# TIME-FREQUENCY MULTIPLEXING FOR TIME-ENCODED SIGNALS FROM BRAIN-COMPUTER INTERFACES

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## ABSTRACT

Brain-computer interfaces (BCI) provide a way to monitor and treat neurological diseases. These interfaces contain an array of sensors that gather and transmit data. Given low-power and no-clock limitations, asynchronous sigma delta modulators (ASDMs) are considered as an alternative to synchronous analog to digital conversion. ASDMs are non-linear feedback systems that enable time-encoding of analog signals. In this paper we provide an efficient reconstruction of time-encoded band-limited signals using a prolate spheroidal waveform (PSW) projection. Furthermore, we show how a modified orthogonal frequency division multiplexing (OFDM) technique using chirp modulation can be used to transmit an array of time-encoded signals emanating from the BCI. Our method generalizes the chirp modulation of binary streams with non-uniform symbol duration.

## 1. INTRODUCTION

Gathering and transmission of data from the brain, for monitoring or treatment, can be done using an array of sensors supported by analog circuitry. Two issues of special interest in the design and implementation of these brain-computer interfaces (BCI) [1] are energy management and use of clocks. The power dissipation due to analog to digital conversion and to wireless transmission is significant. Furthermore, the presence of clocks in BCIs is problematic. In conventional sigma delta modulators, for instance, the required high frequency clocks may cause electromagnetic interference corrupting the analog signal to be sampled [2]. Given the lack of clocks and the low power consumption required in biomonitoring systems, asynchronous data acquisition is a viable alternative to analog to digital conversion [1, 2].

Asynchronous sigma delta modulators (ASDMs) [3] are non-linear feedback systems, without a clock, that transform amplitude information into time information to represent analog signals in a discrete form. Their simple circuitry allows them to operate at low power levels. A band-limited signal can be reconstructed from the zero crossings of the ASDM binary signal [2], just like the amplitude information in a sampled signal allows to recover the analog signal in Shannon's sampling theory. In this paper, we present a reconstruction of the signal by means of the prolate spheroidal waveform (PSW) projection presented in [4]. This projection is based in the approximation of the sinc function in terms of the PSWs giving a lower order representation than the complex exponential-bases used in [2].

As the brain implant collects data from several ASDMs, it is necessary to multiplex these data for transmission. The power consumption in the transmission can be reduced by using the skin as a short-range communication channel [5]. However, the non-uniformity of the zero-crossings of the time-encoded signals makes otherwise very efficient methods such as Orthogonal Frequency Division Multiplexing (OFDM) not applicable. We propose a combination of chirp and localized modulation of the ASDM time-encoded signals to achieve an efficient transmission with a modified OFDM system. OFDM is a multi-carrier communication technique that divides the bit stream into sub-streams that are more efficiently transmitted. Given that the communication channel is modeled as a linear time-varying system, chirp modulation and time-frequency processing of the signals in such a system is more appropriate than the conventional linear time-invariant modeling and Fourier domain processing [6].

A sequence of ortho-normal chirps can be used to transmit multichannel data in an efficient way and with robustness to additive noise. In [6, 7] it is shown that the transmission of a sequence of binary symbols $\{b_u(t)\}$, $u = 1, \cdots, U$, with uniform duration of $T$ seconds and corresponding to $U$ users, can be efficiently done by modulating each of the binary signals with a set of orthonormal chirps. The orthonormality of these chirps can be obtained using the kernel of the fractional Fourier transform (FrFT) [8]. If the symbol duration is not constant, the ortho-normality of the chirps is not sufficient to recover the transmitted signal from a multiplexed version of it. As we will show it is necessary to create a localized set of chirps capable of representing each of the non-uniform pulses.

## 2. ASYNCHRONOUS DATA COLLECTION AND WIRELESS TRANSMISSION FOR BCI

In this section, we will show how the data collection in the BCI can be accomplished without a clock using ASDMs, and how the data from a set of ASDMs can be multiplexed and transmitted using chirp modulation.

### 2.1 Asynchronous Sigma Delta Modulators

An ASDM is a nonlinear feedback system that operates at low power. It can be used to encode a band-limited analog signal into a continuous-time signal with discrete amplitudes. The zero-crossing times of this signal permit recovery of the original band-limited signal.

The structure of an ASDM is similar to that of the better-known synchronous sigma-delta modulator but it differs in that no sampling is done in the ASDM and as such no quantization noise is input into the modulator. Recently, the ASDM shown in Fig. 1, consisting of an integrator and a non-inverting Schmitt trigger, has been proposed for bio-monitoring [2]. This type of ASDM transforms amplitude information into time information by the limit cycles of the non-linear component.

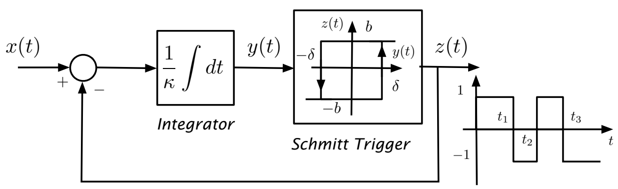


Figure 1: Example of ASDM.

The operation of the ASDM in Fig. 1 can be related to the non-uniform sampling of a band-limited signal $x(t)$. Different from the uniform sampling, to reconstruct $x(t)$ from non-uniform samples requires knowledge not only of the samples of the signal but also of the times at which they occur. Although reconstruction from non-uniform samples can be posed as a generalization of the sinc interpolation of the Nyquist-Shannon sampling theorem, the problem is not well defined due to the infinite dimension of the matrices and vectors involved, and to the ill-conditioning of the matrix with sinc entries.

Perfect reconstruction of $x(t)$ from non-uniform samples can be achieved provided that the time sequence $\{t_k\}$ at which the samples occur satisfies the condition [2]:

$$\max_k (t_{k+1} - t_k) \leq T_N \tag{1}$$

where $T_N = \pi/\Omega_{max}$ is the Nyquist sampling period. In [2] it has been shown that the input signal $x(t)$ of the ASDM can be reconstructed from the zero-crossings of the binary output signal $z(t)$. Indeed, for a bounded signal $x(t)$

$$|x(t)| \leq c < b \tag{2}$$

for a certain value of $\kappa$ the output of the integrator, $y(t)$, is also bounded, i.e., $|y(t)| < \delta$ for all $t$, and the output of the feedback system is binary,

$$z(t) = b(-1)^{k+1} \qquad t_k \leq t \leq t_{k+1}.$$

If at a time $t_{k+1} > t_k$ the output of the integrator is $y(t_{k+1}) = y(t_k) + 2\delta$ and $z(t_k) = b(-1)^{k+1}$, then we have

$$y(t_{k+1}) - y(t_k) = \frac{1}{\kappa}\left[\int_{t_k}^{t_{k+1}} x(\tau)d\tau - b(-1)^{k+1}(t_{k+1} - t_k)\right]$$

After replacing the right hand-side term by $2\delta$, it becomes

$$\int_{t_k}^{t_{k+1}} x(\tau)d\tau = (-1)^k\left[-b(t_{k+1} - t_k) + 2\kappa\delta\right] \tag{3}$$

Furthermore, from $|x(t)| \leq c$ and condition (1) we have

$$\frac{2\kappa\delta}{b+c} \leq t_{k+1} - t_k \leq \frac{2\kappa\delta}{b+c} \leq T_N \tag{4}$$

which gives us the way to choose the parameters $\delta$, and $\kappa$ in terms of the Nyquist sampling rate. The train of rectangular pulses $z(t)$ displays non-uniform transition times depending on the input signal amplitude. Indeed, approximating the integral by the trapezoidal rule we have that if $\Delta = (t_{k+1} - t_k)/D$ for an integer $D > 1$ (the larger this value the better the approximation), we have that

$$\int_{t_k}^{t_{k+1}} x(\tau)d\tau \approx \Delta\left[\frac{x(t_k)}{2} + \sum_{\ell=1}^{D-1} x(t_k + \ell\Delta) + \frac{x(t_{k+1})}{2}\right]$$

Representing the band-limited signal by its PSW projection [4]

$$x(t_\ell) = \sum_{k=0}^{L-1} \gamma_k s_k(t_\ell) \tag{5}$$

where $s_k(t)$ are the prolate spheroidal functions and $L$ is the order of the projection chosen according to the maximum frequency $\Omega_{max}$ of the band-limited signal $x(t)$. Using the trapezoidal approximation of the integral we obtain the following reconstruction algorithm:

$$\begin{aligned} &(i) && \mathbf{v} = \mathbf{Q}\mathbf{x} = \mathbf{Q}\mathbf{P}\gamma \\ &(ii) && \gamma = [\mathbf{Q}\mathbf{P}]^{\dagger}\mathbf{v} \\ &(iii) && \mathbf{x} = \mathbf{P}\gamma \end{aligned}$$

where $\mathbf{v}$ is the right term in (3), $\mathbf{Q}$ is the matrix for the trapezoidal approximation, $\mathbf{x} = \mathbf{P}\gamma$ is the PSW projection, and $\dagger$ indicates pseudo-inverse. Thus the signal $x(t)$ can be reconstructed from the zero crossings $\{t_k\}$ of the output of the ASDM $z(t)$.

### 2.2 Chirp Modulation for ASDM Signals

In the rest of the paper we consider the transmission of binary signals $\{z_n(t)\}$, $n = 1, \cdots, N$ from an array of $N$ ASDM's conforming a BCI. These signals need to be transmitted in the most efficient way from the BCI to an intermediate personal digital assistant (PDA) capable of transmitting the signal to a server where the signal analysis is performed. Each of the signals to transmit is a train of pulses with non-uniform zero-crossings. We explore the application of OFDM using orthonormal chirp basis for the modulation of the $N$ time-encoded signals.

### 2.3 Uniform symbol period

Chirp modulation has been applied successfully in OFDM [6, 7], a multi-carrier technique that transmits data by dividing the bit stream into several parallel streams. This chirp modulation has been shown to mitigate the effects of the channel Doppler frequency shifts (due to a moving receiver or transmitter) and to be robust to the presence of noise in the transmitted signal.

In the transmission of source symbols $+1$ or $-1$ with a uniform period $T$, if we have ortho-normal chirps $c_k(t)$ for users $k = 1, \cdots, U$ the baseband transmitted signal for user $k$ is given by

$$s_k(t) = b_k(t)c_k(t) \tag{6}$$

where $b_k(t)$ is either 1 or $-1$ for $t_0 \leq t \leq t_0 + T$. Assuming perfect synchronization between transmitter and receiver, and that the only channel effect is addition of Gaussian noise $\eta(t)$, the baseband received signal is

$$r(t) = \sum_{k=1}^{U} s_k(t) + \eta(t) \tag{7}$$

To recover the source symbols, multiplying the received signals by the conjugate of the chirps, $c_k^*(t)$, we obtain a decision variable for user $k$, $y_k$, by integrating over a period and using the orthogonality of the chirp signals:

$$\begin{aligned} y_k &= \int_{t_0}^{t_0+T} r(t)c_k^*(t)dt \\ &= \sum_{n=1}^{U} b_n(t) \int_{t_0}^{t_0+T} c_n(t)c_k^*(t)dt + \int_{t_0}^{t_0+T} \eta(t)c_k^*(t)dt \\ &= b_k(t) + \int_{t_0}^{t_0+T} \eta(t)c_k^*(t)dt \qquad t_0 \leq t \leq t_0 + T \end{aligned} \tag{8}$$

The value $b_k(t)$, which is either 1 or $-1$, is estimated by a thresholder. The ortho-normality of the chirps mitigates the multiple-access interference caused by users different from the user we are interested in.

Consider a set of frequency-modulated linear chirps $\{c_k(t)\}$ with instantaneous frequencies

$$\phi_k(t) = \theta t + 2f_k \qquad k = 1, \cdots, U \tag{9}$$

where $\theta$ is the chirp rate, common for all the chirps, and $f_k = k/T$ is a multiple of the frequency corresponding to the symbol period $T$. The chirps are given by

$$c_k(t) = e^{j\pi t\phi_k(t)} = e^{j\pi\theta t^2} e^{j2\pi f_k t}$$

The orthonormality of the chirps $\{c_k(t)\}$ depends on the orthonormality of the $\{e^{j2\pi f_k t}\}$ terms. Indeed, the common chirp rate makes it so that

$$\begin{aligned} \frac{1}{T}\int_{t_0}^{t_0+T} c_k(t)c_n^*(t)dt &= \frac{1}{T}\int_{t_0}^{t_0+T} e^{j2\pi(f_k-f_n)t}dt \\ &= \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases} \end{aligned} \tag{10}$$

In [6, 7] the orthonormal chirps are obtained from the properties of the kernel of the fractional Fourier transform, but such relation is unnecessary as shown above.

### 2.4 Non-uniform symbol period

Applying the chirp-modulated OFDM for the transmission of the time-encoded signals obtained from $N$ ASDM's is complicated by the fact that the pulses, corresponding to the symbols, do not have a uniform period as before. Indeed, the duty-cycle modulation that is being used to get $z(t)$ from $x(t)$ gives that the pulse width, $\alpha_k(t)$, and the pulse period, $\tau_k(t)$, of two consecutive pulses give a duty-cycle

$$\frac{\alpha_k(t)}{\tau_k(t)} = \frac{1 + x_k(t)}{2}$$

for $x_k(t)$ in $[t_k, t_{k+2}]$. Thus only when $x(t) = 0$ we would have uniform pulse periods.

In this case we will again consider chirps with a common chirp rate $\theta$, but with frequencies $f_n = 1/\hat{T}$ where

$$\hat{T} = \min\{T_n(k)\}$$

and $T_n(k) = t_n(k+1) - t_n(k)$ are the time intervals from the signals $\{z_n(t),\, n = 1, \cdots, N\}$. The bandwidth allocated to the $n^{th}$-ASDM, $F_n = f_{n+1} - f_n$, is divided into $M$ sub-bands with frequencies

$$f_n(m) = f_n + \frac{F_n}{M} m \qquad m = 0, \cdots, M-1 \tag{11}$$

Using these frequencies and the zero crossings $\{t_n(k)\}$ from $z_n(t)$ we create an array of chirps with instantaneous frequencies

$$\phi_{n,m}(t) = \theta t + 2f_n(m) \tag{12}$$

when $t \in [t_n(m), t_n(m+1)]$ and $-\infty$ otherwise (so that the chirp is zero outside $[t_n(k), t_n(k+1)]$ ). Thus the chirp

$$c_{nm}(t) = e^{j\pi t\phi_{nm}(t)} = e^{j\pi\theta t^2} e^{j2\pi f_n(m)t} \tag{13}$$

for $t_n(m) \leq t \leq t_n(m+1)$ and zero otherwise.

Considering an analysis time segment $t_0 \leq t \leq t_0 + T_f$, where $T_f = \beta\hat{T}$ for a small integer $\beta$, the orthonormality of the chirps $c_{nm}(t)$ is kept by the common chirp rate and by the orthogonality of the complex exponentials with frequencies $\{f_n(m)\}$. Each consecutive pulse in $z_n(t)$ is multiplied by a chirp with an increasing frequency $f_n(m)$.

Assuming again that the effect of the channel is only the addition of Gaussian noise, the received signal is now

$$\begin{aligned} r(t) &= \sum_{n=1}^{N}\sum_{m=0}^{M-1} s_{nm}(t) + \eta(t) \\ &= \sum_{n=1}^{N}\sum_{m=0}^{M-1} z_n(t)c_{nm}(t) + \eta(t) \end{aligned} \tag{14}$$

If we multiply this signal by $e^{-j\pi\theta t^2}$ the resulting signal is

$$\begin{aligned} y(t) &= r(t)e^{-j\pi\theta t^2} \\ &= \sum_{n=1}^{N}\sum_{m=0}^{M-1} z_n(t)e^{j2\pi f_n(m)t} + \eta(t)e^{-j\pi\theta t^2} \end{aligned} \tag{15}$$

and when we pass this signal through a band-pass filter of bandwidth $F_n$ gives

$$\tilde{y}_n(t) = \sum_{m=0}^{M-1} z_n(t) e^{j2\pi f_n(m)t} + \tilde{\eta}(t) \quad (16)$$

which is a combination of sinusoids in the bandwidth assigned to channel $n$, and $\tilde{\eta}(t)$ is the noise within that band-width.

If we express $z_n(t)$ for $t_0 \leq t \leq t_0 + T_f$ as a concatenation of rectangular pulses using the unit-step signal $u(t)$ and let $d_\ell = \pm 1$ for the subchannels being occupied and zero for those that are not, we get

$$z_n(t) = \sum_{\ell=0}^{M-1} d_\ell [u(t - t_n(\ell+1)) - u(t - t_n(\ell))] \quad (17)$$

The Fourier transform of $z_n(t)$ is

$$Z_n(\omega) = \sum_{\ell=0}^{M-1} d_\ell \int_{t_n(\ell)}^{t_n(\ell+1)} e^{-j\omega t} dt \quad (18)$$

and then the Fourier transform of $\tilde{y}_n(t)$ is given by

$$\tilde{Y}_n(\omega) = \sum_{m=0}^{M-1} Z_n(\omega - 2\pi f_n(m)) + \tilde{\eta}(\omega)$$

If we filter $\tilde{Y}_n(\omega)$ with a band-pass filter of center frequency $f_n(m)$ and determine the value of this function at the frequencies $f_n(m)$, for $m \in [0, \cdots, M-1]$ we obtain

$$\begin{aligned} \hat{Y}_n(f_n(m)) &= Z_n(0) + \tilde{\eta}(f_n(m)) \\ &= d_m [t_n(m+1) - t_n(m)] \\ &\quad + \tilde{\eta}(f_n(m)) \end{aligned} \quad (19)$$

so that $|\hat{Y}_n(f_n(m))| \approx t_n(m+1) - t_n(m)$. We thus have that for the $m$-subchannel in the $n^{th}$-ASDM outputl with high signal to noise ratio the corresponding period is

$$T_n(m) = t_n(m+1) - t_n(m)$$

and the magnitude of $\hat{Y}_n(f_n(m))$ is $d_m$.

## 3. SIMULATIONS

To illustrate the duty-cycle modulation performed by the ASDM we consider four different signals. When $x(t) = 0$, the output of the integrator is a symmetric triangular signal and the output $z(t)$ of the ASDM is a train of square pulses of uniform symbol duration. In any other case we obtain rectangular pulses with a duty cycle depending on the value of the amplitude of the input signal. Figure 2 shows the cases when the input is zero, a positive constant, a ramp and an arbitrary signal. A characteristic of these cycles is that the average of the input signal $x(t)$ equals the average of the output signal $z(t)$ in each of the intervals $[t_k, t_{k+2}]$.

The transmission of four outputs $\{z_n(t),\ n = 1, 2, 3, 4\}$, assumed to come from arbitrary signals, is illustrated in Fig. 3. To illustrate the performance of our procedure a Monte Carlo simulation with 500 trials for each signal to noise ratio (SNR) between $-10$ and 10 dBs (with increments of 5 dBs) was implemented. Gaussian noise is added to the chirp-modulated signal to obtain the different SNR's. The binary signals $\{z_n(t),\ n = 1, 2, 3, 4\}$ in a window of 4 msec are shown in the top plot of Fig. 3 displaying different widths for the two pulses in each $z_n(t)$. The magnitudes $|\hat{Y}_n(f_n(m))|$ corresponding to different frequencies in the middle plot are estimates of the width of the pulses in each of the $\{z_n(t),\ n = 1, 2, 3, 4\}$. The axis showing this information is labeled symbol duration. The horizontal axis displays the frequency at which the chirp originates. The effect of the noise (this corresponds to an SNR of 10 dBs) is shown. Thus our algorithm provides the duration of each of the symbols in seconds and also the value $\pm 1$, both of these provide the data necessary to reconstruct the original signals in each of the channels. The plot at the bottom of Fig. 3 displays the error probability when estimating the width of each of the pulses in the binary signals for each of the SNR used in the Monte-Carlo simulation.

## 4. CONCLUSION

In this paper we consider the time-encoding of signals using ASDMs. The advantages of using ASDM's are the low-power consumed and the lack of clocks. For the transmission of the outputs of a number of ASDM's we propose using chirp modulation OFDM. Since the conventional approach cannot be implemented given the non-uniformity of the pulses, we propose a novel approach that uses a sequence of localized linear chirps that are orthonormal. The results are encouraging, especially its robustness to noise.

## REFERENCES


[1] C. Kaldy, A, Lazar, E. Simonyi and L. Toth, "Time encoded communications for human area network biomonitoring," Technical Report, Columbia U., 2007.

[2] A. Lazar and L. Toth, "Perfect recovery and sensitivity analysis of time encoded bandlimited signal," *IEEE Trans. on Circuits and Sys.,* pp. 2060-2073, Oct. 2004.

[3] E. Roza, "Analog-to-digital conversion via duty-cycle modulation ," *IEEE Trans. on Circuits and Sys.,* vol. 44, pp. 907-914, Nov. 1997.

[4] S. Senay, L. F. Chaparro and L. Durak , "Reconstruction of nonuniformly sampled time-limited signals using Prolate Spheroidal Wave functions ," In press, *Signal Proc.*

[5] P. Roche, M. Sun and R. Sclabassi,"Signal multiplexing and modulation for volume conduction communication," *IEEE ICASSP 2005*, pp. 157-160, 2005.

[6] M. Martone, "A multicarrier system based on the Fractional Fourier Transform for time-frequency-selective channels," *IEEE Trans. Comm.*, pp. 1011-1020, Jun. 2001.

[7] Y. Ju and B. Barkat, "A new efficient chirp modulation technique for multi-user access communication systems," *IEEE ICASSP 2004,* , pp. 937-940, May 2004.

[8] M. Ozaktas, Z. Zalevsky and M. Kutay. *The Fractional Fourier Transform with Application in Optics and Signal Processing,* John Wiley & Sons, New York, 2001.

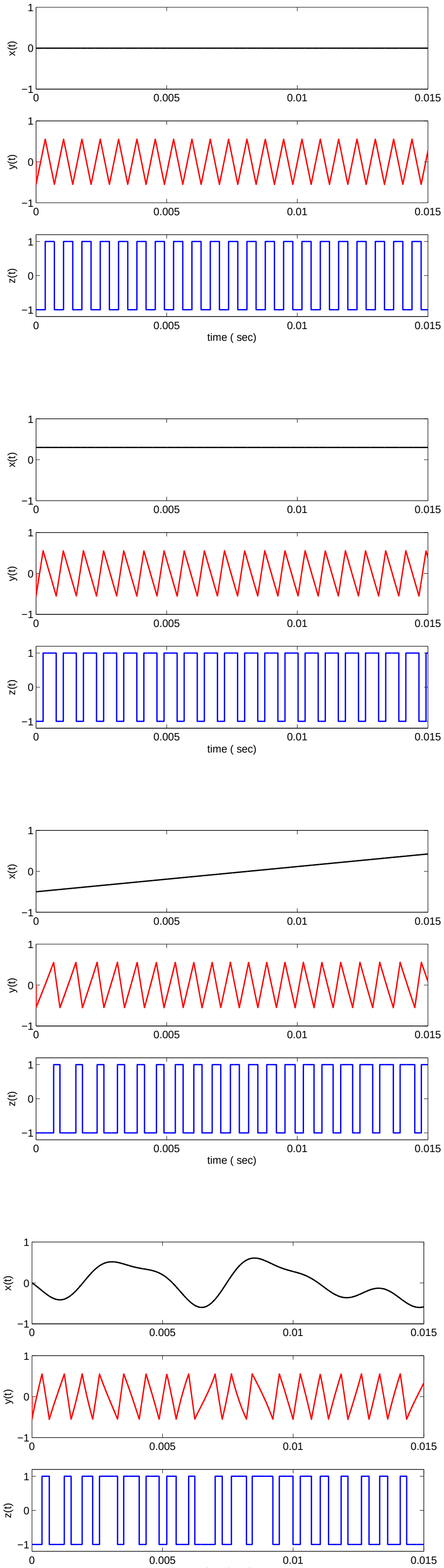


Figure 2: Examples of the processing of ASDM for different inputs. From top to bottom, the input to the ASDM is zero, a constant, a ramp and an arbitrary signal.

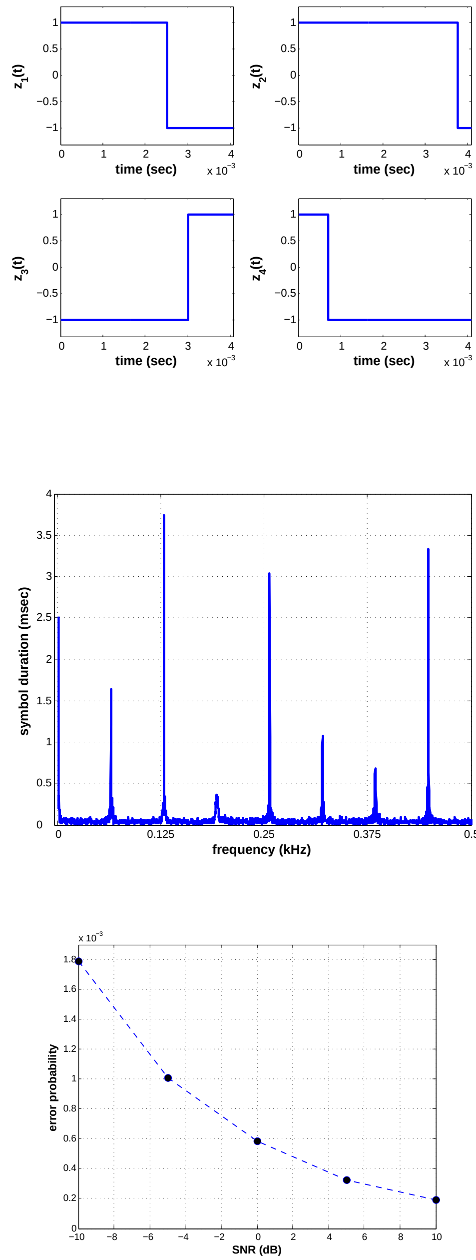


Figure 3: Results of Monte-Carlo simulation for the transmission of four channel ASDM binary signals $\{z_n(t),\ n = 1, 2, 3, 4\}$. Top figure illustrates the non-uniform widths of the binary signals $z_n(t)$. Middle figure shows the estimated widths for each of the pulses in $\{z_n(t),\ n = 1, 2, 3, 4\}$. The bottom figure shows the error probability of the estimation of the widths when noise is added (SNR=10 dB).