

A MULTISCALE ERROR DIFFUSION ALGORITHM FOR GREEN NOISE DIGITAL HALFTONING

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ABSTRACT

Multiscale error diffusion (MED) is superior to conventional error diffusion algorithm as it can eliminate directional hysteresis completely and possesses good blue noise characteristics. However, due to its filter design, it is not suitable for printing processes which suffer instable dot generation and large dot gain. This paper presents a multiscale error diffusion algorithm for green noise digital halftoning. Analysis and simulation results show that the proposed algorithm not only provides outputs of green noise characteristics, but eliminates pattern artifacts and preserve image details as well.

1. INTRODUCTION

Digital halftoning is a technique used to turn a gray-level image into a bi-level image and has been widely used in printing applications [1]. Basically, there are two types of halftoning schemes. One is amplitude modulation (AM) halftoning [2] in which a halftone is produced by varying the size of printed dots arranged along a regular grid. The other one is frequency modulation (FM) halftoning [3-5] in which a halftone is produced by varying the relative dot density of fixed-size printed dots.

As compared with AM halftoning, FM halftoning produces halftones of higher spatial resolution and better image quality [6]. Unlike outputs of AM halftoning, halftones produced by FM halftoning are free of moiré artifacts as neighboring pixels have been taken into account during error diffusion.

However, halftones produced by FM halftoning are sensitive to the dot gain of a printer, which is the increase in size of the printed dot relative to the intended dot size. In practice, the size and the shape of a printed dot are not as perfect as they are expected and the printed halftone generally appears darker than expected. When the variation in dot size and shape from printed dot to printed dot is small, printing can rely on dot gain compensation technique to minimize this distortion [7]. However, dot gain compensation does not work effectively when the reliability to produce isolated dots is not stable. In such a case, other than using AM halftoning, using a halftoning method which produces clustered dots helps to reduce the dot gain distortion as a cluster has a lower perimeter to area ratio as compared with an isolated dot.

AM-FM halftoning is a hybrid version of AM and FM halftoning which aims at producing clustered dots. For example, Levien [8] proposed an output-dependent feedback error diffusion algorithm to form dot clusters of adjustable size to increase the printing stability. As a result, the output halftone image can be tuned to have larger

cluster dots in a high dot gain situation to cater for different printing systems.

Through a systematic study on the halftones produced with various algorithms, Lau found that a halftone bearing green noise characteristics which contains only mid-frequency spectral components is less susceptible to image degradation from non-ideal printing devices [9]. Based on Lau's idea, Damera-Venkata et al. proposed an adaptive threshold modulation framework to improve the halftone quality by optimizing error diffusion parameters in the least squares sense and derived an adaptive algorithm to provide halftones of green noise characteristics [10]. A block error diffusion algorithm which allows a user to determine the dot size and shape directly was later proposed by the same research group [11].

Multiscale error diffusion (MED) has already been proven to be superior to conventional error diffusion algorithms in terms of various measuring criteria [12-14]. The comprehensive quantitative analysis provided in [14] shows that the outputs of MED algorithms are free of directional hysteresis and possess good blue noise characteristics. Its success in providing a blue-noise halftone motivates us to explore whether the MED framework can also be used in green noise halftoning which aims at providing halftones of green noise characteristics.

The organization of this paper is as follows. In Section 2, a MED algorithm is briefly reviewed. Its diffusion filter is redesigned and the design philosophy is provided. Section 3 presents the proposed green noise halftoning algorithm and shows how it changes output cluster size according to input gray level. An analytical study based on the spectral statistics of the proposed green-noise MED algorithm is given in Section 4 and some empirical simulation results are provided in Section 5 to evaluate the performance of the proposed algorithm. Finally, a brief conclusion is given in Section 6.

2. INSPIRATION

In this section, we first briefly review a MED algorithm [13] which form the basis of the proposed algorithm. Then, we explain why it does not provide halftones of green noise characteristics and suggest a modification to its diffusion filter to make it do it.

Without loss of generality, consider we want to halftone an input gray-level image X of size $2^k \times 2^k$, where k is a positive integer, to obtain an output binary image B . The values of X are within $[0,1]$. Here we assume that the maximum and the minimum intensity values are, 1 and 0

respectively. Note that there is no limitation of the input image size when using MED algorithms. The mentioned size is for easier illustration only.

The feature-preserving MED algorithm (FMED) proposed in [13] is a two-step iterative algorithm. At the beginning, an error image E is initialized to be the gray-level input image X. Pixels of B are then picked iteratively to determine their intensity values until the termination criterion is satisfied. At each iteration, a pixel in B is selected via the “extreme error intensity guidance” based on the most updated E. Then a dot is introduced to the selected pixel in B by assigning a corresponding value (0 or 1) to $b_{i,j}$, and the error ($= e_{i,j} - b_{i,j}$) is diffused to $e_{i,j}$'s neighbors to update the error image E. Here, $e_{i,j}$ and $b_{i,j}$ denote, respectively, the (i,j) th elements of E and B, and (i,j) denotes the coordinates of the selected pixel. The value of $e_{i,j}$ is reset to 0 after the diffusion. These procedures are repeated until the sum of all elements of E is bounded in absolute value by 0.5.

The output of FMED bears good blue-noise characteristics [14], which is good to a stable printing situation. However, when the dot gain is high and dots cannot be consistently reproduced, clustered dots are preferred in the halftoning output to compensate for the dot-gain distortion.

As a matter of fact, conventional MED algorithms [12-13] are prone to produce isolated dots as they all use small diffusion filters (3x3 in most cases). When $b_{i,j}$ is located and quantized, the quantization error ($e_{i,j} - b_{i,j}$) is diffused to $e_{i,j}$'s nearest neighboring pixels. This encourages the formation of isolated dots.

As an example, when one puts a black dot to pixel (i,j) by assigning $b_{i,j}$ to 0, an error diffusion with a 3x3 diffusion filter increases the intensity values of pixels $(i,j\pm 1)$, $(i\pm 1,j\pm 1)$ and $(i\pm 1,j)$ unless $e_{i,j}$ is 0 already before the diffusion. This intensity increase increases the likelihood of assigning white dots to $b_{i,j\pm 1}$, $b_{i\pm 1,j}$ and $b_{i\pm 1,j\pm 1}$ in the future. Accordingly, it is more likely that the black dot at (i,j) will be surrounded by white dots in the final halftoning output.

If it is desirable to form a cluster of dots for pixel (i,j) , the error should not be diffused to the closest neighboring pixels with a 3x3 filter. Instead, it should be diffused to the pixels that are farther away from pixel (i,j) . By doing so, the intensity values of the closest neighboring pixels are not affected by the diffusion and hence it does not directly affect the chance of assigning a particular type of dots to them after the diffusion. In contrast, the intensity values of the outer neighboring pixels to which the error is diffused are increased, which increases these pixels' chance of having white dots in the future. As the number of white dots to be assigned to the output is fixed and now the outer neighboring pixels are more likely to be white, the inner neighboring pixels are actually more likely to be black indeed. Consequently, it encourages the formation of a dot cluster centered at pixel (i,j) .

Based on the aforementioned idea, one can redesign the diffusion filter for FMED to produce halftones with green noise characteristics. In our proposal, a set of filters $\{H_k | k>0\}$ are defined as

$$H_k(m,n) = \begin{cases} 0 & \text{if } (m,n) \in \Omega_{k-1} \\ (1/S) \cdot (1/(m^2 + n^2)) & \text{if } (m,n) \in \Omega_k \setminus \Omega_{k-1} \end{cases} \quad \text{for } k=1,2,\dots \quad (1)$$

where $H_k(m,n)$ is the (m,n) th filter coefficient of filter H_k ,

$$\Omega_k = \{(x,y) | 0 \leq |x|, |y| \leq k\} \quad (2)$$

and $S = \sum_{(m,n) \in \Omega_k \setminus \Omega_{k-1}} (1/(m^2 + n^2))$ is a normalization factor

which makes the sum of all filter coefficients be 1. As an example, when $k=2$, we have

$$H_2 = \begin{bmatrix} w_{2,-2} & w_{2,-1} & \cdots & w_{2,2} \\ w_{1,-2} & w_{1,-1} & & \\ \vdots & & \ddots & \vdots \\ w_{-2,-2} & w_{-2,-1} & \cdots & w_{-2,2} \end{bmatrix} = \frac{10}{31} \begin{bmatrix} 1/8 & 1/5 & 1/4 & 1/5 & 1/8 \\ 1/5 & 0 & 0 & 0 & 1/5 \\ 1/4 & 0 & 0 & 0 & 1/4 \\ 1/5 & 0 & 0 & 0 & 1/5 \\ 1/8 & 1/5 & 1/4 & 1/5 & 1/8 \end{bmatrix} \quad (3)$$

The determination of the filter coefficients is based on the idea that, in an isotropic diffusion process, the intensity at a particular point away from the source is inversely proportional to the square of the distance from the source.

Filter H_k is k -dependent. Its size is of $(2k+1) \times (2k+1)$ pixels. The larger the k value is, the larger the filter size is and the larger the clusters can be produced in the halftone output for a fixed gray-level input. For reference purpose, the parameter k associated with filter H_k is referred to as the order of filter H_k hereafter.

3. PROPOSED ALGORITHM

The proposed algorithm is a two-step iterative algorithm. In each iteration, it selects a pixel to assign a dot in the 1st step and then diffuse the error to update the error plane E in the 2nd step. E is initialized to be X at the beginning. The 1st step of the proposed algorithm is exactly the same as the 1st step of FMED except that parameters x_{off} and y_{off} , the random shifts added to the starting searching window, are bounded by $[-2,0,2]$ instead of $[-1,0,1]$.

Assume that pixel (i,j) is selected and a white dot is assigned to it by making $b_{i,j}=1$ in the 1st step. In the 2nd step, in order to support green noise digital halftoning, the error between $b_{i,j}$ and $e_{i,j}$ is diffused with a filter selected from $\{H_k | k \geq k_{min}\}$ or its adapted version, where $k_{min}>1$ is a predetermined integer value used to adjust the desirable cluster size for a particular input gray level. $H_{k_{min}}$ is used as the default error diffusion filter for all pixels. Its adjusted versions or other H_k are used only when necessary.

For example, when the default diffusion filter is H_2 (i.e. $k_{min}=2$), the error image E is updated as

$$e'_{x,y} = \begin{cases} 0 & \text{if } (x,y) = (i,j) \\ e_{x,y} - w_{x-i,y-j} m_{x,y} (1 - e_{i,j}) / s & \text{if } (x-i,y-j) \in \Omega_2 \setminus \Omega_1 \end{cases} \quad (4)$$

where $m_{x,y} = \begin{cases} 0 & \text{if } b_{x,y} \text{ has been assigned a value} \\ 1 & \text{else} \end{cases}$ and

$$s = \sum_{(x-i,y-j) \in \Omega_2, \Omega_1} w_{x-i,y-j} m_{x,y} \quad (5)$$

In the case when $s=0$, we gradually increase the value of k to pick a larger H_k to make $s \neq 0$ and keep the algorithm work.

In green noise halftoning, cluster size should adapt to input gray level. A simulation was performed to study how the proposed algorithm changes the cluster size according to different gray levels. In our study, for each particular gray level, a constant gray-level image of size 128x128 was generated and halftoned with the proposed algorithm to produce 10 halftones. The average cluster size M_g in the resultant halftones was then measured.

The blue curve in Fig. 1 shows how the average cluster size M_g changes according to the input gray level g when filter H_2 is used as the default filter in the proposed algorithm. This curve can be approximated with a 2nd order polynomial $M_g = -6.1847g^2 + 11.4813g + 0.7920$. The green curve shows the case when filter H_3 is used as the default filter, which can be approximated by polynomial $M_g = -4.7590g^2 + 29.4427g + 0.1721$.

Two observations can be obtained in Fig. 1. First, the higher the order of filter H_k , the larger the clusters can be produced in the halftone output for a fixed gray-level input. One can control the desirable cluster size by selecting an appropriate k_{min} to pick an appropriate default error diffusion filter. Theoretically, larger cluster of dots reduces more dot gain distortion but at the same time it reduces the effective spatial resolution of the output halftone. Second, for a fixed value of k , the cluster size varies with the input gray level.

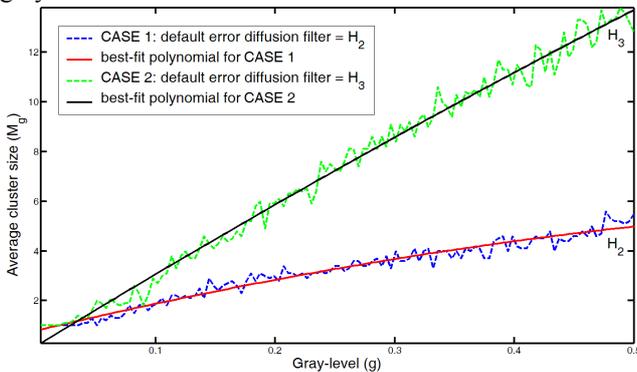


Fig. 1 Connection between cluster size and input gray level

4. PERFORMANCE ANALYSIS

This section provides an analysis on the performance of the proposed algorithm based on radically averaged power spectrum density (RAPSD) and anisotropy [6]. Both measures are developed to analyze the spectral characteristics of a halftone pattern.

RAPSD is defined as the average power in an annular ring band of the frequency spectrum of the halftone of a constant gray-level input. In formulation, we have

$$P(f_p) = \frac{1}{N(R(f_p))} \sum_{f \in R(f_p)} \hat{P}(f) \quad (6)$$

where $R(f_p)$ is an annular ring band of width Δ_p and center radius f_p in the spectral domain, $N(R(f_p))$ is the number of frequency samples in $R(f_p)$, and $\hat{P}(f)$ is the magnitude square value of frequency f .

Anisotropy is defined as

$$A(f_p) = \frac{1}{N(R(f_p)) - 1} \sum_{f \in R(f_p)} \frac{(\hat{P}(f) - P(f_p))^2}{P^2(f_p)} \quad (7)$$

It provides the variance of frequency samples of $\hat{P}(f)$ in $R(f_p)$ and is used to measure the extent of directional artifact.

In our analysis, various green noise error diffusion algorithms were applied to a set of constant gray-level images of size 256x256 each and the dot distribution in their outputs was studied in terms of $P(f_p)$ and $A(f_p)$. Levien's [8], Damera-Venkdata's [10], Damera-Venkdata's [11] and the proposed algorithm were included in the comparison. In the realization of [8] and [10], serpentine scanning is used and the hysteresis constant H is set to 1.0. The hysteresis filter and the error filter used in [8] are, respectively, fixed to be $[0, 0.6, 0; 0.4, \underline{0}, 0]$ and $[0, 0.5, 0; 0.5, \underline{0}, 0]$. These two filters are adaptive in [10] and they are, respectively, initialized to be $[0, 0.6, 0; 0.4, \underline{0}, 0]$ and $[0, 0.5, 0; 0.5, \underline{0}, 0]$ as well. The dot shape used in simulating [11] is a 2×2 cluster ($= [1, 1; 1, 1]$). Their settings are selected to make all of them produce clusters of comparable size at the output when gray-level input $g=0.5$ is provided.

Fig. 2 shows the performance of various algorithms in terms of anisotropy. As mentioned in [6], when $A(f_p) > 0\text{dB}$ happens, directional components are considered to be strong or noticeable to human eyes. To provide a reference to study the performance of the algorithms, a surface defined as $A(f_p) = 0\text{dB}$ is added in each of the plots. The plots show that the proposed algorithm is better than the other algorithms. Its anisotropy values are well below 0dB .

Green noise halftoning is characterized by a distribution of clusters which are formed by a group of pixels as homogeneously as possible [9]. It is visually pleasant as it does not clash with the structure of an image. Clusters distributed in this way create an aperiodic and isotropic pattern. In other words, for a constant gray-level input, an ideal green noise generator should produce a halftone which has no low- and high-frequency spectral components and a spectral peak at the input-dependent green noise principle frequency [9].

Fig 3 shows the performance of various algorithms in terms of RAPSD. To have a clear picture on the performance of the algorithms, a white surface which marks the green noise principle frequency f_g for a particular gray level is added in each of the plots as a reference for comparison. We can see that all the tested algorithms have green noise characteristics.

5. SIMULATION RESULTS

Simulation was carried out to evaluate the performance

of the proposed algorithm. In our simulation, a number of *de facto* standard 8-bit gray-level images including *Mandrill*, *Barbara*, *Boat*, *House*, *Lena*, *Man* and *Peppers* were used. Each of them is of size 256×256 . Green noise halftoning algorithms [8], [10] and [11] were also included in our simulation for comparison. The same set of realization parameters mentioned in Section 4 were used in their realization.

MSE_v is one of the halftone visibility metrics [16] used to measure the distortion observed by a human viewer between an original gray-level image X and its binary halftone B . In particular, it is defined as

$$MSE_v = \frac{1}{N \times N} \|hvs(X, vd, dpi) - hvs(B, vd, dpi)\|^2 \quad (7)$$

where hvs is the HVS filter function defined in [15], vd is the viewing distance in inches and dpi is the printer resolution. In our simulations, the viewing distance is fixed at 50 inches and printer resolution of 600dpi is considered.

Universal Objective Image Quality Index (UQI) [16] is another measure used in our study. In the measurement of UQI, the larger the value, the better the performance. The value of UQI is bounded by -1 and 1.

Tables 1a and 1b show the performance of various green noise halftoning algorithms in terms of MSE_v and UQI respectively. One can see that the proposed algorithm performs better. For subjective evaluation, Fig. 4 shows the halftone results of various algorithms for *Barbara* of size 512×512 . The proposed algorithm preserves the details of the stripe patterns better than the others.

	Image	[8]	[10]	[11]	ours
(a)	Mandrill	26.9585	27.8279	24.2370	21.1674
	Barbara	42.8652	43.8482	29.2730	22.5430
	Boat	26.9577	27.6888	30.3380	20.5762
	House	34.1923	34.7439	28.7891	18.9034
	Lena	41.9335	43.5121	27.7417	20.6478
	Man	33.7160	34.6094	27.4531	19.9675
	Peppers	45.9688	46.5153	30.9943	20.9107
	Average	36.0846	36.9637	28.4037	20.6737
(b)	Mandrill	0.0306	0.0300	0.0927	0.1749
	Barbara	0.0598	0.0599	0.0960	0.1440
	Boat	0.0559	0.0556	0.0962	0.1375
	House	0.0352	0.0366	0.0573	0.1136
	Lena	0.0496	0.0506	0.0756	0.1083
	Man	0.0516	0.0532	0.0809	0.1290
	Peppers	0.0771	0.0773	0.0955	0.1186
	Average	0.0514	0.0519	0.0849	0.1323

Table 1. The measurements of (a) MSE_v and (b) UQI

6. CONCLUSIONS

In this paper, a multiscale error diffusion (MED) algorithm for green noise digital halftoning is proposed. Its performance is studied empirically. Study results show that the proposed algorithm can provide good green noise characteristics and provide better halftoning performance as compared with other evaluated green noise error diffusion algorithms in terms of anisotropy, MSE_v and UQI.

As a final note, when the proposed algorithm H_1 is used as the default diffusion filter (i.e. $k_{min}=1$), it becomes FMED to provide halftones of blue noise characteristics. In

other words, the proposed algorithm is actually a generalized version of MED with FMED as its special case. Based on the practical printing condition, one can change parameter k_{min} to adjust the cluster size to compensate for the dot overlap distortion.

7. ACKNOWLEDGEMENT

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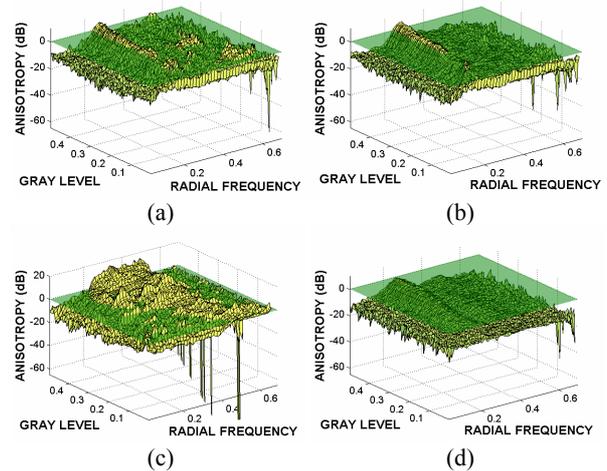


Fig 2. Performance in terms of anisotropy with (a) [8], (b) [10], (c) [11], (d) ours

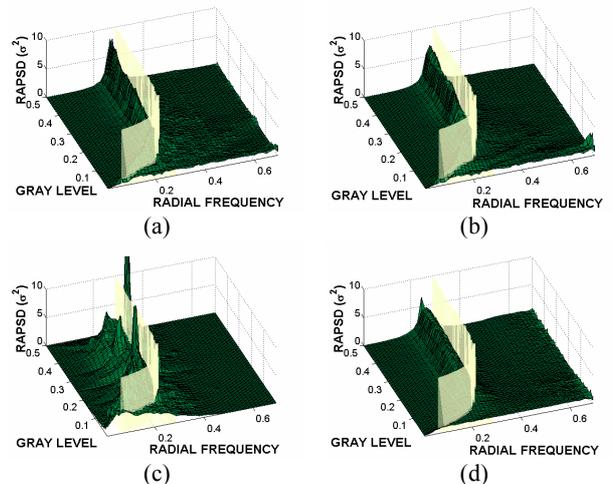


Fig 3. Performance in terms of RAPS D with (a) [8], (b) [10], (c) [11], (d) ours

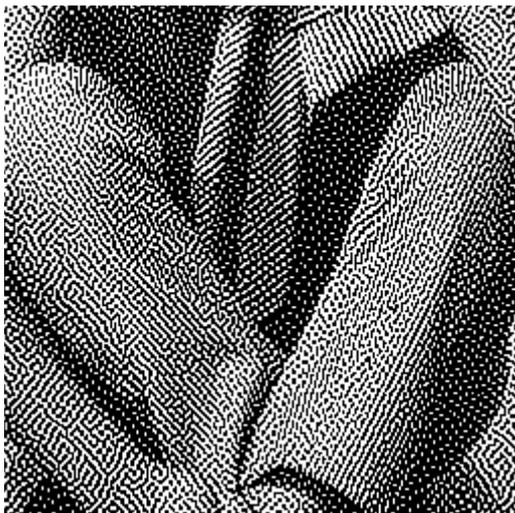
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(a) Original



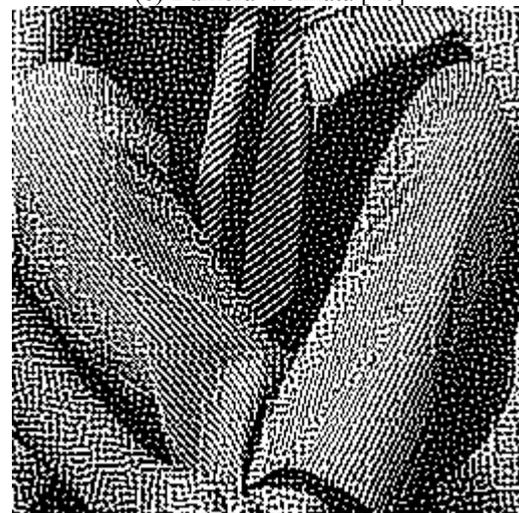
(b) Levien [8]



(c) Damera-Venkata [10]



(d) Damera-Venkata [11]



(e) Ours

Fig. 4 Halftones produced with various algorithms