Adaptive-Rate Transmission Schemes for Two-hop Multiple Access Relay Networks

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***Abstract*—In this paper we present and analyze link adaptation schemes that maximize the average throughput for two-hop relay network with single and multiple source terminals. The relay terminals are constrained by half-duplex assumption and a limit on the maximum total transmit power. We present two practical link adaptation schemes, *asymmetric rate transmission* (ART) for single-source and *layered space-time ART* (LST-ART) for multiple-source scenario. ART adapts the modulation modes and the transmission duration in the two hops separately such that the throughput is maximized. For ease of implementation the transmission duration of the hops are selected from a pre-defined look-up table (LUT). LST-ART allows for simultaneous transmission from the source terminals to the relay and from the relay to the destination through spatial multiplexing (SM) and it also adapts the transmission duration for each hop based on the combined data rate than can be achieved in that hop. Analytical expressions are derived for average spectral efficiency for both single and multiple source scenarios. From the performance analysis, ART is shown to offer higher throughput compared to that of symmetric rate transmission with fixed hop duration by a gap of 2dB between their achievable spectral efficiency.**

I. INTRODUCTION

Future wireless systems require higher data rate services and better coverage. This necessitates for fundamental enhancements in the systems including advanced transmission techniques, multi-antenna technologies and modifications in the network architecture. Future networks also require integration of variety of networks including co-operative systems, multi-hop networks, hybrid wireless networks, cognitive radio etc.,.

The multi-hop networks with relaying nodes in between helps to combat shadowing and allows for radio range extensions in larger cells and shorter communication links with higher capacity due to reduced path loss. Also transmission between two end terminals through direct and multi-hop transmissions allows for additional spatial diversity at the receiver.

The systems under investigations are (1) A single source two-hop relay network and (2) Multiple source multiple access two-hop relay network. The relay terminals use decode and forward (DF) based relaying. The throughput is maximized for a fixed quality of service (QoS) by allowing the nodes to adapt the modulation modes to match with the variations in the link conditions. The presence of multiple relays between the source and the destination terminal allows for the relay selection which further enhances the link quality in addition to the data rate adaptations. It is assumed that the nodes have a constraint on the maximum transmit power.

Rate adaptation allows users to choose the modulation modes based on the current state of the transmission link [1]. A practical example is the widely popular IEEE 802.11 systems, which allows the nodes to operate at multiple data rates [2]. Adaptive modulation for multi-hop and co-operative systems has been studied recently in [4], [5], [8]. In [4], the throughput performances of different relay-assisted transmission schemes with adaptive modulation were compared. The focus was on the rate adaptations to maximize the throughput without taking QoS and delays into considerations, hence desgined to allow higher modulation modes to be adapted at the cost of larger number of re-transmissions thus longer delays due to a higher bit error rate (BER). Our work differs from [4] in two aspects: First, a practical scheme that adapts the modulation modes in the hops independently and selects asymmetric transmission duration that maximizes the throughput for single and two source relay networks. Second, the adaptation of modulation modes in the individual links is carried out with the knowledge of BER requirements, hence avoiding retransmissions.

We present two link adaptation schemes: (1) *asymmetric rate transmission* (ART) for a source and (2) *Layered space-time ART* (LST-ART) for multiple source network setup with relay terminals having single and multiple antennas. Unlike conventional adaptive modulation schemes that adapt the modulation mode supported by the weakest of the two hops, the ART adapts the modulation modes in the two hops separately. For practical implementation to suit the modulations of the hops, the transmission duration of each hop is selected from a pre-defined look-up table (LUT), which maps different pairs of modulation modes to the respective transmission durations. With fixed duration for the two-hop transmission, ART introduces a flexibility in adapting the transmission duration of the hops, such that the average throughput is maximized. With multiple relays, the relay that jointly maximizes the sum rate achieved in the two hops is selected.

LST-ART enables simultaneous transmission through spatial multiplexing (SM) from the virtual MIMO channel present between the multiple source terminals and the multiple antennas at the relay terminal in the first hop and between the multiple antennas in the relay and destination terminals in the second hop. LST-ART adapts the modulation modes for each user (or layer) based on the post-detection SNR achieved at the receiver such that the combined throughput is maximized. Subsequently, based on the throughput achieved in the source to relay links and in the relay to the destination link, it carries out the ART procedure to select the transmission duration of the two hops. We will demonstrate through analysis and numerical simulations, the average spectral efficiency achieved by AM, ART and LST-ART schemes for a two-hop relay network. Due to space limitations, we restrict the analysis to the network configurations that serves to the problem on hand.

The rest of the paper is organized as follows. In section II, we discuss the system model. Section III presents the various link adaptation schemes for single and multiple source two-hop networks. In Section IV, we show the numerical results on the throughput performance. Section V concludes our work.

II. SYSTEM MODEL

We consider two-hop multiple access relay network with multiple source $\{\mathrm{S}\}$, multiple relay $\{\mathrm{R}\}$ and destination terminal D. The links exhibit path loss and i.i.d. flat Rayleigh

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fading. The instantaneous received SNR, γ=$\bar{\gamma}|h|^2$ is exponentially distributed with average SNR $\bar{\gamma}$, which depends on the transmit power P_i, distance d, pathloss exponent α, channel power gain $|h|^2$ and noise variance N_0, given by $\bar{\gamma}$=$\frac{P_i}{2N_0}\frac{1}{d^\alpha}$. The instantaneous received SNR in S→R and R→D links are given by $\gamma_{\rm SR}$ and $\gamma_{\rm RD}$ respectively. The additive white noise is distributed as $\mathbb{CN}(0, \sigma^2)$. The slowly time-varying flat fading channels are assumed constant for two consecutive phases.

All terminals operate in a half-duplex mode. Relay terminals employ DF based protocol [10], where signals received from the source terminal are decoded and then re-encoded before transmission to the destination. It is assumed that the relays have full local channel state information (CSI) (backward channels from the sources and the forward channel to the destination). Thus the transmission scheme with a relay terminal has the CSI to perform rate adaptation, and the transmission scheme with M relays have the CSI to perform opportunistic relay selection (RS) and rate adaptation. The destination terminal is assumed to have a perfect CSI of the R→D link. The total transmit power at the relay terminals is limited to P for a fair comparison among different schemes.

Rate adaptation is carried out with the discrete-rate M-QAM signal constellation on a frame by frame basis by splitting the SNR range into $K+1$ intervals, $[\gamma_k, \gamma_{k+1})_{(k=0,\ldots,K)}$, where $\gamma_0 = 0$, $M_k = 2^k$ and K is the number of M-QAM constellations. Thus, the instantaneous SNR in a link, γ is quantized into one of those $K+1$ levels. The SNR intervals are calculated based on the target bit error rate (BER) for the above modulation modes. The BER of coherent M-QAM with two dimensional gray coding and equally likely symbols over the AWGN channel is well approximated by [6].

$$P_b(k, \gamma) \approx 0.2\exp(-g_k\gamma),\ k = 1, \ldots, K \qquad (1)$$

where γ=P/N_0, g_k is a constellation dependent constant, equal to $1.5/(M_k{-}1)$ for square M-QAM and to $6/(5M_k{-}4)$ for rectangular M-QAM. The selected mode corresponds to the maximum achievable rate for which the BER can be assured.

A. Two phase communication

Communication between S and D is carried out through R in two phases (or one frame), from S to R in phase I and from R to D in phase II. The rate adaptation for both the links is done by the relay terminal. The adapted modes are informed to the source and destination terminal using feedback channels in each frame. The two phase transmission is given below. A block of $N_{\rm SR}$ symbols $\mathbf{x}_1$ with k_1 bps is transmitted from S to R in phase I. The received signal at R is given as

$$\mathbf{y}_{\rm R} = \sqrt{P_{\rm S}}\ h_{\rm SR}\mathbf{x}_1 + \mathbf{w}_{\rm R} \qquad (2)$$

where $\mathbf{y}_{\rm R}$, and $\mathbf{w}_{\rm R}$ are of size $N_{\rm SR}{\times}1$. R decodes the received signal and re-encodes them as a block of $N_{\rm RD}$ symbols with k_2 bps before forwarding it in phase II. D thus receives

$$\mathbf{y}_{\rm D} = \sqrt{P_{\rm R}}\ h_{\rm RD}\hat{\mathbf{x}}_1 + \mathbf{w}_{\rm D} \qquad (3)$$

where $\mathbf{y}_{\rm D}$, $\hat{\mathbf{x}}_1$ and $\mathbf{w}_{\rm D}$ are of size $N_{\rm RD}{\times}1$. The transmit powers, $P_{\rm S}$ and $P_{\rm R}$ are set to $P/2$. Block length is varied according to the modulation selected in each link, such that the total transmitted bits in two hops remains the same.

B. Symmetric rate transmission

For the case of symmetric rate transmission (SRT), the transmission duration of hops are fixed, $N_{\rm SR}$=$N_{\rm RD}$=N. The data rate, $T(n)$ is adapted based on the value of $\gamma_{\min}$, where $\gamma_{\min} = \min\{\gamma_{\rm SR}, \gamma_{\rm RD}\}$. The effective data rate achieved in S→R→D link is based on the modulation mode supported by the weakest of the two links S→R and R→D. In other words, rate adaptation for symmetric transmission incurs a loss of data rate when observed from the point of the stronger link.

C. Asymmetric rate transmission

The proposed ART provides a solution for the above problem of loss of data rate arising from the inequality of the link qualities and the fixed transmission duration. Instead of selecting the minimum achievable rate in the combined link, ART adapts the modulation modes, $M_{\rm SR}$ and $M_{\rm RD}$ in S→R and R→D links separately. Based on the modulation modes selected, the transmission interval is varied as given by

$$N_{\rm SR} = \left(\frac{k_2}{k_1 + k_2}\right)2N \ \text{ and } \ N_{\rm RD} = \left(\frac{k_1}{k_1 + k_2}\right)2N \qquad (4)$$

where k_1=log$_2(M_{\rm SR})$, k_2=log$_2(M_{\rm RD})$. When $M_{\rm SR}$=$M_{\rm RD}$, ART is as same as SRT ($N_{\rm SR}$=$N_{\rm RD}$). When $M_{\rm SR}$<$M_{\rm RD}$, the duration of phase I is increased and is compensated with a reduced duration in phase II ($N_{\rm SR}$>$N_{\rm RD}$) and vice versa for $M_{\rm SR}$>$M_{\rm RD}$. For e.g., assume $2N$=60 and the modulation modes selected based on the individual link SNRs are $M_{\rm SR}$=16 and $M_{\rm RD}$=2. The effective data rate achieved by SRT for the frame will be $2N\log_2(\min\{\rm M_{SR}, M_{RD}\})\frac{1}{2}$=30 bits. In the case of ART, $N_{\rm SR}$ and $N_{\rm RD}$ are adapted based on $M_{\rm SR}$ and $M_{\rm RD}$ to 12 and 48 symbols respectively. The data rate achieved will be equal to $N_{\rm SR}\log_2(\rm M_{SR})$=$N_{\rm RD}\log_2(\rm M_{RD})$ = 48 bits and that is 60% improvement over SRT. Actually, the performance of ART is more effective compared to SRT when the difference between the link SNRs $\gamma_{\rm SR}$ and $\gamma_{\rm RD}$ is large. Table. I shows an example LUT for two-hop transmission having five modulation modes (L) and a total frame duration ($2N$) of 60 symbols.

TABLE I
A SAMPLE ART LOOK UP TABLE: $2N = 60$ AND $L = 5$

$M_{\rm SR}$	$M_{\rm RD}$	$N_{\rm SR}$	$N_{\rm RD}$	$M_{\rm SR}$	$M_{\rm RD}$	$N_{\rm SR}$	$N_{\rm RD}$
2	4	40	20	4	16	40	20
2	8	45	15	4	32	40	16
2	16	48	12	8	16	32	24
2	32	50	10	8	32	35	21
4	8	36	24	16	32	30	24

D. Implementation Issues

It is assumed that all the terminals have the knowledge of the LUT used. The link layer at the relay terminal undertakes a two step process. First, the relay acquires the CSI for the forward and backward channels. In the second step, the relay terminal broadcasts the LUT index (which identifies the pair of modulation modes selected) to the source and the destination terminal. A finite rate feedback of L bits is used as the LUT index. It is assumed that the feedback channels are error free.

E. Performance metrics

We consider two metrics, namely the average spectral efficiency (ASE) and the average bit error rate (ABER) to assess the performance of the discrete-rate adaptive M-QAM used. ASE using discrete-rate adaptive M-QAM is the sum of information rates of each modulation level weighted by the probability a_n that the received SNR falls within the corresponding SNR interval.

$$T = \sum_{k=1}^{K} T_k = \sum_{k=1}^{K} k \cdot a_k \qquad (5)$$

where $a_k = \int_{\gamma_k}^{\gamma_{k+1}} p_\gamma(\gamma) d\gamma = \left[\mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}}\right)\right]$ and $\mathcal{P}_m(x)$ designates the Poisson distribution: $\mathcal{P}_m(x) = \sum_{l=0}^{m-1} \frac{x^l}{l!} e^{-x}$. ABER is the ratio of the average number of bits received in error over the total average number of transmitted bits.

$$\langle BER \rangle = \frac{\sum_k^K T_k \cdot \langle BER \rangle_k}{\sum_k^K T_k} = \sum_k^K \frac{k \cdot a_k}{\sum_k^K k \cdot a_k} \langle BER \rangle_k \quad (6)$$

where $\langle BER \rangle_k = \int_{\gamma_k}^{\gamma_{k+1}} P_b(k, \gamma) p_\gamma(\gamma) d\gamma$.

III. TRANSMISSION SCHEMES

In this section, transmission schemes for a single source and two source terminals with link adaptations are presented.

A. Single Source - Single Relay

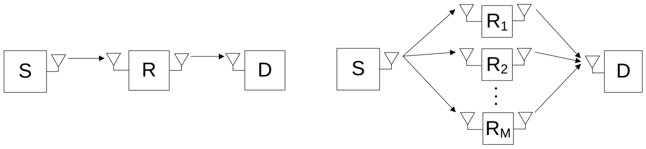

Fig. 1. A source with single-relay and multiple-relay network configuration

Consider the two system setup with a source, single or M relays and a destination terminal as shown in Fig. (1).

1) Symmetric rate transmission: Since the transmission interval is fixed for both phases to be N, the total transmitted bits in two hops must be equal. The relay terminal adapts the overall modulation mode for the two-hop transmission from the value of γ_{AM}, where $\gamma_{\text{AM}} = \min\{\gamma_{\text{SR}}, \gamma_{\text{RD}}\}$. Applying order statistics, the pdf of γ_{AM} can be calculated to be

$$p_{\gamma_{\text{AM}}} = 2 \cdot [1 - P_\gamma(\gamma)] \cdot p_\gamma(\gamma) = \frac{1}{\bar{\gamma}/2} e^{-\frac{\gamma}{\bar{\gamma}/2}} \quad (7)$$

substituting (7) into (5), the ASE is given by,

$$T_{\text{AM}} = \sum_{k=1}^{K} \frac{1}{2} \cdot k \cdot a_k \quad (8)$$

where $a_k = \left[\mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}/2}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}/2}\right)\right]$. Similarly, substituting (7) into (6), the ABER is then given by,

$$\langle BER \rangle_k = \int_{\gamma_k}^{\gamma_{k+1}} 0.2 \exp(-g_k \gamma) \cdot \frac{1}{\bar{\gamma}/2} e^{-\frac{\gamma}{\bar{\gamma}/2}} d\gamma \quad (9)$$

$$= 0.2 \ \frac{1}{b_k \bar{\gamma}/2} \cdot \left[\mathcal{P}_1\big(b_k \gamma_k\big) - \mathcal{P}_1\big(b_k \gamma_{k+1}\big)\right] \quad (10)$$

where $b_k = \frac{1}{\bar{\gamma}} + g_k, \ k = \{1, \ldots, K\}$.

2) Asymmetric rate transmission: For ART mode of operation, the relay terminal independently adapts the modulation modes M_{SR} and M_{RD} based on γ_{SR} and γ_{RD} respectively. Subsequently, using M_{SR} and M_{RD}, the relay terminal selects the block durations N_{SR} and N_{RD} for transmission in S→D and R→D links respectively from a pre-defined LUT. Since, ART independently adapts the rate in each hop, the pdf of γ_{ART} is given by $p_{\gamma_{\text{ART}}} = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}$. The ASE can be calculated as

$$T_{\text{ART}} = \sum_{k=1}^{K} a_k \left[\sum_{j=1}^{K} a_j \cdot \left(\frac{j}{k+j}\right) k \right] \quad (11)$$

where $a_{k|j} = \left[\mathcal{P}_1\left(\frac{\gamma_{k|j}}{\bar{\gamma}}\right) - \mathcal{P}_1\left(\frac{\gamma_{k|j+1}}{\bar{\gamma}}\right)\right]$, $k|j$ is k or j ABER of ART will be same as that of a direct link transmission as the channel power and the modulation schemes in the individual links vary independently. ABER of ART will be equal to that of the ABER of S→R or R→D link.

$$\langle BER \rangle_k = \int_{\gamma_k}^{\gamma_{k+1}} 0.2 \exp(-g_k \gamma) \cdot \frac{1}{\bar{\gamma}} \exp\Big(-\frac{\gamma}{\bar{\gamma}}\Big) d\gamma \quad (12)$$

$$= 0.2 \ \frac{1}{b_k \bar{\gamma}} \cdot \left[\mathcal{P}_1\big(b_k \gamma_k\big) - \mathcal{P}_1\big(b_k \gamma_{k+1}\big)\right] \quad (13)$$

B. Single Source - Multiple Relay

Consider the system setup in Fig. (1) with M relay terminals ($\text{R}_i, \ i{=}[1 \ldots M]$). Define $A = [\text{R}_1 \ldots \text{R}_M]$. We consider an opportunistic relaying scheme by selecting the relay ($\text{R}_{\text{best}} = \text{R}_i{:}\arg\max_i(\min(\gamma_{\text{SR}_i}, \gamma_{\text{R}_i\text{D}}))$, $\text{R}_i \in A$) that has the best channel gain with the S and the D. The signalling mechanism that enables the best relay to inform the source and the destination is given in [11]. The selected relay R_{best} then participates in the two-hop communication with rate adaptation in S→R_{best}→D link using both the SRT and ART modes of operation as given above for the single relay scenario.

1) Symmetric rate transmission: Applying order statistics twice, first to select the weakest of two hops in each link followed by selecting the best of M links i.e., $\gamma_{\text{AM}} = \arg\max_i(\min(\gamma_{\text{SR}_i}, \gamma_{\text{R}_i\text{D}}))$. The pdf of γ_{AM} is given by

$$p_{\gamma_{\text{AM}}} = \frac{M!}{(M-1)!} [\bar{P}_\gamma(\gamma)]^{M-1} \bar{p}_\gamma(\gamma) \quad (14)$$

where $\bar{p} = \frac{1}{\bar{\gamma}/2} e^{-\frac{\gamma}{\bar{\gamma}/2}}$ from (7). For e.g., assuming $M = 3$,

$$p_{\gamma_{\text{AM}}}(\gamma_{\text{AM}}) = \frac{3}{\bar{\gamma}/2} e^{-\frac{\gamma}{\bar{\gamma}/2}} - \frac{3}{\bar{\gamma}/4} e^{-\frac{\gamma}{\bar{\gamma}/4}} + \frac{1}{\bar{\gamma}/6} e^{-\frac{\gamma}{\bar{\gamma}/6}} \quad (15)$$

ASE is given by $T_{\text{AM}} = \sum_{k=1}^{K} \frac{k \cdot a_k}{2}$, where a_k is given by (16).

2) Asymmetric rate transmission: ART is carried out in the S→R_{best}→D link similar to that given for single relay scenario given before. The ASE can be calculated as in (11), with a_k and a_j given in (17). From the ASE of ART and SRT presented in the previous subsection and from the numerical simulations it is concluded that the throughput performance of ART is better than that of SRT. Hence, only ART is considered for the multiple source scenario presented in the next subsection.

C. Multiple Source - Multiple Antenna Relay

Here we address the rate adaptation for a system setup with L source terminals simultaneously communicating with the destination through the relay terminal as shown in Fig. (2).

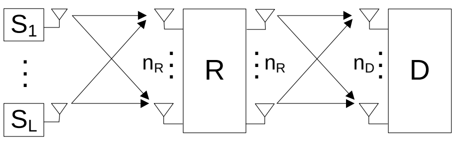

Fig. 2. Single source network configuration.

Let n_{R} and n_{D} be the number of antennas at the relay and the destination terminals respectively. Here it is assumed that $n_{\text{D}} = n_{\text{R}} = L$. L independent input data streams are transmitted in parallel from L single antenna source terminals to the relay terminal in the first hop as in V-BLAST. At the relay, the antennas receive the data streams, which are mixed and superimposed by noise. By applying optimal ordering, and

$$a_k = 3\left[\mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}/2}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}/2}\right)\right] - 3\left[\mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}/4}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}/4}\right)\right] + \left[\mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}/6}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}/6}\right)\right] \tag{16}$$

$$a_k, a_j = 3\left[\mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}}\right)\right] - 3\left[\mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}/2}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}/2}\right)\right] + \left[\mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}/3}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}/3}\right)\right] \tag{17}$$

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sequential interference nulling and cancellation, the relay separates the data from each source. The relay adapts the modulation modes for each source (or layer) based on the post-detection SNR achieved at the receiver with ZF-V-BLAST such that the combined throughput is maximized.

A similar transmission is carried out in the second hop between the relay and the destination terminal. Subsequently, based on the combined throughput (sum of throughput achieved in L layers) that can be achieved in the source to relay links in the first hop and the combined throughput that can be achieved in the relay to the destination link in the second hop, LS-ART carries out the ART procedure to adapt the transmission duration of the two hops.

1) Channel model: We consider the communication between the L source terminals and the relay terminal in the first hop. Let $\mathbf{H}_{\text{SR}}$ be the $L \times L$ channel matrix whose entries are i.i.d. and circularly symmetric Gaussian random variable, i.e., $h_{ij} \sim \mathbb{CN}(0, 1)$. We also assume that the transmitted data streams are independent and have uniform transmit power, i.e., $E[\mathbf{x}\mathbf{x}^*] = P\mathbf{I}$, where $E[.]$ stands for the expected value. The sampled baseband signal is given by

$$\mathbf{y}_{\text{R}} = \mathbf{H}_{\text{SR}}\mathbf{x} + \mathbf{z}_{\text{R}} \tag{18}$$

where $\mathbf{y}_{\text{R}} \in \mathbb{C}^{L\times 1}$ is the received signal, $\mathbf{x} \in \mathbb{C}^{L\times 1}$ is the transmitted signal with each elements corresponds to the signal from each source and $\mathbf{z}_{\text{R}} \in \mathbb{C}^{L\times 1}$ is the received noise.

The ZF-V-BLAST scheme can be represented by the QR decomposition $\mathbf{H}_{\text{SR}} = \mathbf{QR}$, where $\mathbf{R}$ is a $L \times L$ upper triangular matrix and $\mathbf{Q}$ is a $L \times L$ orthonormal matrix whose columns form ZF interference nulling vectors. Multiplying by $\mathbf{Q}^*$ on both sides of (18) yields,

$$\tilde{\mathbf{y}}_{\text{R}} = \mathbf{Rx} + \tilde{\mathbf{z}}_{\text{R}} \tag{19}$$

For details on the performance analysis of V-BLAST, please see [7]. The post-processing SNR of l^{th} layer is given by

$$\gamma_l^{\text{ZF}} = r_{l,l}^2\bar{\gamma}, \quad l = 1, \ldots, L \tag{20}$$

where $r_{l,l} \sim \chi^2_{(L-l+1)}$. The pdf of γ_l^{ZF} is given by

$$p_{\gamma_l^{\text{ZF}}} = \frac{\gamma^{L-l}}{\Gamma(L-l+1)}\left(\frac{1}{\bar{\gamma}}\right)^L e^{-\frac{\gamma}{\bar{\gamma}}}, \quad \gamma \geq 0 \tag{21}$$

For e.g., assuming $L = 2$, the pdf of two layers are given by

$$p_{\gamma_1^{\text{ZF}}} = \gamma\left(\frac{1}{\bar{\gamma}}\right)^2 e^{-\frac{\gamma}{\bar{\gamma}}}, \quad \text{and} \quad p_{\gamma_2^{\text{ZF}}} = \frac{1}{\bar{\gamma}}e^{-\frac{\gamma}{\bar{\gamma}}}, \quad \gamma \geq 0 \tag{22}$$

The transmission duration of each hop is given as

$$N_{\text{SR}} = \left(\frac{j_1 + j_2}{k_1 + k_2 + j_1 + j_2}\right)2N \quad \text{and} \quad N_{\text{RD}} = 2N - N_{\text{SR}}$$

The ASE of LS-ART is then given by

$$T_{\text{LS-ART}} = \sum_{k_1=1}^{K} a_{k_1}\left[\sum_{k_2=1}^{K} a_{k_2}\left[\sum_{j_1=1}^{K} a_{j_1}\left[\sum_{j_2=1}^{K} a_{j_2}\lambda\right]\right]\right] \tag{23}$$

where $\lambda = \left(\frac{j_1 + j_2}{k_1 + k_2 + j_1 + j_2}\right)(k_1 + k_2)$

$$a_{k_1} = \mathcal{P}_2\left(\frac{\gamma_k}{\bar{\gamma}}\right) - \mathcal{P}_2\left(\frac{\gamma_{k+1}}{\bar{\gamma}}\right), \quad a_{k_2} = \mathcal{P}_2\left(\frac{\gamma_k}{\bar{\gamma}/2}\right) - \mathcal{P}_2\left(\frac{\gamma_{k+1}}{\bar{\gamma}/2}\right)$$

$$a_{j_1} = \mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}}\right), \quad a_{j_2} = \mathcal{P}_1\left(\frac{\gamma_k}{\bar{\gamma}/2}\right) - \mathcal{P}_1\left(\frac{\gamma_{k+1}}{\bar{\gamma}/2}\right)$$

IV. SYSTEM PERFORMANCE

We show the simulation results of SRT, and the proposed ART and LST-ART schemes for the system setups in Fig. (1), (2). We present the ASE (bits/s/Hz) achieved for different values of SNR (P_{S}/N_0). Rate adaptations are done for a target BER of 10^{-3} and 10^{-6}. Since two-hop network involves two phase transmission which takes twice the duration for transmitting the same amount of information compared to that of direct transmission, the ASE achieved are halved. The path loss exponent, α is set to 3. No power control is assumed at the source or relay terminals. The length of a frame, $2N$ is set to 60. The number of M-QAM constellation, K is set to 5. For all the schemes, the total transmit power at the relay terminal is fixed to P to maintain a fairness in comparison. We have presented the ASE for all the schemes discussed based on the derived analytical expressions.

A. Single Source with Single Antenna Relays

Fig. 3 shows the ASE vs. SNR for a network with a source and one or more relay terminals, all having single antenna i.e., $L = n_R = n_D = 1$ and $M = 1, 3$. The link adaptations are carried out using SRT and ART for $P_b = 10^{-3}$ and 10^{-6}. We present here a comparison of ASE obtained through analytical expressions and from the monte-carlo simulations.

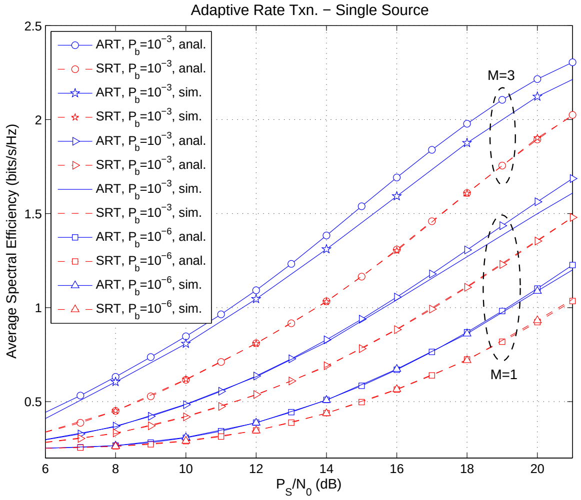

Fig. 3. ASE of SRT and ART for single antenna relay and destination terminals for P_b=10^{-3} and P_b=10^{-6} (sim.-simulated, anal.-analytical).

We observe from Fig. 3, ART leads to higher throughput than SRT for all SNR's and this can be explained as follows. Using SRT, the throughput achieved in every frame depends on the modulation mode adapted for the weakest link (S→R or R→D). In other words, the strongest link reduces its modulation mode to match with that of the weakest link and hence SRT fails to fully utilize the strongest link. The loss of throughput increases with an increase in the difference between the average SNRs in S→R and R→D. On the other hand, ART first adapts the modes of S→R and R→D links separately and subsequently selects the transmission durations. It increases the transmission duration ($>N$) for the link with a lower SNR at the expense of the stronger link.

For the case of $M = 1$ and $P_b = 10^{-3}$, for the same ASE, ART has a SNR gain of 1.5 dB compared to that of SRT or for the same SNR, ART achieves 20% higher ASE. For the case of $P_b = 10^{-6}$, the higher BER target results in the selection of lower modulation modes. Hence, the performance gain of ART achieved is relatively lesser than that of $P_b = 10^{-3}$ for a moderate SNR range, but is similar for the high SNR range. For low SNR values, the link adaptation selects BPSK and hence it dominates the ASE performance of the schemes.

For $M \geq 1$, relay selection chooses the best 1 out of M links from S to D. Subsequently, link adaptation using SRT and ART is carried out on the S→R_{best} and R_{best}→D links. For the case of $M = 3$ and $P_b = 10^{-3}$, for the same ASE, ART has a SNR gain of around 2 dB compared to that of SRT or for the same SNR, ART achieves 30% higher ASE. For the same ASE, a SNR gain of around 4 dB can be achieved between relay selection and no selection for $M = 3$.

B. Multiple Source with a Multiple Antenna Relay

Fig. 4 shows the ASE vs. SNR for the network setting with a relay terminal (M=1) having multiple antennas ($n_R \geq 1$). The bottom curve in Fig. 4 is presented for the sake of comparison. The results are shown for two different system setups.

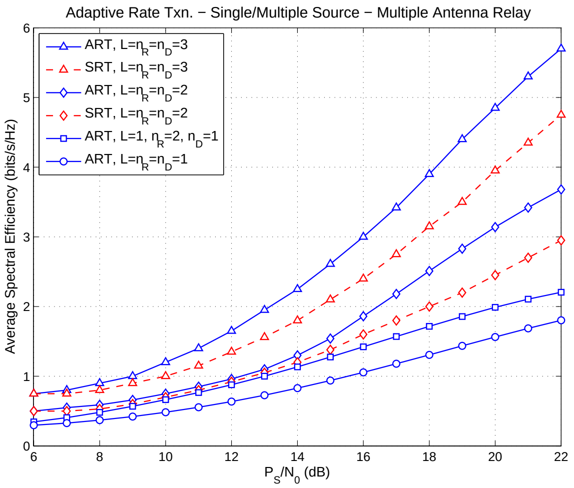

Fig. 4. ASE (analytical) of LST-ART for multiple source with $P_b = 10^{-3}$.

[L=1, n_R=2, n_D=1] corresponds to source and destination both having single antenna communicating through a relay having two antennas. With multiple antennas at the relay, the S→R link resembles a SIMO channel, hence the transmission in first hop can avail the receive diversity and the R→D link resembles that of a MISO channel, hence the transmission in second hop can avail the transmit diversity. With a constraint on the total transmit power at the relay, the ASE performance of this network setup is similar to that of a network which selects one of two relays each having single antenna (M=2 and n_R=1) as mentioned in the previous subsection.

System with [L=n_R=n_D] corresponds to L single antenna source terminals communicating to the destination through a relay terminal where both relay and destination have L antennas. S→R link corresponds to that of a virtual MIMO channel formed out of L source terminals and the R→D link corresponds to that of a standard MIMO channel. Link adaptation on L parallel transmissions in each hop is done using LST-ART for a $P_b = 10^{-3}$. From the analytical expression in (24) and from the results shown in Fig. 4, we can conclude that the ASE of the system with [L=n_R=n_D] scales proportionally to number of L participating source terminals.

Due to space limitations, we have restricted to ZF-V-BLAST for signal detection. Another common detection scheme is the MMSE-V-BLAST. It is given in [7], both the schemes have similar diversity gain, with MMSE-V-BLAST having a fixed SNR gap over the ZF-V-BLAST. But the fixed SNR gap would result in a fixed performance gap between the ASE achieved with each detection scheme and which will have a equal impact for both SRT and ART based rate adaption. Hence performance advantage of ART over SRT still holds.

V. CONCLUSION

We have presented two rate adaptation schemes, ART and LST-ART and provided analytic expressions to calculate the average spectral efficiency. The flexibility of ART in adapting the transmission duration of the each hop results in higher ASE compared to that with fixed transmission duration. Further gains are achieved with multiple relays using relay selection. LST-ART uses spatial multiplexing for multiple source and also adapts the transmission duration for each hop based on the combined data rate achieved in each hop. From the results, it is evident that the throughput achieved scales proportionally to the number of source terminals. From the results, we conclude that the proposed schemes achieve an increase in average throughput of around 20% for the network with single relay and around 30% for the network with relay selection. For a fixed average throughput the proposed scheme achieves a SNR gain around 2.0 dB. The proposed schemes will have further performance gain when there is an inequality between the average SNR of the links. One possibility could be due to the relay at an unequal distance from the source and destination.

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