

# EDGE DETECTION AND SKELETONIZATION USING QUANTIZED LOCALIZED PHASE

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## ABSTRACT

Localized phase conveys much more information about image structure than magnitude does. For example, phase is used in image reconstruction, edge detection and analysis of textures. We take advantage of the fact that phase is more important on edges and contours than it is in smooth, edge-free regions. Based on this observation, we detect edges as follows: we first calculate the local spatial-frequency transform. We then reconstruct the image, using the magnitude and the quantized phase. The effect of phase quantization on the reconstruction error is negligible in smooth areas, while it is very significant around edges. The reconstruction error provides therefore an excellent map of the edges and a skeleton of the image in the sense of primal sketch.

## 1. INTRODUCTION

One of the most important and widely used tools for image representation and analysis is the spatial frequency transform. Since image content is not stationary, the localized frequency analysis has become an important and powerful tool in image representation [1], [2]. The importance of phase in images was shown first in the context of global phase [3]. This can be most vividly demonstrated by swapping the phase and magnitude of two images (Figure 1), whereby the reconstructed images appear to be more similar to the one whose phase was used in the reconstruction.

Moreover, as was shown, the localized (Gabor)-phase is sufficient for image reconstruction and, in fact, the raw data of Gabor phase depicts the contour information already in the first iteration of the reconstruction [4]. The importance of phase information in images has inspired its application in edge and corner detection and segmentation and in complementing magnitude information [5],[6]. Phase is highly immuned to noise and distortions, features that are very desirable in image processing.

We address the fundamental problem of edge detection and present a new innovative algorithm based on localized phase quantization.

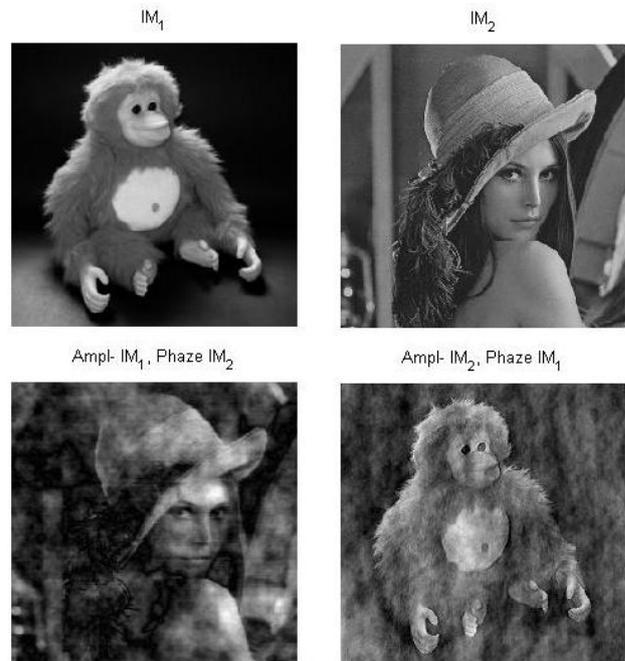


Figure 1: Swapping phase and amplitude during reconstruction.

## 2. LOCALIZED PHASE CALCULATION

The Fourier transform of a 1D function is given by:

$$X(w) = \int_{-\infty}^{\infty} x(t) \cdot e^{-iwt} dt. \quad (1)$$

The global phase of the Fourier transform can be immediately calculated:

$$\angle X(w) = \tan^{-1} \left( \frac{\text{Im}\{X(w)\}}{\text{Re}\{X(w)\}} \right). \quad (2)$$

Our goal is to use the phase information in order to detect significant features, i.e. edges. However, one of the drawbacks of Fourier analysis is the lack of localization in either the time or position spaces [7]. The issue of localization is addressed by the information uncertainty principle [8]. Several methods were proposed to deal with this issue: the Short Time Fourier Transform (STFT), Gabor analysis and Wave-

lets. We adopt the STFT, as it offers a simple implementation, though not an optimal one in terms of localization and computational effort. The localization is achieved by multiplying the signal by a window  $g(t)$  prior to the Fourier analysis.

$$X(\tau, w) = \int_{-\infty}^{\infty} x(t) \cdot g(t-\tau) \cdot e^{-iwt} dt. \quad (3)$$

In this case, the local phase is:

$$\angle X(\tau, w) = \tan^{-1} \left( \frac{\text{Im}\{X(\tau, w)\}}{\text{Re}\{X(\tau, w)\}} \right). \quad (4)$$

### 3. RECONSTRUCTION USING LOCALIZED PHASE

As mentioned, global and local phase based representations are attractive signal analysis tools. While the global phase suffers from lack of localization, the localized phase provides information in both the time (position) and frequency spaces up to their combined uncertainty. This is why localized phase is used in various applications [9], [10], [11].

Localized phase information is also useful in signal reconstruction, and there are studies where a signal is reconstructed from its localized phase components only, while the amplitude information is totally ignored [4]. Such a reconstruction, using localized phase, is more efficient computationally than a reconstruction based on global phase. We do not address the computational efficiency (since our goal was ease of implementation and understanding, at the price of computational efficiency) at the present time, but the reader is encouraged to review the relevant papers on field-programmable gate array (FPGA) oriented implementation [12], [13]. These papers address the issue of computational efficiency and its parallel FPGA based implementation.

The synthesis scheme we address here is the Inverse Short Fourier Transform (ISFT):

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(\tau, w) \cdot e^{iwt} d\tau dw. \quad (5)$$

Note that in order to obtain an accurate reconstruction for digital systems, the window and step size should be carefully selected (for further details see [14]).

### 4. LOCAL PHASE QUANTIZATION

Although it is possible to exactly reconstruct the signal using its phase information, this is not what we aim for here. We would like to quantize the local phase values resulting from the STFT analysis. The phase values are distributed between  $[-\pi:\pi]$ . We wish to quantize them to  $K$  levels, while preserving the original values of the amplitude. In the proposed quantization scheme, the local phase values will be replaced by their nearest quantization value:

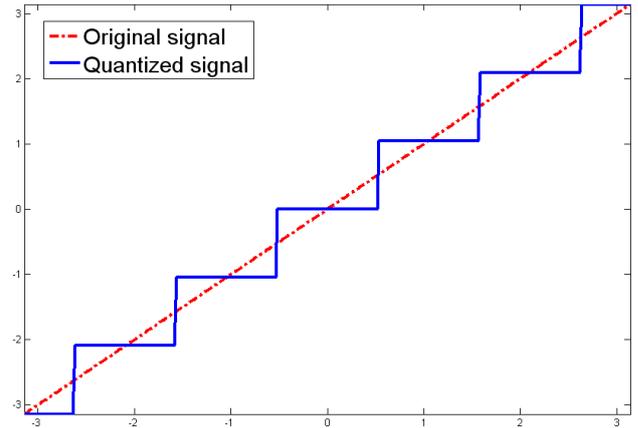


Figure 2: Angle quantization for  $K=6$ .

$$\hat{\phi} = K \left\lfloor \frac{\phi}{Q} \right\rfloor, Q = \frac{2\pi}{K}, \quad (6)$$

where the  $\lfloor \bullet \rfloor$  operator represents the “nearest integer” function.

Quantization of a signal is obviously a destructive operation, and once applied to the local phase, results in an error in the reconstructed signal (for global phase quantization effects see [15]). It is well known that the localized phase data is highly correlated with the “edges” of the signal. The “local energy” term, first coined in [16] describes “in phase” Fourier components- i.e. highly congruent phase components. Phase congruency is used as an efficient edge detector in [9]. Accordingly, impairing the localized phase data will mostly influence the signals' edges. We now wish to define a feature detector, derived from the root-mean-squared (RMS) error of the signal reconstruction with quantized localized phase, compared to the original image:

$$\begin{aligned} err &= x(t) - ISTFT \left[ \hat{X}(\tau, w) \right] \\ \hat{X}(\tau, w) &= \|X(\tau, w)\| \cdot e^{i\hat{\phi}} \\ \hat{\phi} &= K \left\lfloor \frac{\phi}{Q} \right\rfloor, \phi = \angle X(\tau, w), Q = \frac{2\pi}{K}. \end{aligned} \quad (7)$$

Next, we show that this RMS error may serve as a good “edge” detector.

## 5. RESULTS

### 5.1 One-dimensional signals

We examine how the reconstruction error of our scheme is influenced by signal features. We have chosen a rectangular window of 31 samples and a step size of 5 samples in our STFT analysis. Use of the Gaussian windows slightly improves the detection of edges, but we wish to keep the scheme as simple as possible at this stage.

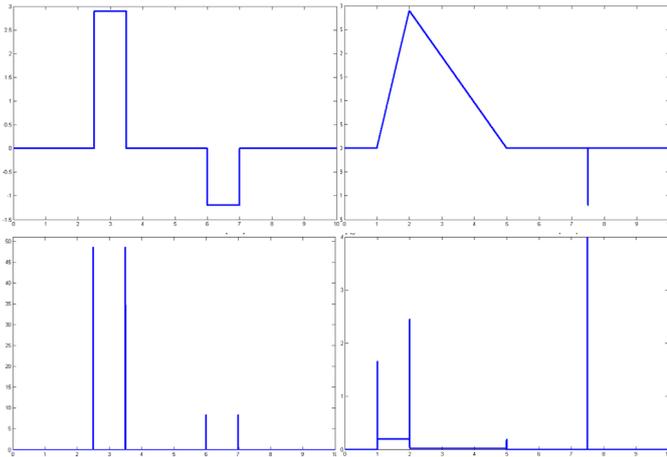


Figure 3: Feature detection in 1D. Analyzed signals (top row) and their local reconstruction error RMS (bottom row) are presented.

Results such as those shown in Figure 3 indicate that the reconstruction error is capable of detecting signal features such as positive and negative steps and ramps<sup>1</sup>. The reconstruction error is obviously influenced by the value of  $K$ ; Lower quantization levels will result in a higher error. We wish to use the error as a feature detector, thus we will maximize it using the minimal quantization level  $K=2$ . Figure 4 demonstrates the relation between signal RMS error and  $K$  derived empirically from a realistic data (column wise Lena image in this example).

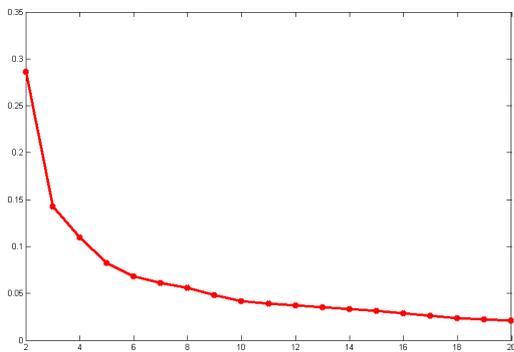


Figure 4- RMS error vs.  $K$  – typical behavior example.

Next, we create a 1D signal from a 2D image (column-wise stacking), and then attempt to detect its 1D features. Finally, we create a 2D signal from the resulting RMS error vector, and present it as an image. Obviously, the features that we can expect to detect are in the vertical direction only. Let us begin with an artificial image that consists of various geometrical shapes (Figure (5), left). Then we shall continue with a realistic image of Lena.

In Figure 5 we can see that the scheme seems to detect most features with components in the vertical direction. If we apply the same scheme to a real image (Figure 6) this also seems to work quite well.

<sup>1</sup>The reconstruction error in Figure 3 that is related to the spike in the right hand side was scaled down to fit into the figure.

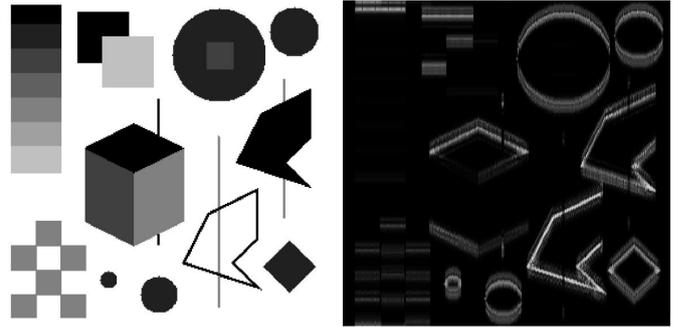


Figure 5: 1D scheme performance with artificial image.



Figure 6: 1D scheme performance with Lena image.

## 5.2 Two-dimensional signals

We use 2D simplified versions of the STFT (full 2D STFT implementation results in large 4D matrices) and ISTFT in the same manner. The window applied here is a 5-by-5 pixels rectangular window, and the step size is one pixel, in order to minimize the location uncertainty. Again, as it is in the 1D case, we wish to achieve a maximal reconstruction error, thus applying a minimal quantization level  $K=2$ . The following figures present the edges detected using the described method. As a means to evaluate the quality of the detected edges, the results obtained from the application of the Phase Congruency (PC)<sup>2</sup> [9] and the Canny methods [20] to same images are presented. The PC feature extraction scheme is also phase-based (“local energy”), and was adopted by many researchers [17], [18],[19] due to its good qualities. Some of these qualities are according to [9]: abilities to deal with noise and contrast changes and lack of thresholds and parameters. The last feature makes PC a very desirable reference edge detector, since no parameter fitting is needed to get a good edges detection. Canny edge detector is a gradient based method, well known for its good performance. We wish to compare our scheme versus this edge detector, to demonstrate some strengths (Figure 8) of phase-based methods.

<sup>2</sup> MATLAB implementation for PC used here is available at Peters Kovesi web page:

<http://www.csse.uwa.edu.au/~pk/Research/MatlabFns/index.html>



Figure 7: Edge detection in a real image using (from left to right): Error RMS, PC, Canny method.

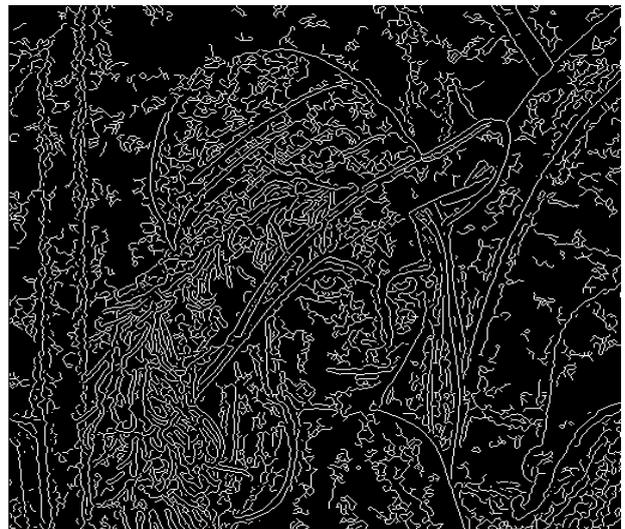
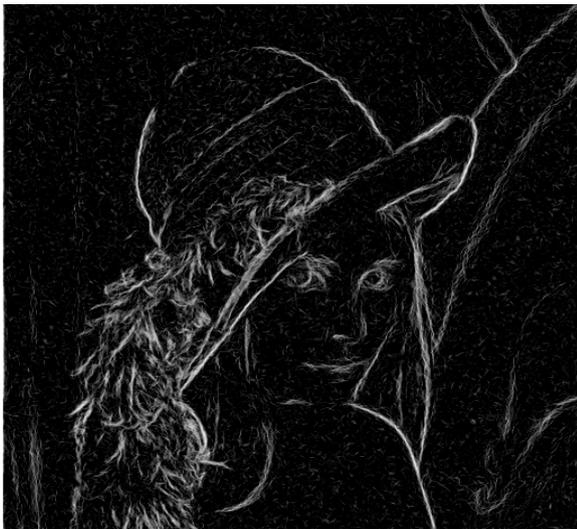


Figure 8: Edge detection in a noisy Lena image. Top (left to right): noisy image, Error RMS. Bottom (left to right): PC, Canny method.

Comparing the resulting edge maps in Figure 7 highlights the better performance of the quantized-phase-based scheme over PC. Moreover, the quantized-phase model is simpler to understand and implement than the PC model, which serves as a significant computational advantage. The Canny method seems to perform best at this point, but since it is gradient based, it is expected to fail in images with changing contrasts and/or noise. In addition, since it uses thresholds, edge strength cannot be evaluated, in contrast to PC and Error RMS methods. Figure 8 demonstrates one of the advantages of phase-based schemes, i.e. a better performance in the case of noisy signals.

## 6. DISCUSSION

Although phase is a non-intuitive feature that is therefore often overlooked, we have shown that it is extremely significant for a high quality feature extraction for both one-, and two-dimensional signals.

In this work, we are using basic tools such as rectangular windows and time frequency analysis schemes, in order to show the robustness of our approach. Further application of Gabor wavelets, and phase only reconstruction is expected to improve the performance of the feature detection. We did not address the advantage of the proposed approach insofar in terms of computational effort reduction. This is an important issue to be addressed elsewhere.

## ACKNOWLEDGEMENT

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