IN-NETWORK COOPERATIVE SPECTRUM SENSING

Mehmet E. Yildiz†, *Tuncer C. Aysal*† *and Kenneth E. Barner*‡

†School of Electrical and Computer Engineering, Cornell University, Ithaca, NY, 14850
‡Electrical and Computer Engineering Department, University of Delaware, Newark, DE, 19716

ABSTRACT

This paper proposes a distributed average consensus algorithm in order to solve the cooperative spectrum sensing task without a cognitive base station. The proposed consensus algorithm converges to the optimal decision statistic in the limit. Since in practice the iteration number has to be finite, we derive high probability bounds on the iteration number at which all the CRs are at most ϵ away from the optimal decision statistic. Moreover, we compute the performance characteristics of the proposed in-network cooperative spectrum sensing at a given iteration.

1. INTRODUCTION

Cognitive radio, a paradigm originated by Mitola [1], has emerged as a promising technology for maximizing the utilization of limited bandwidth while accommodating the increasing amount of services and applications in wireless networks. A cognitive radio (CR) transceiver is able to adapt to the dynamic radio environment and the network parameters to maximize the utilization of limited resources while providing flexibility in wireless access [2]. By detecting particular spectrum holes and exploiting them rapidly, CRs can significantly improve spectrum utilization. To guarantee high spectrum utilization efficiency, while avoiding interference to licensed users, CR must rapidly adapt to spectrum conditions.

Hence, important capabilities to be provided by a CR include spectrum sensing, dynamic frequency selection and transmit power control [3]. Co-existence of multiple cognitive networks is highly probable by virtue of the cognitive radio paradigm. However, the coexistence of multiple CRs generates mutual interference, leading to the hidden terminal problem. This problem typically occurs when a cognitive radio is shadowed, in severe fading or with high path loss, while a primary user (PU) is in vicinity [2, 3]. In order to address the hidden terminal problem, CRs can cooperate to sense and share the spectrum without introducing harmful interference to the PU. Thus, one of the most important and critical components of CR is spectrum sensing and accordingly, detection of PUs. However, the communication model adopted in the cooperative spectrum sensing (CSS) literature assumes noise-free communications between CRs and the cognitive base station (CBS) [2–5], which is clearly not the case in realistic CSS scenarios. Only very recently, this model has been extended to admit imperfect channels [6, 7].

In this paper, we propose a network in which CRs can detect the spectrum cooperatively without the need of a CBS. Assuming non-cooperative primary signaling, we adopt an energy-based detection scheme at the CRs. Moreover, we utilize distributed averaging algorithms to fuse the soft local CR energy decisions over the network. We derive high probability bounds on the iteration number at which all the CRs are ϵ close to the optimal decision statistic. Also presented, for a given iteration step, are the probability of detection, miss detection and false alarm of the proposed in-network cooperative spectrum sensing.

2. PROBLEM FORMULATION

This section summarizes the energy-based centralized cooperative spectrum sensing and presents the distributed average consensus algorithms that recently received a lot of attention. Finally, this section concludes with a brief discussion on the motivations of this paper.

2.1 Cooperative Spectrum Sensing

Formally, the extended CSS model considered in this paper is as follows. We assume that each CR performs local spectrum sensing independently. The CR local spectrum sensing is to decide between the following two hypotheses [2, 3, 5]:

$$H_0 : x_i(k) = w_i(k) \quad (1)$$
$$H_1 : x_i(k) = s(k) + w_i(k) \quad (2)$$

for $i = 1, 2, \ldots, N$, $k = 1, 2, \ldots, K$, (K being the PU complex signal length) where $x_i(k)$, $s(k)$, and $w_i(k)$ denote the complex observed signal at the i-th CR, the complex signal transmitted from the PU, and the additive complex Gaussian white noise ($w_i(k) \sim \mathcal{CG}(0, \sigma^2)$). Note that this model is especially valid when the CRs are close to each other, and their relative distances are smaller than their distances to the PU, so that they observe almost identical source signal. Moreover, the local spectrum sensing SNR is defined as [6]

$$\gamma = \frac{1}{\sigma^2}\sum_{k=1}^{K}|s(k)|^2 \triangleq \frac{E_S}{\sigma^2}. \quad (3)$$

We assume that the primary signaling is unknown and we adopt energy detection as the building block for the cooperative spectrum sensing scheme. Thus, each cognitive obtain their local decision statistic, denoted as q_i, as [2, 6],

$$q_i = \sum_{k=1}^{K}|x_i(k)|^2. \quad (4)$$

CRs transmit their local *soft decisions* to a cognitive base station (CBS) through control channels allowing the transmitted values to be seen error-free. The CBS then averages the local soft decisions to obtain the network-wide decision statistic and the decision on the spectrum occupancy is made as [2, 5, 6]

$$\overline{q} \triangleq \frac{1}{N}\sum_{i=1}^{N} q_i > (\leq)\tau \;\Rightarrow\; \text{Spectrum is (un)occupied} \tag{5}$$

where τ denotes the decision threshold providing trade-off between the probability of detection and false alarm performance of the CBS. The final decision made at the CBS is then transmitted back to the CRs through high SNR cognitive pilot channels.

2.2 Distributed Average Consensus

Consider a network where each CR measures a physical phenomenon that is represented by a real scalar value $q_i(0) \equiv q_i$ where i denotes the CR index. Distributed average consensus (DAC) is a decentralized method allowing all CRs to compute the average of the initial observations, *i.e.*, $\overline{q}(0) \triangleq 1/N \sum_{i=1}^{N} q_i(0)$, in an iterative fashion via only near neighbors' communications [8–12]. Different types of consensus protocols and their performance characteristics have been studied extensively in the literature. We consider in our analysis the *synchronous* linear consensus algorithms where every CR, simultaneously, updates its own state value by a weighted sum of its neighbors' values and its own value [8, 9]:

$$q_i(t+1) = W_{ii}q_i(t) + \sum_{j\in\mathcal{N}_i} W_{ij}q_j(t) \tag{6}$$

where W is the weight matrix with non-negative entries and $\mathcal{N}_i$ is the neighbor set of CR i, *i.e.*, node $j \in \mathcal{N}_i$ if j is within the transmission radius of node i. It assumed that the transmission is below the capacity within the neighborhood of each node allowing CRs to receive error-free state values.

In this case, the network-wide update is given by $q(t+1) = Wq(t)$, where $q(t) = [q_1(t)\dots q_N(t)]^T$. The state values converge to a consensus, namely the initial node measurements' average, regardless of the initial state values if [8]:

$$\mathbf{1}^T W = \mathbf{1}^T,\; W\mathbf{1} = \mathbf{1},\; \rho(W - \mathbf{1}\mathbf{1}^T/N) < 1 \tag{7}$$

where $\mathbf{1}$ is the all ones vector and $\rho(\cdot)$ denotes the spectral radius of its argument. It is also shown that if the graph is connected, then any doubly stochastic matrix W satisfy the spectral radius condition. Of note is that it is easy to see that average consensus protocols can be modified to compute any function $f(q_1, q_2, \dots, q_N)$ with a linear synopsis.

2.3 Spectrum Sensing without a CBS

CSS techniques vastly considered in the current literature rely on the presence of a CBS capable of performing the tasks required to fuse the received local CR decisions. The CBS based CSS has however direct implications to consider in practical applications; to name a few:

- High transmission power required at each CR to transmit its local information to the CBS
- Lack of robustness in case of CBS failures
- Limited bandwidth available for coordination
- CBS needs to broadcast the final decision to the co-operating CRs.

It is intuitive to consider a CSS scheme with a CBS from both theoretical and practical point of view as a first step to understand the issues underlying CSS, and subsequently CRNs. However, innovation of algorithms capable of sensing the spectrum occupancy without the need of a CBS is of significant interest for applications envisioned for future CRNs. Providing in-network algorithms capable of sensing the spectrum co-operatively will open new directions and possibilities to CRNs. Thus, it is important to create models and algorithms capable of cooperatively sensing the spectrum without the need of a CBS as distributed CSS will take the CR paradigm one step further.

In this work, we adopt the consensus algorithms to achieve the in-network spectrum sensing task. Unfortunately, consensus algorithms achieve the average of the initial measurements in the limit; a constraint that clearly cannot be satisfied in current cognitive networks. Thus, in this work we are interested in the trade-off between the iteration number and the system performance.

3. IN-NETWORK SPECTRUM SENSING

DAC guarantees us that every CR will, almost surely, have the desired decision statistic in the limit, *i.e.*,

$$\Pr\left\{\lim_{t\to\infty} q_i(t) = \overline{q}(0)\right\} = 1 \tag{8}$$

for $i = 1, 2, \dots, N$. Since DAC will not run infinite many iterations, CRs need to stop at a finite time step $t < \infty$. Clearly, when the iterations are stopped, we would like all the CRs to have the same decision regarding the spectrum occupancy. Thus, we would like to have $\max_i |q_i(t) - \overline{q}(0)|$ to be as small as possible in order to guarantee the mentioned constraint. This can be achieved by keeping the number of iterations large. On the other hand, we want the communication cost and the number of transmissions between the nodes to be low. This is achieved by keeping the number of iterations small. Thus, there is a clear trade-off between the system performance and communication cost.

3.1 Cognitive Network Convergence

Note that the local decision statistics $q_i(0)$ are, for large enough K (asymptotically but for $K \geq 10$ in practice [6]), modeled as Gaussian [6, 13]:

$$H_k : q_i(0) \sim \mathcal{G}\{\mu_k, \delta_k\} \tag{9}$$

for $i = 1, 2, \dots, N$ and $k \in \{0, 1\}$, where

$$\mu_0 = K\sigma^2,\; \delta_0 = 2K\sigma^4 \tag{10}$$

and

$$\mu_1 = (K+\gamma)\sigma^2,\; \delta_1 = 2(K+2\gamma)\sigma^4 \tag{11}$$

where μ_k and δ_k denotes the mean and the variance of the Gaussian distribution, respectively.

Recall that we need to ensure that $\max_i |q_i(t) - \overline{q}(0)|$ is small enough so that all the CR's make the same (better) decision with regard to the spectrum occupancy. In other words, we are interested in iteration numbers at which *all* the CRs decision statistic $q_i(t)$ is ϵ (in the absolute value) close to the optimal cooperative decision statistic $\overline{q}(0)$. The following Proposition gives a bound on such iteration number.

Proposition 1 *For any* $\epsilon > 0$ *and* $t \geq T(\epsilon)$*, we have that*

$$\Pr\{\max_{1 \leq i \leq N} |q_i(t) - \overline{q}(0)| \geq \epsilon | H_k\} \leq \epsilon \tag{12}$$

where $k \in \{0, 1\}$ *and*

$$T(\epsilon) = \frac{3/2 \log \epsilon^{-1} + 1/2 \log(\mathbb{E}\{||q(0)||_2^2 | H_k\})}{1 - \rho(W - J)}. \tag{13}$$

and $J \triangleq 1/N \mathbf{1}\mathbf{1}^T$*.*

Proof 1 *We start our proof by noting that* $\max_{1 \leq i \leq N} |q_i(t) - \overline{q}(0)| = ||q(t) - \overline{q}(0)\mathbf{1}||_\infty$*. Thus, we have*

$$\begin{aligned} &\Pr\{||q(t) - \overline{q}(0)\mathbf{1}||_\infty \geq \epsilon | H_k\} \\ &\qquad = \Pr\{||q(t) - \overline{q}(0)\mathbf{1}||_\infty^2 \geq \epsilon^2 | H_k\} \quad (14) \\ &\qquad \leq \frac{\mathbb{E}\{||q(t) - \overline{q}(0)\mathbf{1}||_\infty^2 | H_k\}}{\epsilon^2} \quad (15) \end{aligned}$$

where the second line follows from the Markov inequality. Now note that $||q(t) - \overline{q}(0)\mathbf{1}||_\infty^2 \leq ||q(t) - \overline{q}(0)\mathbf{1}||_2^2$*. Moreover, we have almost surely that [8]*

$$||q(t) - \overline{q}(0)\mathbf{1}||_2^2 \leq \rho^{2t}(W - J)||q(0)||_2^2 \tag{16}$$

where in the above $J = 1/N\mathbf{1}\mathbf{1}^T$*. Furthermore, this implies that* $\mathbb{E}\{||q(t) - \overline{q}(0)\mathbf{1}||_\infty^2 | H_k\} \leq \rho(W - J)^{2t}\mathbb{E}\{||q(0)||_2^2 | H_k\}$ *yielding*

$$\begin{aligned} &\Pr\{||q(t) - \overline{q}(0)\mathbf{1}||_\infty \geq \epsilon | H_k\} \\ &\qquad \leq \frac{\rho^{2t}(W - J)\mathbb{E}\{||q(0)||_2^2 | H_k\}}{\epsilon^2}. \quad (17) \end{aligned}$$

Since $\rho(W - J)^{2t}\mathbb{E}\{||q(0)||_2^2 | H_k\} = \epsilon^3$*, we have*

$$t = \frac{3/2 \log \epsilon^{-1} + 1/2 \log(\mathbb{E}\{||q(0)||_2^2 | H_k\})}{\log(\rho(W - J))^{-1}}. \tag{18}$$

Finally nothing that $\log(1 + u) \leq u$ *(a bound which is tight for small* u*), we obtain the claimed result.*

Proposition 1 gives the iteration number $T(\epsilon)$ guaranteeing us to have $\max_i |q_i(t) - \overline{q}(0)|$ arbitrarily small with very high probability. Namely, with probability at least $1 - \epsilon$. However, as expected $T(\epsilon)$ depends on the initial conditions which also depend on the spectrum occupancy. Since we do not have access to this information, in the following we provide an upper-bound.

Lemma 1 *The following statement holds:*

$$\mathbb{E}\{||q(0)||_2^2 | H_0\} < \mathbb{E}\{||q(0)||_2^2 | H_1\} \tag{19}$$

where

$$\mathbb{E}\{||q(0)||_2^2 | H_0\} = N[2K\sigma^4 + (K\sigma^2)^2] \tag{20}$$

and

$$\mathbb{E}\{||q(0)||_2^2 | H_1\} = N\sigma^4[2K + 4\gamma + (K + \gamma)^2]. \tag{21}$$

Proof 2 *Recall that for a random variable* u*, we have*

$$\mathbb{E}\{u^2\} = \mathrm{Var}\{u\} + (\mathbb{E}\{u\})^2. \tag{22}$$

We also have that $\mathrm{Var}\{q_i(0)|H_k\} = \mathrm{Var}\{q_j(0)|H_k\}$ *and* $\mathbb{E}\{q_i(0)|H_k\} = \mathbb{E}\{q_j(0)|H_k\}$ *for all* i, j *and* $k \in \{0, 1\}$ *indicating that* $\mathbb{E}\{||q(0)||_2^2 | H_k\} = N[\mathbb{E}\{q_i^2(0)|H_k\}]$ *for* $k \in \{0, 1\}$*. Moreover, observe the following set of (in)equalities:*

$$\begin{aligned} \mathbb{E}\{||q(0)||_2^2 | H_0\} &= N[\delta_0 + (\mu_0)^2] && (23) \\ &= N[2K\sigma^4 + (K\sigma^2)^2] && (24) \\ &< N[2K\sigma^4 + 4\gamma\sigma^4 + [(K + \gamma)\sigma^2]^2] && (25) \\ &= N[\delta_1 + (\mu_1)^2] && (26) \\ &= \mathbb{E}\{||q(0)||_2^2 | H_1\} && (27) \end{aligned}$$

where the inequality holds since $\gamma_S > 0$*, for* $\{E_S, \sigma\} > 0$ *and the result of the Lemma follows.*

Now, given the result of the Lemma 1 and noting that $\log(\cdot)$ is a monotonic function implying that $u_1 \leq u_2 \Rightarrow \log(u_1) \leq \log(u_2)$, we have

$$T(\epsilon) \leq \frac{3/2 \log \epsilon^{-1} + 1/2 \log(N\sigma^4[2K + 4\gamma + (K + \gamma)^2])}{1 - \rho(W - J)} \tag{28}$$

which depends on the desired accuracy ϵ, the signal length K, the number of CRs N, the noise variance σ^2, the signal energy E_S and the network connectivity $\rho(W - J)$. Thus, for given these set of parameters, we can guarantee that the probability that *all* the CRs decision statistics are at most ϵ away (in absolute value) from the optimal decision statistic, is at least $1 - \epsilon$.

Of note is that, the primary signaling energy has little (logarithmic) effect where the network connectivity has a great impact on how fast the CRs converge to the optimal cooperative decision. This is due to the fact that

- Network connectivity directly influences (inversely proportional to $(1 - \rho(W - J))$) the speed of information fusion over the network where increasing network connectivity allows CRs to reach more CRs at each iteration.
- Increasing primary signaling energy changes the initial conditions of the system and increases the second moment of the soft decisions that are fed to the in-network iterative algorithm. Initial conditions has little impact (logarithmic) on the convergence speed.

3.2 Performance Characterization

Next we analyze the decision of the CRs at time step t. Note that the CRs makes decision by simply comparing their current state value to a decision threshold τ, *i.e.*,

$$q_i(t) > (\leq)\tau \Rightarrow \text{Spectrum is (un)occupied.} \quad (29)$$

For a given time step t, the probability of detection, the probability of miss detection and the probability of false alarm of the i-th CR are, respectively, given by,

- $P_D(i;t) \triangleq \Pr\{q_i(t) > \tau | H_1\}$
- $P_M(i;t) \triangleq \Pr\{q_i(t) \leq \tau | H_1\}$
- $P_F(i;t) \triangleq \Pr\{q_i(t) > \tau | H_0\}$

for all $i = 1, 2, \ldots, N$ and $t \geq 0$.

Recall that the DAC iterations yield $q(t) = W^t q(0)$ indicating that

$$q_i(t) = [W^t]_i q(0) \Rightarrow q_i(t) = \sum_{j=1}^{N} [W^t]_{ij} q_j(0) \quad (30)$$

where $[W^t]_i$ denotes the i-th row of the matrix W^t and $[W^t]_{ij}$ denotes the elements of the matrix W^t. Note that $q_j(0)$'s are independent Gaussian random variables for a given hypothesis as defined in (9). Moreover, $W^t\mathbf{1} = \mathbf{1}$ for all $t \geq 0$ since $W\mathbf{1} = \mathbf{1}$. Thus, $q_i(t)$, which is a (normalized) weighted average of Gaussian distributed random variables, is also Gaussian distributed, *i.e.*,

$$H_k : q_i(t) \sim \mathcal{G}\{\mu_k, \delta_k ||[W^t]_i||_2^2\} \quad (31)$$

where $||\cdot||_2$, as usual, denotes the ℓ_2 norm. The probability of detection at the i-th node and time step t is given by

$$P_D(i;t) = \overline{F}\left(\tau; \mu_1, \delta_1 ||[W^t]_i||_2^2\right) \quad (32)$$

where $\overline{F} = 1 - F$ and $F(u; x, y)$ denotes the cumulative distribution function (CDF) of a Gaussian random variable with mean x and variance y. Since $P_M(i;t) = 1 - P_D(i;t)$, we have

$$P_M(i;t) = F\left(\tau; \mu_1, \delta_1 ||[W^t]_i||_2^2\right). \quad (33)$$

Finally, using similar steps, we have shown that

$$P_F(i;t) = \overline{F}\left(\tau; \mu_0, \delta_0 ||[W^t]_i||_2^2\right). \quad (34)$$

Thus, for a given $T(\epsilon)$, we can exactly calculate the per-node performance of the system by characterizing the probabilities of interest.

The following discusses the limiting behavior of the above computed probabilities.

Corollary 1 *As the iteration number tends to infinity, the performance converges to* $\lim_{t\to\infty} P_D(i;t) = \overline{F}(\tau; \mu_1, \delta_1/N)$, $\lim_{t\to\infty} P_M(i;t) = F(\tau; \mu_1, \delta_1/N)$ *and* $\lim_{t\to\infty} P_F(i;t) = \overline{F}(\tau; \mu_0, \delta_0/N)$ *for all* $i = 1, 2, \ldots, N$ *and where* N *is the number of CRs in the network.*

Proof 3 *The result follows by using the facts that* $F(\cdot)$ *and* ℓ_2 *norm are continuous functions,* i.e.,

$$\lim_{t\to\infty} P_D(i;t) = \overline{F}\left(\tau; \mu_1, \delta_1 \lim_{t\to\infty} ||[W^t]_i||_2^2\right) \quad (35)$$

and $\lim_{t\to\infty}[W^t]_i = 1/N\mathbf{1}^T$ *for all* $i = 1, 2, \ldots, N$ *since* $\lim_{t\to\infty} W^t = 1/N\mathbf{1}\mathbf{1}^T$ *[8]. Similar steps are applied to* $P_M(i;t)$ *and* $P_F(i;t)$ *in order to obtain the results shown in the above corollary.*

As expected, the above results, *i.e.*, the limiting behavior of the in-network cooperative spectrum sensing, corresponds to the performance of a centralized system with a CBS. Before we further proceed, an interesting Lemma is in order. Of note is that consensus system improves the quality of the state value estimates by decreasing their variances through iterations and effects the probabilities through the variance parameter in the CDFs.

Lemma 2 $P_D(i;t+1) > P_D(i;t)$ *and* $P_F(i;t+1) < P_F(i;t)$ $\forall i$ *and* $t \geq 0$ *if* $\tau \in (\mu_0, \mu_1)$.

Proof 4 *Note that for fixed* u *and* y *such that* $u > y$, *we have* $F(u; y, z_1) > F(u; y, z_2)$ *for* $z_1 < z_2$. *Moreover,* $P_D(i;t)$ *is in the form of* $1 - F$, *and* $||[W^t]_i||_2^2$ *is strictly decreasing for all* i *through iterations. Therefore, we have,*

$$P_D(i;t+1) > (<)\{=\}P_D(i;t) \text{ if } \tau < (>)\{=\}\mu_1 \quad (36)$$

Similarly,

$$P_F(i;t+1) > (<)\{=\}P_F(i;t) \text{ if } \tau < (>)\{=\}\mu_0 \quad (37)$$

Noting that we want $P_D(i;t)$ *to increase whereas* $P_F(i;t)$ *to decrease through iterations concludes the proof.*

According to the Lemma 2, in order to guarantee performance improvements through iterations both in terms of detection (which occurs when $\tau < \mu_1$) and false alarm (which occurs when $\tau > \mu_0$), the decision threshold should satisfy $\tau \in (\mu_0, \mu_1)$.

Consider a random graph on $[0,1] \times [0,1]$ where the CRs are uniformly placed in the unit square. The connectivity radius is 0.4. Moreover, we form the weight matrix of the in-network consensus system as $W = I - \nu L$ where I, L and $\nu = 2/(\lambda_1(L) + \lambda_{N-1}(L))$ denote the identity matrix, graph laplacian and the constant edge weight, respectively [8]. Of note is that the particular values of the entries of W is not directly important as the proposed system does not depend on particular W (although the proposed system depends on the spectral gap of W which varies depending on the algorithm utilized to design the weights). Figure 1 depicts the probability of detection and false alarm of the CRs through iterations. Note that they all converge to the same probabilities directed by the optimal decision statistics. For reference, we have also denote the universal upper bound on $T(\epsilon)$ with a vertical dashed line. The derived bound on the iteration number clearly ensures the convergence of the CRs to the optimal decision statistic.

3.3 Universal Bounds on the Performance

The following Lemma, that will be useful in the subsequent of this section, gives a universal bound on the rows of the matrix W^t.

Lemma 3 *The following statement holds:*

$$||[W^t]_i||_2^2 \leq N^{-1} + \rho^{2t}(W - J) \quad (38)$$

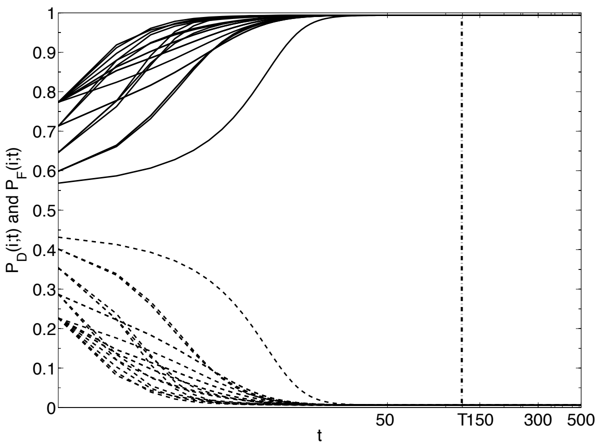

Figure 1: The behavior of the $P_D(i;t)$ (solid) and $P_F(i;t)$ (dashed) through iterations where the network connectivity $\rho(W-J) = 0.91$, $\tau = 25$, $N = 20$, $\epsilon = 10^{-2}$, $K = E_S = 20$ and $\sigma^2 = 1$. For given set of parameters, $T(\epsilon) \leq 134 \triangleq T$.

for all $i = 1, 2, \ldots, N$.

Proof 5 *Observe the following set of (in)equalities:*

$$||[W^t]_i||_2^2 = ||[W^t]_i - N^{-1}\mathbf{1}^T + N^{-1}\mathbf{1}^T||_2^2 \quad (39)$$

$$=^{(a)} ||[W^t]_i - N^{-1}\mathbf{1}^T||_2^2 + N^{-1} \quad (40)$$

$$\leq^{(b)} ||W^t - J||_2^2 + N^{-1} \quad (41)$$

$$\leq^{(c)} \rho^{2t}(W-J) + N^{-1} \quad (42)$$

where (a) *follows since* $\left([W^t]_i - N^{-1}\mathbf{1}^T\right) N^{-1}\mathbf{1} = 0$ *as* $W\mathbf{1} = \mathbf{1} \Rightarrow W^t\mathbf{1} = \mathbf{1}$ *and since* $||1/N\mathbf{1}^T||_2^2 = 1/N$, (b) *follows from the fact that the norm of any row of* $W - J$ *is smaller than the norm of the matrix* $W - J$ *and* (c) *is due to the fact that* $W^t - J = (W-J)^t$ *and* $||W - J|| = \rho(W-J)$*, and norm inequality.*

In the limit, the upper bound becomes an equality since $\lim_{t\to\infty} ||[W^t]_i||_2^2 = 1/N$ for all $i = 1, 2, \ldots, N$. Moreover, given the above Lemma and considering the region of interest for the decision threshold, *i.e.*, $\tau \in (\mu_0, \mu_1)$, we have the following universal bounds:

$$P_D(i;t) \geq \overline{F}\left(\tau; \mu_1, \delta_1\left(N^{-1} + \rho^{2t}(W-J)\right)\right) \quad (43)$$

and

$$P_M(i;t) \leq F\left(\tau; \mu_1, \delta_1\left(N^{-1} + \rho^{2t}(W-J)\right)\right). \quad (44)$$

Finally, using similar steps, we have that

$$P_F(i;t) \leq \overline{F}\left(\tau; \mu_0, \delta_0\left(N^{-1} + \rho^{2t}(W-J)\right)\right). \quad (45)$$

The above bounds relates the system performance to the spectral radius of the weight matrix and provides bounds valid for all CRs in the network.

4. CONCLUDING REMARKS

We proposed a distributed average consensus system that is capable of solving the cooperative spectrum sensing task without the presence of a base station. We provided bounds on the iteration number allowing all the CRs to be at most ϵ away from the optimal decision statistic. Moreover, we have characterized the performance of the algorithm through iterations and shown the reliability of the bound.

REFERENCES

[1] J. Mitola III, "Cognitive radio for flexible mobile multimedia communications," *Mobile Networks and Applications*, vol. 6, no. 5, September 2001.

[2] E. Hossain and V. K. Bhargava, *Cognitive Wireless Communication Networks*. New York, NY: Springer Science Business Media, LLC, 2007.

[3] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, February 2005.

[4] A. Ghasemi and E. S. Sousa, "Coolaborative spectrum sensing for opportunistic access in fading environments," in *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks*, Baltimore, MD, November 2005.

[5] S. M. Mishra, A. Sahai, and R. Brodersen, "Cooperative sensing among coginitive radios," in *IEEE International Conference on Communications*, Istanbul, Turkey, June 2006.

[6] Z. Quan, S. Cui, and A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE Journal on Selected Areas in Signal Processing*, vol. 2, no. 1, February 2008.

[7] T. C. Aysal and K. E. Barner, "Blind decentralized estimation for bandwidth constrained wireless sensor networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 5, May 2008.

[8] L. Xiao and S. Boyd, "Fast linear iterations for distributed averaging," *Systems and Control Letters*, vol. 53, pp. 65–78, 2004.

[9] J. Tsitsiklis, "Problems in decentralized decision making and computation," Ph.D. dissertation, Dept. of Electrical Engineering and Computer Science, M.I.T., Boston, MA, 1984.

[10] T. C. Aysal, M. J. Coates, and M. G. Rabbat, "Distributed average consensus using probabilistic quantization," in *Proc. IEEE Statistical Signal Processing Workshop*, Madison, WI, Aug. 2007.

[11] ——, "Distributed average consensus using dithered quantization," *IEEE Trans. on Signal Processing*, Oct. 2008.

[12] G. Scutari, S. Barbarossa, and L. Pescosolido, "Distributed decision through self-synchronizing sensor networks in the presence of propagation delays and asymmetric channels," *IEEE Transactions on Signal Processing*, Apr. 2008.

[13] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. Mc–Graw Hill, 1984.