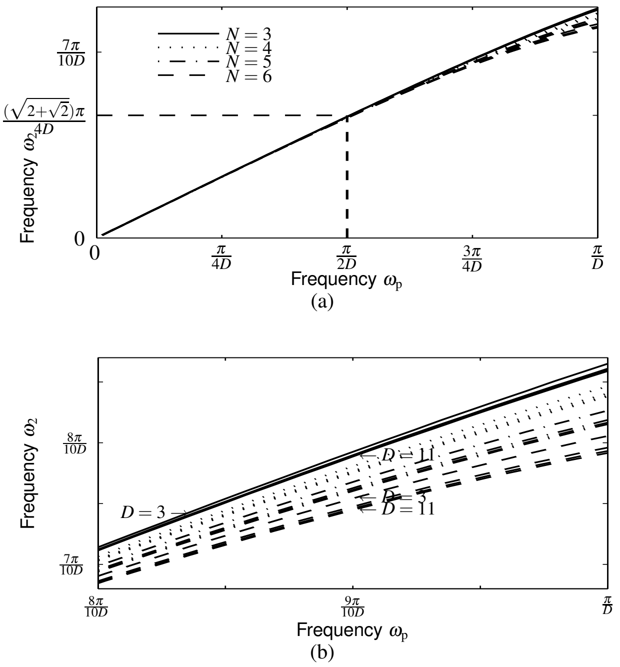

Figure 5: Frequency ω_2 as a function of the frequency ω_p for $N = 3,4,5,6$, $D = 3,5,7,11$, and $\nu = 4$.

the relations between ω_1 and ω_p and ω_2 and ω_p are approximately linear in the band $[0,\pi/2D]$ and that ω_1 and ω_2 are practically independent of N, D, and ν in this band.

Solving equations (15)–(18) and knowing that for small values of ω,

$$\cos(D\omega) \approx 1 - \frac{D^2\omega^2}{2}, \tag{19}$$

and

$$H(\omega) \approx 1 - \frac{H''(0)\omega^2}{2}, \tag{20}$$

where $H''(0)$ is the second derivative of $H(\omega)$ evaluated at $\omega = 0$, we obtain the closed form equations for ω_1 and ω_2, that is,

$$\omega_1 \approx \frac{\sqrt{2-\sqrt{2.2}}}{2}\omega_\mathrm{p}, \tag{21}$$

$$\omega_2 \approx \frac{\sqrt{2+\sqrt{2}}}{2}\omega_\mathrm{p}. \tag{22}$$

3. DISCUSSION OF RESULTS

Based on the results of Section 2, we have the procedure for the design of GCF compensation filter as follows:

Step 1. For a given value of passband frequency ω_p, compute the values of the frequencies ω_1 and ω_2 using (21) and (22), respectively.

Step 2. Use ω_1 and ω_2 to get the filter coefficients a and b from (11) and (12).

Step 3. Substitute a and b into (4) to obtain the designed filter.

In the following we analyze the passband droop R_p in dB after compensation.

Figures 6(a) and 6(b) illustrate the passband droops as a function of the frequency ω_p for $N = 3,4,5,6$, $D = 3,5,7,11$, and $\nu = 4$. Observe that the passband droop is less than 0.4 dB in the band of interest.

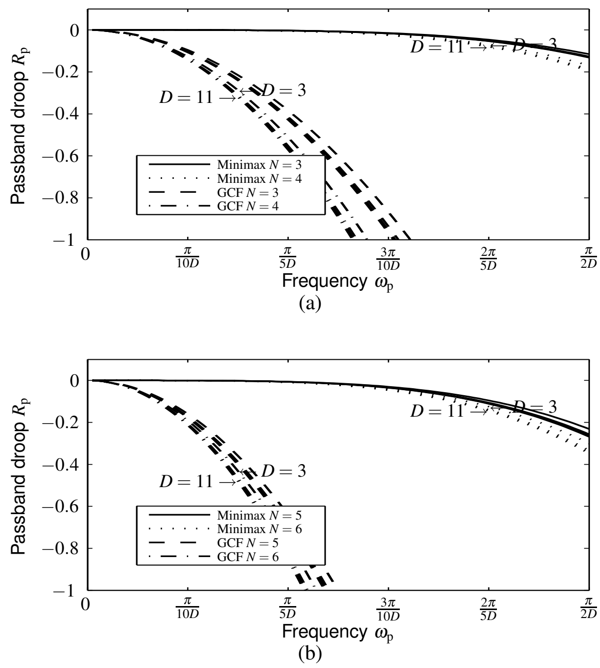

Figure 6: a) Passband droops for $N = 3,4$ and $D = 3,5,7,11$ b) Passband droops for $N = 5,6$, and $D = 3,5,7,11$.

Example. We design a GCF compensation filter with the following design parameters $D = 7$, $N = 5$, $\omega_p = 0.45\pi/D$, and $\alpha_n = q_n\pi/4D$ for $n = 1,2,3,4,5$, where $q_n = [-0.55, -0.93, 0, 0.93, 0.55]$ [6].

The corresponding passband droop of the GCF filter is $R_p = -3.66$ dB, as shown in Fig. 7.

The method is illustrated in the following steps:

Step 1. From (21) and (22) the values of ω_1 and ω_2 are

$$\omega_1 = 0.161743\pi/D,$$
$$\omega_2 = 0.415745\pi/D.$$

Step 2. The corresponding filter coefficients are

$$a = -0.30776,$$
$$b = 1.592821.$$

Step 3. The resulting transfer function of the GCF compensation filter is

$$P(z^D) = -0.30776(1+z^{-2D}) + 1.592821z^{-D}.$$

The results of the design are summarized in Table 1.

	GCF_5	Minimax
R_p	-3.66	-0.17
ω_1		$0.161743\pi/D$
ω_2		$0.415745\pi/D$
a		-0.30776
b		1.592821

Table 1: Parameters in the design example.

The frequency responses along with the passband details are shown in Fig. 7.

From Fig. 7, it is worth highlighting that the reduction of the stopband attenuation impacts the regions $[2\pi k/D + \pi/\nu D, 2\pi(k+1)/D + \pi/\nu D]$, for $k = 1,\ldots,D-2$, (don't care regions [6]) and not the folding bands. However, the passband droop is well compensated resulting in $R_p = -0.17$ dB.

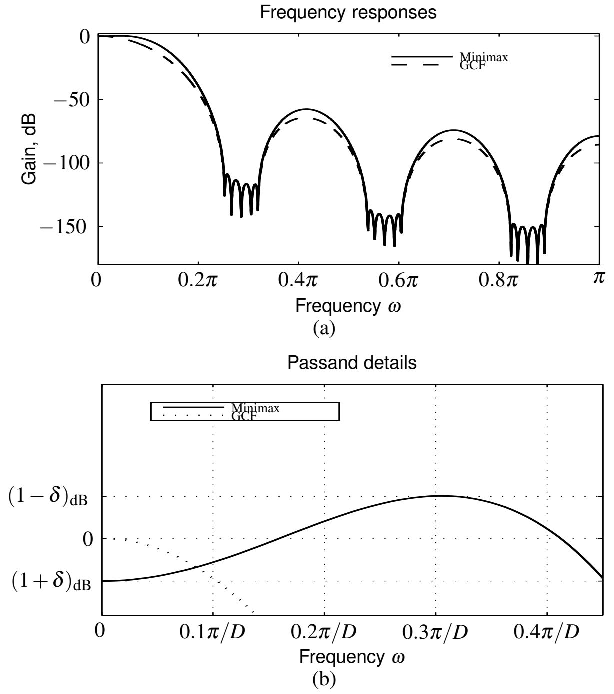

Figure 7: Overall magnitude response of the GCF filter and the compensation filter in the design example.

4. CONCLUSIONS

A novel design technique to the optimum, in the minimax sense, GCF compensation filter is presented. This result can be further generalized to include also least square and maximally flat designs, as explained in [8]. The designed $2D$ order compensation filter becomes a second order filter after moving to a lower rate. The main advantage of the proposed method is that the filter coefficients are obtained by using closed form equations, which depend on the passband frequency ω_p. The designed compensation filter results in a considerable decrease of the passband droop, of GCF filter which is less than 0.4 dB in the passband region of interest. The presented design also includes the CIC compensation filter design, as a special case ($\alpha_n = 0$, $n = 1,\ldots,N$).

Acknowledgements

This work was supported by CONACyT Mexico under grant No. 49640.

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