

SIGNAL-DEPENDENT TECHNIQUES FOR NON-STATIONARY SIGNAL SAMPLING AND RECONSTRUCTION

M. Greitans and R. Shavelis

Institute of Electronics and Computer Science
14 Dzerbenes Str., Riga LV-1006, Latvia
phone: + (371) 67558110, fax: + (371) 67555337, email: greitans@edi.lv, shavelis@edi.lv
web: www.edi.lv

ABSTRACT

The paper describes the processing of non-stationary signals, which takes the advantages offered by the use of signal-dependent techniques in sampling and analysis procedures. The level-crossing approach is exploited for signal sampling, whereby the local sampling density provides information about the local maximum spectral frequency of the signal. The frequency is used to build signal-dependent reconstruction functions required for signal recovery by solving a least squares problem. The results of simulation are presented using speech as an example. The approach developed can be implemented using asynchronous design techniques and could be aimed at application in speech transmission over wireless networks.

1. INTRODUCTION

The spectral contents of signals of practical interest often change with time. Generally, a signal with time-varying spectral bandwidth can be approximated with fewer samples per interval using appropriate non-equidistantly spaced samples than using uniform sampling procedure, where the sampling rate is chosen taking into account the highest signal frequency. Intuitively speaking, the non-stationarity of the signal should be reflected in the process of analog-to digital conversion – the low frequency regions should be sampled at a lower rate than the high frequency regions.

A special class of non-uniform sampling is derived if the sampling process is driven by the signal itself – it is so called signal dependent sampling. Popular types are based on zero-crossing, reference signal crossing, level crossing and send-on-delta concepts. Particular attention should be paid to cases, where the local sampling density derives from the local properties of the signal. One of the sampling approaches with such a quality is level-crossing sampling, which will be used further in the paper as a tool for digital data capture from a continuous time signal.

2. LEVEL-CROSSING SAMPLING

The idea of level-crossing sampling (LCS) is based on the principle that samples are captured when the input signal crosses predefined levels. Such a sampling strategy has quite long history and is exploited for various applications [1, 2]. The quantization levels can be located arbitrarily, however, if there is no special reason, the typical solution is to dispose them uniformly along the amplitude range of the signal.

It has been shown that level-crossing sampling has several interesting properties and is more efficient than traditional sampling in many respects [3]. In particular, it can be

related to the processing of non-stationary signals, because the local density of samples reflects the local characteristics of the signal [4, 5]. If a waveform is changing rapidly, the samples are spaced more closely, and conversely – if a signal is varying slowly, the samples are spaced sparsely. This property allows tracking of the local maximum frequency in the signal spectrum in order to use it for data analysis. Since the level-crossing sampling scheme provides non-equidistantly spaced samples, appropriate processing methods must be developed.

3. RECONSTRUCTION OF SIGNAL WITH TIME-VARYING BANDWIDTH

Several methods for reconstruction of non-uniformly sampled band-limited signals are used. For correct recovery, they typically require that the maximal length of the gaps between the sampling instants does not exceed the Nyquist [6]. If a signal is non-stationary with time-varying spectral bandwidth, the global satisfying of this requirement is not an appropriate decision, because that provides redundant data. The use of level-crossing sampling scheme can reduce the amount of samples, because the intervals between samples are determined by signal local properties and by the number of quantization levels. The quality of processing can be improved if the recovery procedure takes into account the local bandwidth of the signal [7]. In the following subsections will be discussed proposed idea and methods for reconstruction using filters with time-varying bandwidth and for estimation of local maximum frequency of signal from its level-crossing samples.

3.1 Signal-dependent reconstruction functions

The sampling theorem states that every bandlimited signal $s(t)$ can be reconstructed from its equidistantly spaced samples if the sampling rate equals or exceeds the Nyquist rate $2F_{max}$, where F_{max} is the maximum frequency in the signal spectrum. The reconstruction in time domain can be expressed as

$$\hat{s}(t) = \sum_{n=0}^{N-1} s(t_n)h(t-t_n), \quad (1)$$

where $\hat{s}(t)$ denotes reconstructed signal, N is the number of the original signal samples $s(t_n)$ and $h(t)$ is an appropriate impulse response of the reconstruction filter, classically, sinc-function

$$h_1(t) = \text{sinc}(2\pi F_{max}t) \quad (2)$$

As the sampling instants $t_n = \frac{n}{2F_{max}}$, then the impulse response

$$h_1(t - t_n) = h_1(t, t_n) = \text{sinc}(2\pi F_{max}t - n\pi), \quad (3)$$

where $h_1(t - t_n) = h(t, t_n)$ is written as the function of two arguments. The reconstructed signal becomes

$$\hat{s}(t) = \sum_{n=0}^{N-1} s(t_n)h_1(t, t_n) \quad (4)$$

If the signal with time-varying frequency bandwidth $f_{max}(t)$ is considered, then the sampling rate of the signal according to Nyquist must be at least $2F_{max}$, where $F_{max} = \max(f_{max}(t))$. In this case any information about the local spectral bandwidth is ignored during the sampling process. To take it into account, it is proposed instead of $h_1(t, t_n)$ to use more general function

$$h_2(t, t_n) = \text{sinc}(\Phi(t) - \Phi(t_n)) = \text{sinc}(\Phi(t) - n\pi), \quad (5)$$

where $\Phi(t) = 2\pi \int_0^t f_{max}(t)dt$ is the phase of the sinusoid, whose frequency changes in time as $f_{max}(t)$, $t \geq 0$ and sampling instants t_n are chosen such that $\Phi(t_n) = n\pi$. If the signal is stationary and band-limited $f_{max}(t) = \text{const} = F_{max}$, equations (3) and (5) become equivalent. In case of non-constant $f_{max}(t)$ waveform of the reconstruction function $h_2(t, t_n)$ and the desired sampling instants t_n are determined by $f_{max}(t)$. Samples are spaced non-equidistantly and the mean sampling frequency can be less than it is required by Nyquist criterion, which, in this case, should be satisfied rather in local than in global sense.

3.2 Reconstruction algorithm

To apply the formula (4) for reconstructing the signal from its level-crossing samples $s(t_m)$, the recovery procedure involves signal resampling from sampling set $\{t_m\}$ to $\{t_n\}$. The new sampling values $\hat{s}(t_n)$ are found by the method of least squares to ensure the minimal error

$$\sum_{m=0}^{M-1} (s(t_m) - \hat{s}(t_m))^2 = \min, \quad (6)$$

where

$$\hat{s}(t_m) = \sum_{n=0}^{N-1} \hat{s}(t_n)h(t_m, t_n) \quad (7)$$

Considering (6) and (7) the solution in matrix notation is obtained

$$\hat{\mathbf{S}} = \mathbf{S}\mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}, \quad (8)$$

where $\hat{\mathbf{S}} = [\hat{s}(t_0), \hat{s}(t_1), \dots, \hat{s}(t_{N-1})]$, \mathbf{H} is $M \times N$ matrix whose element in row m and column n is $h(t_m, t_n)$ and $\mathbf{S} = [s(t_0), s(t_1), \dots, s(t_{M-1})]$.

In the level-crossing sampling case the values of $\hat{s}(t_n)$ are limited by two corresponding adjacent quantization levels $a_n \leq \hat{s}(t_n) \leq b_n$, where $a_n \in Q$, $b_n \in Q$ and Q is the set of all quantization levels. This restriction can be written by substituting

$$\hat{s}(t_n) = a_n + (b_n - a_n)k_n, \quad (9)$$

where the coefficient $0 \leq k_n \leq 1$. The equation (9) for all samples $\hat{s}(t_n)$ can be written as

$$\hat{\mathbf{S}} = \mathbf{A} + (\mathbf{B} - \mathbf{A}) \circ \mathbf{K}, \quad (10)$$

where $\mathbf{A} = [a_0, a_1, \dots, a_{N-1}]$, $\mathbf{B} = [b_0, b_1, \dots, b_{N-1}]$, $\mathbf{K} = [k_0, k_1, \dots, k_{N-1}]$ and (\circ) denotes Hadamard product of two matrices. From (8) and (10) it follows

$$\mathbf{K} = \frac{\mathbf{S}\mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1} - \mathbf{A}}{\mathbf{B} - \mathbf{A}} \quad (11)$$

The solution (11) may provide coefficient values that lie outside the allowed interval limits of $[0, 1]$. To prevent this the minimization task (6) considering (7) and (9) should be solved for k_n values $0 \leq k_n \leq 1$. As it can be very time-consuming, the coefficients obtained by (11) are roughly limited by (12)

$$k_n = \begin{cases} 0, & \text{if } k_n < 0 \\ k_n, & \text{if } 0 \leq k_n \leq 1 \\ 1, & \text{if } k_n > 1 \end{cases} \quad (12)$$

Further the coefficients are made more precise considering that the reconstructed signal between two successive level-crossings is also limited by two corresponding quantization levels. If we choose the uniform sampling set $\{t_u\}$ with high enough density and indices $u = 0, 1, 2, \dots, U - 1$, then the reconstructed signal according to (4) and (9) is

$$\hat{s}(t_u) = \sum_{n=0}^{N-1} (a_n + (b_n - a_n)k_n)h(t_u, t_n) \quad (13)$$

Every recovered sample $\hat{s}(t_u)$ must lie between two corresponding quantization levels $c_u \leq \hat{s}(t_u) \leq d_u$, where $c_u \in Q$ and $d_u \in Q$. For all samples this condition can be written as

$$\mathbf{C} \leq (\mathbf{A} + (\mathbf{B} - \mathbf{A}) \circ \mathbf{K}) \mathbf{G}^T \leq \mathbf{D}, \quad (14)$$

where $\mathbf{D} = [d_0, d_1, \dots, d_{U-1}]$, $\mathbf{C} = [c_0, c_1, \dots, c_{U-1}]$ and \mathbf{G} is $U \times N$ matrix whose element in row u and column n is $h(t_u, t_n)$. The inequality in (14) is applied element-wise. After the estimation of \mathbf{K} according to (11) and (12) the verification of condition (14) follows, providing indices u' that do not satisfy the requirement. By randomly choosing one of the indices u' the index n is found for which the distance $|t_{u'} - t_n|$ is minimal. Then the coefficient k_n is changed as follows

$$k_n = \begin{cases} k_n(1 - \alpha), & \text{if } \hat{s}(t_{u'}) > d_u \\ k_n(1 - \alpha) + \alpha, & \text{if } \hat{s}(t_{u'}) < c_u \end{cases} \quad (15)$$

where $0 \leq \alpha \leq 1$ determines how fast the coefficient is decreased towards zero or increased towards one (we choose $\alpha = 0.05$). Thereafter steps (13), (14) and (15) are repeated until condition (14) is satisfied for all u or fixed number of iterations is reached. The fixed number should be set (we choose $10N$) because such technique can not guarantee the fulfilment of (14). However, the number of indices u' can be reduced significantly by random selection of u' and change of corresponding coefficient k_n at each iteration. Random selection is preferred since the new coefficient influences all $\hat{s}(t_u)$ values. The number of indices u' can also be reduced if instead of one coefficient two coefficients k_n and k_{n+1} corresponding to $\hat{s}(t_n)$ and $\hat{s}(t_{n+1})$ with t_n and t_{n+1} located most closely to $t_{u'}$ are changed and the condition (14) is made softer by replacing the values c_u and d_u in matrices \mathbf{C} and

D with $c_u - (d_u - c_u)\beta$ and $d_u + (d_u - c_u)\beta$, where $\beta > 0$ (we choose $\beta = 0.05$).

For calculation of **H** and **G** either impulse response (3) or (5) can be used. In level-crossing sampling case better reconstruction result is achieved by $h_2(t, t_n)$ as it depends on instantaneous maximum frequency of the signal. The re-sampling instants t_n are determined by $f_{max}(t)$, that in general case is not known in advance. To solve this problem, an algorithm is developed, which estimates the time-varying instantaneous maximum frequency using information about locations of level-crossings. Please note that instantaneous maximum frequency stands for local bandwidth of the signal and is not the same as instantaneous frequency defined through Hilbert transform.

3.3 Estimation of instantaneous maximum frequency

The local bandwidth of the signal can be estimated by finding its time-frequency representation (TFR) using, for example, short-time Fourier transform, wavelet transform or Wigner-Ville distribution. These methods are developed for uniformly sampled signals, however, there are some modifications in order to find the TFR of non-uniformly sampled signals [8]. The use of such approach is time consuming, thus a simpler method should be considered. In [9] it is shown how to obtain instantaneous frequency of the phase signal sampled by level-crossings. However, signals of practical interest are not so simple, thus the method based on empirical evaluations is proposed.

To estimate the function $\hat{f}_{max}(t)$ from samples $s(t_m)$, starting with the initial index value $m = 0$ two pairs of successive level-crossing samples $s(t_{m'_j}) = s(t_{m''_j+1})$ and $s(t_{m''_j}) = s(t_{m'_j+1})$ are found such that $m''_j > m'_j$ and the difference $m''_j - m'_j$ is minimal. Thereafter the next two pairs are found considering that $m''_{j+1} = m''_j$. For each $j = 1, 2, \dots$ the value $f(t_j)$ is calculated as

$$f(t_j) = \left(t_{m''_j} + t_{m''_j+1} - t_{m'_j} - t_{m'_j+1} \right)^{-1}, \quad (16)$$

where

$$t_j = \frac{1}{4} \left(t_{m''_j} + t_{m''_j+1} - t_{m'_j} - t_{m'_j+1} \right) \quad (17)$$

If a single sinusoid is sampled, then $f(t_j) = f(t_{j+1})$ for all j and it equals the frequency of the sinusoid. If the signal consists of more harmonics, then $f(t_j)$ for different j vary around the average value of $\bar{f} = \frac{1}{J} \sum_{j=1}^J f(t_j)$, where J is the total number of detected pairs within the observation time of the signal. Experiments show that \bar{f} is close to the frequency of the highest component. Thus, the estimate of function of instantaneous maximum frequency $\hat{f}_{max}(t)$ can be obtained by $\{f(t_j)\}$ approximation with piecewise polynomials $p_v^r(t)$ of order r . By choosing the number $L > 1$ the observation interval of signal is divided into subintervals

$$\Delta T_v : t \in [t_{v,1}; t_{v,2}], \quad (18)$$

where $v = 0, 1, \dots$ is the number of subinterval and

$$t_{v,1} = \frac{t_{j=vL} + t_{j=vL+1}}{2}, \quad (19)$$

$$t_{v,2} = \frac{t_{j=(v+1)L} + t_{j=(v+1)L+1}}{2}$$

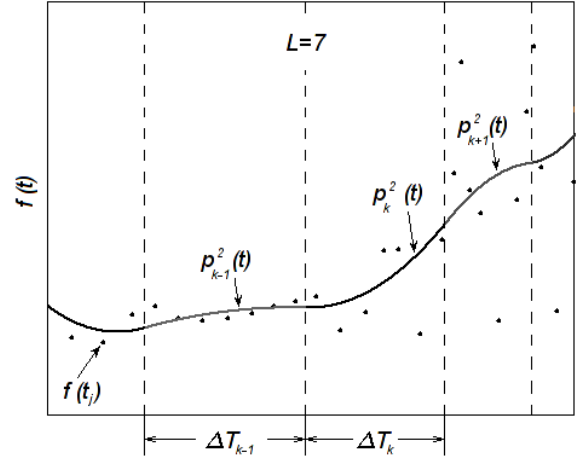


Figure 1: Piecewise polynomial $p_k^2(t)$ approximation (the number of samples per subinterval is $L = 7$).

For each subinterval ΔT_v the coefficients $e_{v,r}, e_{v,r-1}, \dots, e_{v,1}, e_{v,0}$ of polynomial $p_v^r(t) = e_{v,r}t^r + e_{v,r-1}t^{r-1} + \dots + e_{v,1}t + e_{v,0}$ are found to ensure

$$p_{v-1}^r(t_{v,1})^{(0)} = p_v^r(t_{v,1})^{(0)}, p_v^r(t_{v,2})^{(0)} = p_{v+1}^r(t_{v,2})^{(0)}$$

$$p_{v-1}^r(t_{v,1})^{(1)} = p_v^r(t_{v,1})^{(1)}, p_v^r(t_{v,2})^{(1)} = p_{v+1}^r(t_{v,2})^{(1)}$$

$$\vdots$$

$$p_{v-1}^r(t_{v,1})^{(r)} = p_v^r(t_{v,1})^{(r)}, p_v^r(t_{v,2})^{(r)} = p_{v+1}^r(t_{v,2})^{(r)}$$

and the value of expression

$$\sum_{v=0}^{V-1} \sum_{j=vL+1}^{(v+1)L} [f(t_j) - p_v^r(t_j)]^2 = \min \quad (20)$$

is minimal. The denotation $(\dots)^{(r)}$ means the derivative of order r and V is the total number of subintervals. After solving the minimization task using the method of least squares, the coefficients of polynomials $p_v^r(t)$ are obtained and the estimate of instantaneous maximum frequency

$$\hat{f}_{max}(t) = p_v^r(t), \text{ if } t_{v,1} \leq t \leq t_{v,2} \quad (21)$$

depends on the number L of samples $f(t_j)$ per subinterval. To reduce the dependency the final frequency estimate is obtained by averaging $\hat{f}_{max}(t)$ calculated for different L values. The example of piecewise polynomial of order $r = 2$ approximation when $L = 7$ is shown in Fig. 1.

4. APPLICATION TO SPEECH PROCESSING

Speech transmission is one of the most important and common services in telecommunication networks. One of the basic prerequisites for successful speech transmission over data channels is the use of an effective speech encoding technique. It should compress the speech signal at the sender's end and decompress the digital codes to reconstruct the speech with satisfactory quality at the receiver's end. The main concern for system designers is to preserve the best speech quality and at the same time reduce the necessary bit rate of data

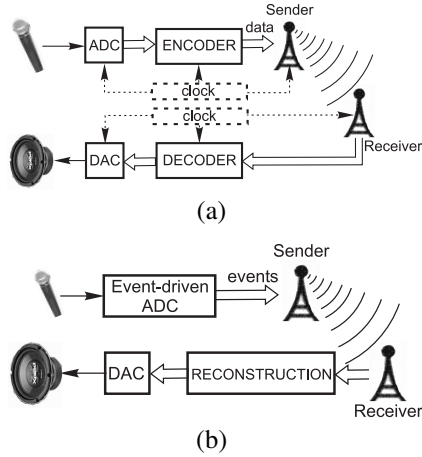


Figure 2: Different structures of speech transmission systems: (a) traditional clock-based; (b) proposed event-driven.

transmission. To achieve such efficiency more and more sophisticated speech-coding algorithms are used that need more memory and computational load.

On the other hand, it is attractive if electronic devices, which perform speech transmission, can be miniaturized with low power consumption, especially in wireless equipment. A simplified block diagram of the "classical" speech transmission approach is illustrated in Fig. 2a. Speech digitizing is based on clock-driven analog-to-digital (A/D) converter, which is followed by a digital signal processing (DSP) block. As a result, the speech data can be compressed approximately ten times, which considerably diminishes the power consumption of the sender (important for wireless system) as well as the load on the data transmission channel (important for VoIP system).

The algorithm described above can be implemented in an alternative structure of speech processing and transmission system, which is based on event-driven A/D conversion and is proposed in [7]. The block diagram of the system is shown in Fig. 2b. It can be seen that this structure provides substantial simplification of the sender part of the system. Application of the method developed to speech processing is motivated by the properties of speech signals. Within the naturally spoken language, pauses and disfluencies occur quite often, which do not provide useful information from the point of view of speech coding. In classical case they are extracted after the A/D conversion during the speech encoding procedure, while level-crossing sampling based A/D converter simply does not capture samples during pauses. In such a way, it is possible to considerably decrease the data flow on the ADC output without additional data encoding. A different application of the idea of signal encoding using level crossings is discussed in [10], where signal is initially pre-filtered using a filterbank and then each output is sampled by level-crossings. Also in this case, the speech signal is taken as an example for illustration of the method.

5. SIMULATION RESULTS

The performance of the signal-dependent algorithm was tested on a speech signal taken from the TIMIT database (/timit/train/dr1/mtpf0/sx335.wav; sampling frequency 16

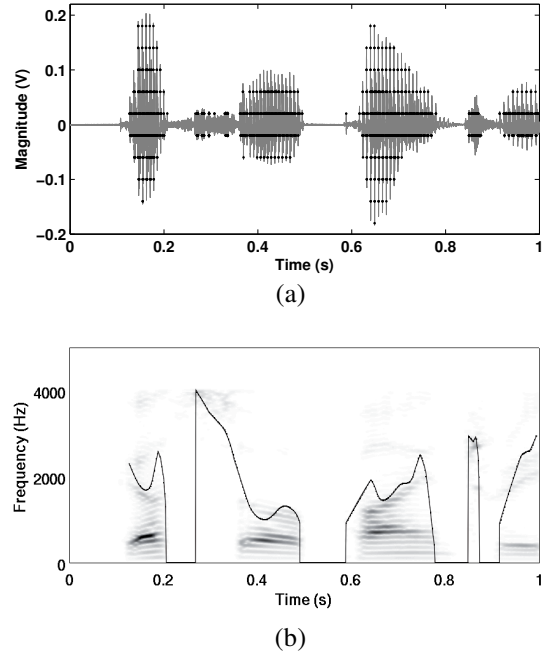


Figure 3: Fragment of the test speech sentence (samples as black points) (a), and its STFT (black line shows the estimated time-varying maximum frequency) (b).

kHz). The signal had been low-pass filtered with a cut-off frequency of 4 kHz, and interpolated by sinc functions to obtain level-crossing samples. Using 10 quantization levels 3301 samples were obtained during 3.8 seconds of the test phrase (mean sampling rate is about 870 samples per second). Uniform sampling at rate of 8 kHz provides 30400 samples. The waveform and the samples captured by LCS are illustrated in Fig. 3a. The time-frequency representation of the signal obtained by STFT is shown in Fig. 3b. The black bold line represents the instantaneous maximum frequency $\hat{f}_{max}(t)$ of the signal estimated according to (21). From the figure follows that the bandwidth of reconstruction filter will vary in the spectral range up to 4 kHz.

After the estimation of $\hat{f}_{max}(t)$ the calculation of k_n according to (11) follows. Only 20% of the coefficients obtained lie inside the allowed interval limits of $[0, 1]$, while the rest 80% are limited by (12). To verify the condition (14) the uniform sampling set $\{t_u\}$ of 64 kHz is chosen. In total, 30% of reconstructed samples $\hat{s}(t_u)$ calculated by (13) do not satisfy (14) and reconstruction error $\sqrt{\frac{1}{U} \sum_{u=0}^{U-1} (s(t_u) - \hat{s}(t_u))^2}$ is 22 mV. However, after $10N$ repetitions of steps (13), (14) and (15) the number is reduced to 8% and the error becomes 15 mV. The fragment of reconstructed speech is shown in Fig. 4a as a solid line, while the dashed line represents the original signal. The error signals $|s(t_u) - \hat{s}(t_u)|$ before and after iterative adjustment of coefficients are shown in Fig. 4b as gray and black solid lines. It can be noticed that the amplitude of the error signal is decreased and does not exceed the value of 40 mV, which is the distance between two quantization levels. The average simulation time used by an ordinary personal computer (CPU frequency 2.66 GHz) for reconstructing the signal is 10 times larger than the length of

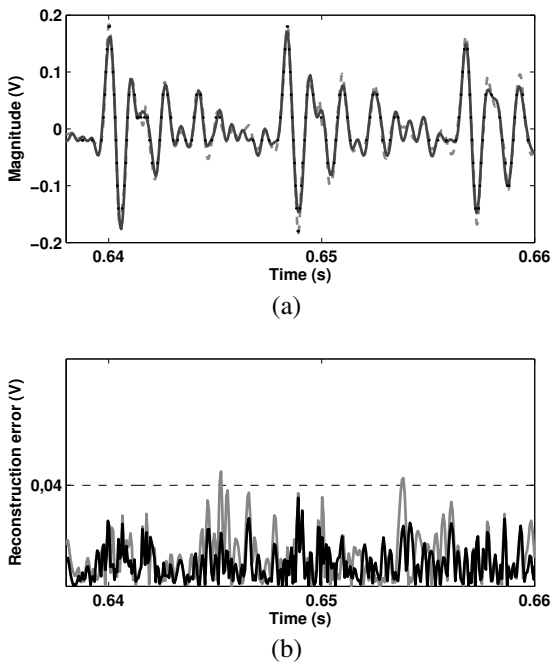


Figure 4: Reconstructed speech signal as black solid line (a), and reconstruction error signals before (gray solid line) and after (black solid line) iterative update of coefficients.

the signal and most of the time (up to 80%) is taken up by iterative adjustment of coefficients.

The reconstruction result improves as the number of quantization levels increases providing more level-crossing samples. If there are 20 quantization levels, then during 3.8 seconds of the test phrase 8509 level-crossing samples are obtained. When $\hat{f}_{max}(t)$ is estimated the calculation of k_n follows. Now 70% of the coefficients obtained by (11) are limited according to (12). In total, 31% of reconstructed samples $\hat{s}(t_n)$ do not satisfy the condition (14) and the reconstruction error is 10 mV. After 10N repetitions of steps (13), (14) and (15) the number is reduced to 9% and the error becomes 6.9 mV.

6. CONCLUSIONS

The proposed approach for non-stationary signal processing uses signal dependent techniques: level crossing sampling for data acquisition and applying of time-varying bandwidth filter for signal reconstruction. The information carried by level-crossing samples is employed in two ways – time instants of samples are used to estimate the instantaneous maximum frequency of the signal, while the amplitude values of samples are used in reconstruction algorithm. The reconstruction procedure is based on solving a least squares problem to find the new samples of the signal at time instants, which are determined by evaluated instantaneous maximum frequency. The adjustment of new sampling values follows by verifying if the reconstructed signal lies between corresponding quantization levels.

Speech signal processing is demonstrated as one of the application areas. Simulation results show advantages of proposed method, which are related to the exclusion of pauses

and disfluencies from processing before A/D conversion as well as to the possibility of decrease in sampling density. In case of 10 quantization levels audio perception remains good, while the number of samples is reduced 9 times in comparison with standard uniform processing.

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