A NEW CHANNEL ORDER ESTIMATION ALGORITHM FOR FIR SIMO CHANNELS

Serkan Karakutuk, T. Engin Tuncer

Electrical and Electronics Eng. Dept., METU, Ankara, Turkey
email: e130522@metu.edu.tr, etuncer@metu.edu.tr

ABSTRACT

Channel order estimation problem is considered for FIR (Finite Impulse Response) modeled single-input multi-output (SIMO) communication systems. The performance of the channel estimation algorithms depends on the accuracy and robustness of the channel order estimation. A new channel order estimation algorithm with high accuracy and robustness is proposed for SIMO systems. The proposed algorithm is based on the least squares smoothing (LSS) algorithm for channel estimation. It is guaranteed to find the true channel order from finite number of samples for noise free case. Several experiments are performed and it is shown that the proposed algorithm significantly enhances the performance in channel order estimation in noisy observations. The comparisons with the alternative techniques show that the proposed method is very effective for the channel order estimation.

1. INTRODUCTION

Channel order estimation is an important problem in many signal processing applications. In this paper, this problem is considered for single-input, multi-output (SIMO) systems. The input and channel coefficients can be estimated accurately when the true channel order is known. When the channel order is underestimated, channel coefficients cannot be found and the result is a complete failure. When the channel order is overestimated, the performance of the algorithms such as subspace (SS), LSS [5] and cross relation (CR) decreases [11]. There are methods that work in a robust manner in case of channel order overestimation [9, 10, 11, 12]. The main disadvantage of such techniques is that their performance is not as good as the SS or LSS algorithm when the true channel order is supplied to those algorithms [11]. Under these circumstances, channel order estimation with high accuracy is important and it determines the performance of the channel estimation algorithms.

There are different algorithms for channel order estimation in the literature. Minimum Description Length (MDL) [1] and Akaike Information Criteria (AIC) [2] algorithms are based on the information theoretic criteria. These algorithms require long observations for accurate extraction of the statistical parameters. It is known that MDL usually performs better than the AIC and AIC has a tendency for channel overestimation [3, 14]. Both of these algorithms are sensitive to deviations from white noise assumption [14]. Liavas algorithm [3] is proposed in order to overcome the limitations of AIC and MDL. This algorithm has a tendency to underestimate the channel order [4]. A review of different channel order estimation algorithms can be found in [8].

One problem in channel order estimation is the fact that cost functions monotonically decrease as the channel order increases. This problem is tried to be solved by using an empirically chosen penalty coefficient [8]. This penalty term leads to over or under estimation in many of the information theoretic techniques. In [4], a new cost function is proposed. This cost function is obtained by combining two cost functions due to channel estimation (ID) and channel equalization (EQ). The main feature of this cost function is its ”convex - like” shape. Therefore channel order estimation can be performed by finding the global minimum. Our motivation is to construct a similar cost function which allows us to obtain the channel order from the global minimum.

Joint least squares smoothing (JLSS) [6] and ID+EQ[4] are the two algorithms that can find the true channel order from finite number of samples in noise free case. Until now, JLSS and ID+EQ were the only algorithm known to have finite convergence property for the channel order estimation.

In this paper, a new channel order estimation algorithm is proposed, namely channel matrix recursion (CMR) algorithm. CMR is based on the properties of the LSS algorithm. Channel matrix is estimated using LSS algorithm for a range of channel orders. The relation between the channel matrices with consecutive channel orders are used to obtain a new cost function for channel order estimation. It has the finite convergence property. Therefore CMR finds the true channel order by using finite number of samples in noise free case. In addition, it has several distinct features which make it one of the most effective algorithm in the literature. Its performance is very good for noisy observations. Furthermore, CMR is robust to different parameters such as the number of channels, channel order and the number of samples.

Several experiments are done in order to compare the proposed algorithm with the alternative techniques such as MDL, AIC, Liavas, ID+EQ [4] and JLSS [6]. It is shown that the proposed algorithm has significantly better performance than the other methods for a variety of cases.

The organization of this paper is as follows. Section 2 describes the system model and the problem. The proposed method is presented in Section 3. The performance of the proposed method is evaluated in Section 4.

2. SYSTEM MODEL AND PROBLEM DEFINITION

The system structure for a single-input multi-output (SIMO) channel is shown in Figure 1. $s(t)$ is the input signal, and there are P channels with channel order L.

The channel output vector can be written as;

$$\mathbf{y}_1(t) = \sum_{k=0}^{L} \mathbf{h}_L(k)\, s(t-k) + \mathbf{n}_1(t) \qquad (1)$$

where

$$\mathbf{y}_1(t) = [\ y_1(t) \quad y_2(t) \quad \cdots \quad y_P(t)\ ]^T$$

$$\begin{aligned}\mathbf{h}_L(k) &= [\ h_{L,1}(k) \quad h_{L,2}(k) \quad \cdots \quad h_{L,P}(k)\ ]^T \\ \mathbf{n}_1(t) &= [\ n_1(t) \quad n_2(t) \quad \cdots \quad n_P(t)\ ]^T\end{aligned}$$

The vector valued quantities $\mathbf{y}_1(t)$, $\mathbf{h}_L(k)$, and $\mathbf{n}_1(t)$ are the received signals, channel impulse response and additive noise respectively. Subindex for channel coefficients represents the channel order and other subindexes indicate the block dimension of the vector. The number of channel is assumed to P, hence $\mathbf{y}_1(t)$, $\mathbf{h}(k)$, and $\mathbf{n}_1(t)$ are P dimensional column vectors. The order of channel is L and the number of snapshots is N for each channel. t is the discrete time index. $y_i(t)$, $h_i(k)$, and $n_i(t)$ are the scalar values of output signal, channel impulse response and additive noise for i^{th} channel respectively. The vector counterpart of the model,

$$\mathbf{y}_1(t) = \mathbf{H}_1 \mathbf{s}_{L+1}(t) + \mathbf{n}_1(t) \tag{2}$$

where,

$$\begin{aligned}\mathbf{H}_1 &= [\ \mathbf{h}_L(L) \quad \mathbf{h}_L(L-1) \quad \cdots \quad \mathbf{h}_L(0)\ ] \\ \mathbf{s}_{L+1} &= [\ s(t-L) \quad \cdots \quad s(t)\ ]^T\end{aligned}$$

To fully utilize the time correlation properties of the SIMO system, M vector samples are studied simultaneously.

$$\mathbf{y}_M(t) = \mathbf{H}_M \mathbf{s}_{M+L}(t) + \mathbf{n}_M(t) \tag{3}$$

where

$$\begin{aligned}\mathbf{y}_M(t) &= [\ \mathbf{y}_1^T(t-M+1) \quad \cdots \quad \mathbf{y}_1^T(t)\ ]^T \\ \mathbf{n}_M(t) &= [\ \mathbf{n}_1^T(t-M+1) \quad \cdots \quad \mathbf{n}_1^T(t)\ ]^T \\ \mathbf{s}_{M+L}(t) &= [\ s(t-L-M+1) \quad \cdots \quad s(t)\ ]^T \\ \mathbf{H}_M &= \begin{bmatrix} \mathbf{h}_L(L) & \cdots & \mathbf{h}_L(0) & & \\ & \ddots & \cdots & \ddots & \\ & & \mathbf{h}_L(L) & \cdots & \mathbf{h}_L(0) \end{bmatrix}\end{aligned}$$

Where, $MP \times (M+L)$ block Toeplitz matrix $\mathbf{H}_M$ is called the channel matrix. The equation (3) can be written as follow, when N channel output vectors $\mathbf{y}_M$ are stacked as the columns of the matrix $\mathbf{Y}$.

$$\mathbf{Y} = \mathbf{H}_M \mathbf{S} + \mathbf{N} \tag{4}$$

Where,

$$\begin{aligned}\mathbf{Y} &= [\ \mathbf{y}_M(t) \quad \cdots \quad \mathbf{y}_M(t+(N-1)M)\ ] \\ \mathbf{S} &= [\ \mathbf{s}_{M+L}(t) \quad \cdots \quad \mathbf{s}_{M+L}(t+(N-1)M)\ ] \\ \mathbf{N} &= [\ \mathbf{n}_M(t) \quad \cdots \quad \mathbf{n}_M(t+(N-1)M)\ ]\end{aligned}$$

Our goal is to estimate the unknown channel order L from the observed data. The following assumptions are used during the order estimation. The channel transfer functions do not share common zeros or the channel matrix is full column rank. Input signal, $s(t)$, has linear complexity greater than $2M+2L$, which is the requirement for the LSS algorithm used in the proposed method.

3. CMR ALGORITHM

Channel matrix recursion (CMR) algorithm is based on the estimation channel matrix via LSS algorithm for different channel orders. Before the explanation of the algorithm details, it is better to indicate some important properties of LSS algorithm for noise free case.

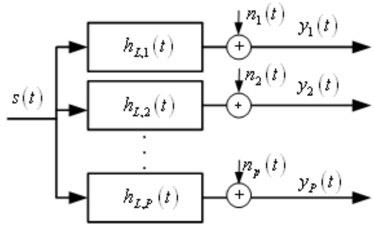

Figure 1: SIMO channel model for channel order, L, and number of channel, P.

- When the channel order is overestimated, LSS results common zeros besides the true channel zeros.
- When the channel order is overestimated, the estimated channel matrix is not full column rank as a result of common zeros.

Properties of LSS algorithm given above are stated by Lemma-1, Lemma-2 in this paper. Proofs are not given due to space limitation.

Lemma-1: Least Squares Smoothing algorithm [5] generates common channel zeros when the channel order is overestimated for noise free case. The remaining zeros are the true channel zeros. If the true channel order is L and the overestimated channel order is $\hat{L}$, the number of common zeros is $\hat{L}-L$.

Lemma-2: If FIR channels of SIMO system have common zeros, then the channel matrix, $\mathbf{H}_M$ whose row size is greater or equal than the column size, is not full column rank.

As a result of Lemma-1, the channel matrix with overestimated channel order, i.e., $\hat{L} = L+m$, $m > 0$, can be written as follows.

$$\mathbf{H}_M^{(L+m+1)} = \mathbf{H}_M^{(L)} \mathbf{H}_c^{(m+1)} \tag{5}$$

where $\mathbf{H}_c^{(m)}$ is a Toeplitz matrix with first row and first column being $[c_m(0) \cdots c_m(m)\ 0 \cdots\ 0]$ and $[c_m(0)\ \ 0 \cdots 0]^T$ respectively. $c_m(k)$ are the coefficients of the transfer function of m common zeros. CMR algorithm uses the relation between the estimated channel matrices with consecutive channel orders. The relation between the channel matrices with channel order $L+m$ and $L+m+1$ can be written as follow.

$$\mathbf{H}_M^{(L+m+1)} = \mathbf{H}_M^{(L+m)} \mathbf{A}_m \tag{6}$$

The proposed algorithm is based on the estimation of the relation matrix $\mathbf{A}_m$, which is defined as follows.

$$\hat{\mathbf{A}}_m = \mathbf{F} \odot \left(\left(\mathbf{H}_M^{(\hat{L})} \right)^{\dagger} \mathbf{H}_M^{(\hat{L}+1)} \right) \tag{7}$$

where, $(\odot)$ is the Hadamard product and $(M+L+m) \times (M+L+m+1)$ matrix $\mathbf{F}$ is a Toeplitz matrix with first row equal to $[1\ \ 1\ \ 0 \cdots 0]$ and first column equal to $[1\ \ 0 \cdots 0]^T$. $(.)^{\dagger}$ indicates the Moore-Penrose pseudoinverse. With the matrix $\mathbf{F}$, only the matrices having Toeplitz structure with two coefficient (as in $\mathbf{H}_c^{(1)}$) go outside to the matrix operation without any distortion. It is shown that $\hat{\mathbf{A}}_m$ has Toeplitz structure only when $m=0$. This property of $\hat{\mathbf{A}}_m$ is used to define a new cost function to estimate the channel order. Using the estimated relation matrix, $\mathbf{H}_M^{(L+m+1)}$ is recalculated and difference between the estimated and calculated channel matrices are compared. The difference between them are taken as the new cost function and it is proven that it has a global minimum at the true channel order in Theorem-1.

Theorem-1 : It is assumed that a SIMO system is given as in Figure 1. For a range of channel order values, $\hat{L} = L+m = L_{min}, \cdots, L_{max}$, the channel coefficients are estimated by the LSS algorithm. Let the estimated channel matrix is given by $\mathbf{H}_M^{\hat{L}}$ for the channel order $\hat{L}$. M is chosen such that channel matrix is a tall matrix. The cost function is defined as,

$$err_{CMR}\left(\hat{L}\right) = \left\|\mathbf{H}_M^{(\hat{L}+1)} - \mathbf{H}_M^{(\hat{L})}\hat{\mathbf{A}}_m\right\|_2 / \left\|\mathbf{H}_M^{(\hat{L}+1)}\right\|_2 \quad (8)$$

$$\hat{\mathbf{A}}_m = \mathbf{F} \odot \mathbf{B}_m \quad (9)$$

$$\mathbf{B}_m = \left(\mathbf{H}_M^{(L+m)}\right)^{\dagger} \mathbf{H}_M^{(L+m+1)} \quad (10)$$

has global minimum at true channel order, $\hat{L} = L$, in the noise free case.

Proof : The proof is given at the Appendix.

In Figure 2, Theorem-1 is also verified for noisy observations through simulation. Channel output error is plotted against the channel order at different SNR values. 200 trials are used in the simulation. Channel order is five and the number of channels is three. Channel coefficients are complex values chosen randomly from a zero mean unit variance Gaussian set. As shown in Figure 2, channel output error has a global minimum at the true channel order, $L=5$.

4. SIMULATIONS

The proposed algorithm (CMR) is compared with different algorithms including MDL, AIC, Liavas , JLSS [6] and ID+EQ [4]. Input signal is chosen as a QPSK signal and input length is 100. Noise is a zero mean Gaussian signal uncorrelated with the input. Channel coefficients are randomly generated complex values. Minimum channel order and maximum channel order are selected as $L_{min} = 1$ and$L_{max} = L+5$ respectively. The block length is chosen as $M = \hat{L}$, which is the lower limit for the LSS algorithm. $\hat{L}$ indicates currently processed channel order in the CMR algorithm. The number of trials used in the simulations is 200.

Table 1 and Table 2 summarize the performance of CMR, MDL, AIC and Liavas algorithms for different number of channel and channel orders. The robustness of the proposed algorithm for different SIMO parameters can be seen easily. For the CMR algorithm, the estimation performance is improved as the number of channels increases and it decreases as the channel order is increased. Overall, CMR algorithm shows the best performance almost all of the cases considered in the tables. Liavas algorithm has the same characteristics as the CMR. In other words, its performance increases with the number of channels and it decreases as the channel order is increased. AIC and MDL show somehow mixed and opposite characteristic. While AIC performance has tendency to fall as the number of channels increases, MDL performance mostly improves except for $L=2$ and it is better in a small region where the number of channels is large.

Figure 3 shows the performance of different algorithms with respect to SNR, when the number of channels, P, is 3, the channel order, L, is 3, and the length of the signal is 100. The proposed algorithm performs much better than the others for all SNR values. When the channel order is increased to $L=5$, a similar characteristics is observed as shown in Figure 4. As the number of channels in the SIMO system is increased from three to five, as shown from the Figure 5, CMR performs much better than the other algorithms in all SNR values.

5. CONCLUSION

A new channel order estimation algorithm is proposed. CMR is based on the LSS algorithm and uses a cost function which has a global minimum at true channel order. It has finite convergence property, i.e., the channel order can be found exactly from the finite number of samples in the absence of noise. Its performance is compared with the algorithms currently available in the literature for different channel settings and it is shown that the performance of the proposed algorithm is significantly better and more robust to the changes in SIMO parameters.

6. APPENDIX

6.1 Proof of Theorem-1

For the proof of the theorem, correctly estimated channel order, overestimated channel order and under estimated channel order cases are studied separately to show that the cost function defined in (8) is zero only when the channel order is correctly estimated in noise free case.

Channel order is estimated correctly: Consider that the channel order is correctly estimated, i.e. $\hat{L} = L$. LSS algorithm finds the true channel coefficients in the noise free case, when the true channel order is given [5, 6]. Therefore, $\hat{\mathbf{H}}_M^{(L)} = \mathbf{H}_M^{(L)}$. We want to show that the cost function defined through equation (8) is equal to zero when $\hat{L} = L$, i.e. $err_{CMR}(L) = 0$.

According to Lemma-1, when the channel order is overestimated by one, one common zero is added to true channel transfer function. In this case, channel matrix $\mathbf{H}_M^{(L+1)}$ can be written as the product of the true channel matrix, $\mathbf{H}_M^{(L)}$, and the matrix, $\mathbf{H}_c^{(1)}$.

$$\mathbf{H}_M^{(L+1)} = \mathbf{H}_M^{(L)}\mathbf{H}_c^{(1)} \quad (11)$$

Since the channel coefficients are estimated exactly for the true channel order, channel matrix, $\mathbf{H}_M^L$, is full column rank under assumption that there is no common zeros between SIMO channels. Therefore,

$$\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L)} = \mathbf{I} \quad (12)$$

The matrix, $\hat{\mathbf{A}}$, can be written as follow.

$$\hat{\mathbf{A}} = \mathbf{F} \odot \left(\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L+1)}\right) \quad (13)$$

$$= \mathbf{F} \odot \left(\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L)}\mathbf{H}_c^{(1)}\right) \quad (14)$$

$$= \mathbf{F} \odot \left(\mathbf{H}_c^{(1)}\right) \quad (15)$$

$$= \mathbf{H}_c^{(1)} \quad (16)$$

Replacing, $\hat{\mathbf{A}}$, in equation (8), the cost function is obtained as follows,

$$\begin{aligned} err_{CMR}(L) &= \left\|\mathbf{H}_M^{(L+1)} - \mathbf{H}_M^{(L)}\mathbf{H}_c^{(1)}\right\|_2 / \left\|\mathbf{H}_M^{(L+1)}\right\|_2 && (17) \\ &= \left\|\mathbf{H}_M^{(L+1)} - \mathbf{H}_M^{(L+1)}\right\|_2 / \left\|\mathbf{H}_M^{(L+1)}\right\|_2 && (18) \\ &= 0 \end{aligned}$$

Channel order is overestimated: Consider that the channel order is overestimated, i.e., $\hat{L} = L + m$ and $m > 0$. As noted before, the function of $\mathbf{F}$ is to extract the Toeplitz form from the matrix $\mathbf{B}_m$. If $\mathbf{B}_m$ is Toeplitz matrix with first row and first column $[b_m(0) \ b_m(1) \ 0 \cdots 0]$ and $[b_m(0) \ 0 \cdots 0]^T$ respectively. Then $\hat{\mathbf{A}}_m = \mathbf{B}_m$. If this is the case, multiplying from left by $\mathbf{H}_M^{(L+m)}$, we obtain,

$$\begin{aligned} \mathbf{H}_M^{(L+m)}\hat{\mathbf{A}}_m &= \mathbf{H}_M^{(L+m)}\mathbf{B}_m \\ &= \mathbf{H}_M^{(L+m)}\left(\mathbf{H}_M^{(L+m)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} \\ &= \mathbf{H}_M^{(L)}\mathbf{H}_c^{(m)}\left(\mathbf{H}_M^{(L)}\mathbf{H}_c^{(m)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} \\ &= \mathbf{H}_M^{(L)}\mathbf{H}_c^{(m)}\left(\mathbf{H}_c^{(m)}\right)^{\dagger}\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} \\ &= \mathbf{H}_M^{(L)}\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} \\ &= \mathbf{H}_M^{(L)}\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L)}\mathbf{H}_c^{(m+1)} \\ &= \mathbf{H}_M^{(L)}\mathbf{H}_c^{(m+1)} \\ &= \mathbf{H}_M^{(L+m+1)} && (19) \end{aligned}$$

Note that Moore-Penrose pseudoinverse property is used for the fourth row equation. With the equality in (19), $err_{CMR}(\hat{L})$ in (8) becomes zero. $err_{CMR}(\hat{L})$ is different than zero as long as $\hat{\mathbf{A}}_m$ is not equal to $\mathbf{B}_m$ or $\mathbf{B}_m$ is not a Toeplitz matrix with two coefficients. In this proof, it is shown that $\mathbf{B}$ can not become a Toeplitz matrix for $m > 0$.

The proof will be by contradiction. First assume that $\mathbf{B}$ is Toeplitz matrix with the formation described in the previous paragraph. In (19) it is also shown that $\mathbf{H}_M^{(L+m+1)} = \mathbf{H}_M^{(L+m)}\mathbf{B}_m$. Rewriting 10 and replacing $\mathbf{H}_M^{(L+m+1)}$ with $\mathbf{H}_M^{(L+m)}\mathbf{B}_m$,

$$\begin{aligned} \mathbf{B}_m &= \left(\mathbf{H}_M^{(L+m)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} && (20) \\ &= \left(\mathbf{H}_M^{(L+m)}\right)^{\dagger}\mathbf{H}_M^{(L+m)}\mathbf{B}_m && (21) \\ &= \mathbf{P}\mathbf{B}_m && (22) \end{aligned}$$

where $\mathbf{P}_m = \left(\mathbf{H}_M^{(L+m)}\right)^{\dagger}\mathbf{H}_M^{(L+m)} \neq \mathbf{I}$, because $\mathbf{H}_M^{(L+m)}$ is not full column rank as a result of Lemma-1 and Lemma-2. Therefore if $\mathbf{B}_m$ is a Toeplitz matrix, $\mathbf{P}\mathbf{B}_m$ can not be a Toeplitz matrix with same formation for $m > 0$, i.e. $\mathbf{P}\mathbf{B}_m \neq \mathbf{B}_m$. Equality only holds when $\mathbf{P} = \mathbf{I}$, which corresponds to $m = 0$ case. Hence, it contradicts with the assumption about $\mathbf{B}_m$. Hence $err_{CMR}(L+m) > 0$ for $m > 0$.

Channel order is underestimated: When the channel order is underestimated, the channel coefficients are not correctly estimated [6] and this leads to nonzero cost function for underestimated channel orders.

REFERENCES

[1] J. Rissanen, "Modelling by shortest data description" *Automatica*, Vol.14, pp.465-471, 1978.

[2] H. Akaike, "A new look at the statistical model identification," *IEEE Trans. Automat. Contr.*, Vol. AC-19, Dec. 1974.

[3] A. P. Liavas, P. A. Regalia, "On the behaviour of information theoretic criteria for model order selection," *IEEE Trans. on Signal Proc.*, Vol. 49, No. 8, Aug. 2001.

[4] Javier Via, Ignacio Santamaria, Jesus Perez, "Effective channel order estimation based on combined identification/equalization", *IEEE Trans. on Signal Proc.*, Vol. 54, No. 9, Sept. 2006.

[5] G. Xu, H. Liu, L. Tong and T. Kailath, "A least squares approach to blind channel identification," *IEEE Trans. on Signal Proc.*, Vol. 43, Dec 1995.

[6] Lang. Tong and Quing Zhao, "Joint order detection and blind channel estimation by leat squares smoothing," *IEEE Trans. on Signal Proc.*, Vol. 47, Sept. 1999.

[7] Adi. Ben-Isreal and Thomas N.E. Greville, "Generalized inverses. Theory and applications," 2nd ed. New York, NY:Springer, 2003.

[8] Petre Stoica, Yngve Selen, "Model order selection: A review of information criterion rules.", *IEEE Signal Proc. Magazine,* July 2004.

[9] Houcem Gazzah, Phillip A. Regalia, Jean-Pierre Delmas, Kerim Abed-Meraim, "A blind multichannel identification algorithm robust to order overestimation," *IEEE Trans. on Signal Proc.*, Vol.50, No.6, June 2002.

[10] D. Slock, "Blind fractionally-spaced equalization, perfect reconstruction filterbanks, and multilineer prediction," *Proc. IEEE ICASSP Conf.*, Adalaide, Australia, Apr. 1994.

[11] K. Abed-Meraim, R. Moulines, and P. Loubaton, "Prediction error method for second order blind identification," *IEEE Trans. Signal Proc.*, Vol.45, pp.694 – 705, Mar. 1997.

[12] Z. Ding, "Matrix outer product decomposition method for blind multiple channel idetification," *IEEE Trans. Signal Proc.*, Vol.45, pp.3053 – 3061, Aug. 2000.

[13] R.L. Kashyap, "Inconsistency of the AIC rule for estimating the order of AR models," *IEEE Trans. Automat. Contr.*, Vol.25, no.5, pp.996 – 998, Oct. 1980.

[14] Q.-T. Zhang, K. M.Wong, P. C. Yip, and J. P. Reilly, "Statistical analysis of the performance of information theoretic criteria in the detection of the number of signals in array processing," *IEEE Trans. Acoust. Speech Signal Processing,* vol. 37, pp. 1557 – 1567, Oct. 1989.

Table 1: CMR and MDL performances (percentage of true channel order estimate)for different channel order and number of channel. SNR is $15dB$.

Channel order L	Number of channels, P											
	CMR						MDL					
	2	3	4	5	6	7	2	3	4	5	6	7
2	92.2	99.4	99.8	99.8	100	100	67.8	66.2	69	65.4	68.8	65.6
3	80.4	95.4	98.8	98.6	99.4	99.4	69.0	19.8	76.2	74.4	80.0	79.4
4	74.0	92.8	98.2	98.4	98.8	99.6	35.0	67.4	76	83.4	79.8	85.4
5	66.6	86.8	93.4	96.8	98.4	98.4	21.8	66.2	82.8	88.0	90.8	91.8
6	54.8	79.8	90.6	93.8	97.2	97.8	11.0	57.2	77.8	86.0	90.2	89.4
7	42.2	69.8	83.8	90.4	93.4	94.6	5.4	46.6	74.2	85.8	89.8	91.4
8	37.0	62.4	78.6	86.6	90.8	93.0	2.6	38.4	71.6	85.0	93.2	93.6
9	30.4	55.4	69.4	79.4	84.8	90.2	1.0	27.6	61.2	82.6	89.0	94.4
10	22.0	43.4	61.8	70.2	77.4	83.8	0	16.0	51.8	73.4	85.2	90.8

Table 2: AIC and Liavas performances (percentage of true channel order estimate)for different channel order and number of channel. SNR is $15dB$.

Channel order L	Number of channels, P											
	AIC						Liavas					
	2	3	4	5	6	7	2	3	4	5	6	7
2	52.6	23.4	14.0	7.8	6.0	4.8	18.6	44.2	62.4	82.2	90.0	92.2
3	44.2	29.2	17.4	10.0	11.2	10.0	7.4	30.4	45.0	63.8	73.6	80.6
4	44.0	35.6	19.4	16.8	11.8	8.0	3.2	17.2	33.8	44.2	58.8	68.8
5	36.4	34.2	23.0	17.4	13.6	10.2	1.6	9.6	25.4	37.6	46.8	60.0
6	32.2	40.2	27.6	21.4	14.2	12.0	0.2	6.8	14.4	23.6	32.0	41.4
7	22.6	40.0	30.2	22.2	16.0	12.2	0	2.0	6.6	14.2	19.2	26.0
8	24.8	39.0	31.6	22.2	20.2	15.2	0	0.6	2.6	8.6	14.0	16.6
9	16.4	42.6	33.0	24.6	19.0	13.4	0	0.2	2.6	5.4	8.0	12.6
10	0	38.6	34.0	23.0	20.4	17.0	0	0	0.8	1.6	5.0	6.8

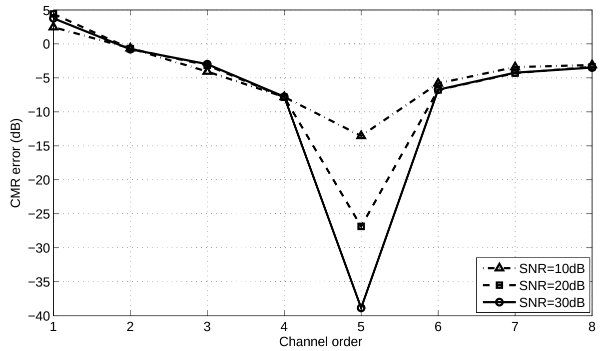

Figure 2: Channel order versus channel output estimation error. True channel order is 5.

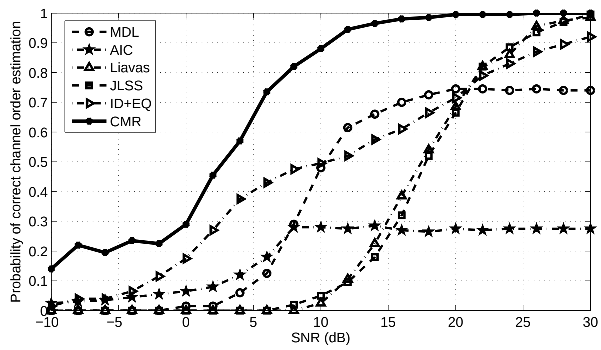

Figure 3: Channel order estimation performance comparison for $L = 3$, $P = 3$.

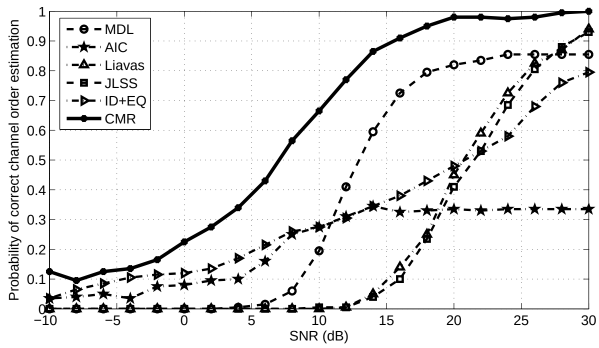

Figure 4: Channel order estimation performance comparison for $L = 5$, $P = 3$.

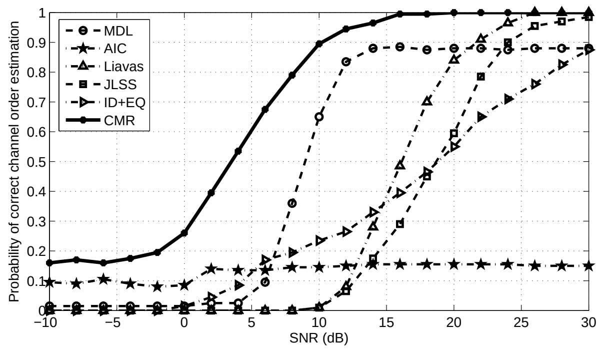

Figure 5: Channel order estimation performance comparison for $L = 5$, $P = 5$.