M-ARY MUTUALLY ORTHOGONAL COMPLEMENTARY GOLD CODES

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ABSTRACT

A method to obtain large sets of periodic conjugate symmetric sequences with perfect periodic autocorrelation functions is presented. Each of these perfect sequences can be transformed into two real sequences which are perfectly orthogonal for any cyclic shift. In this way, mutually orthogonal complementary (MOC) sequences derived from bipolar Gold codes can be transformed into M-ary MOC Gold codes. Ternary MOC Gold codes are the simpler implementation case and exist in large number. These Ternary MOC Gold codes provide a greater immunity to multi-path interferences than other codes, like Hadamard codes, Orthogonal Gold codes, Gold codes, and Complementary Golay codes, when a periodic or aperiodic autocorrelation function is used for asynchronous bit detection.

1. INTRODUCTION

Ideally, direct sequence code division multiple access (DS-CDMA) sequences should have a perfect periodic autocorrelation function [1]-[4]. Some solutions may be found with complex sequences defined by some authors as Small or Large Alphabet Polyphase sequences [1], [5]-[7], Unimodular Perfect sequences [8], Phase Shift Pulse Codes [9], Perfect Root-of-Unity sequences [10], Bent Function sequences [11], or simply as Perfect sequences [12], [13].

A perfect sequence is, by definition, a complex sequence with perfect periodic autocorrelation function equal to the unit impulse function, $\delta(n)$. A variety of perfect sequences has been proposed in the literature [1]-[13]. The lower bound of the maximum absolute value of periodic cross-correlation (MaxCC) is a constant and equals $\sqrt{N}$ [7], [14], [15].

Complementary sequences have ideal autocorrelation and cross-correlation properties. It is for this reason that they are found in some applications, such as spectrometry, acoustics, radar, and CDMA communication systems, since early 1960. Golay introduced first the concept of complementary sequence pairs [16], when an ideal aperiodic autocorrelation function is required. Generalized complementary sets and mutual orthogonality between complementary sets have been introduced by Tseng and Liu [17]. Suehiro and Hatori defined complete complementary sets generated through N-shift cross-orthogonal sequences [18]. The Polyphase complementary sequences [19], [20] were studied by Sivaswamy and Budisin. The Biphase scalable complete complementary sets of sequences [21] were proposed by X. Hang and Y. Li.

Mutually orthogonal complementary sets are interesting for digital communications, offering important potential advantages over traditional CDMA codes [5], [22]. For example, they can be useful to eliminate the inter symbol interference (ISI) and multiple access interference (MAI) in a direct sequence (DS) CDMA system [23], [24]. Such communication system scheme results in an easily parallelized receiver architecture that may be useful in nonfading coherent channels, such as the optical fiber channel or the Rician wireless channel with a strong line-of-sight component. Pairs of mutually orthogonal polyphase complementary sequences have also been proposed for use in ultrasound imaging [25].

Usually, mutually orthogonal complementary (MOC) sequences found in the literature are defined through the aperiodic autocorrelation function, and most of them derive from the Golay sequences. However, we prefer to identify new MOC sequences through the periodic autocorrelation function, and to simulate these new sequences in both periodic and aperiodic cases. New pairs of MOC sequences can be derived from any complex periodic conjugate symmetric sequences. For example, our MOC sequences can be derived from perfect sequences of Gold codes, and can be seen as a special family of Generalized Mutually Orthogonal Complementary Signals [26]. Our MOC Gold codes may be integrated into a multicarrier code division multiple access (MC-CDMA) system. These systems are a promising choice for enhanced quality of service and, especially, for increased transmission rate [26].

The next section presents a mathematical property that can be used to find perfect sequences that are periodic conjugate symmetric sequences. These perfect sequences can be transformed into MOC sequences. In section III, we present some simulation results obtained with the periodic and aperiodic autocorrelation functions for Gold codes, orthogonal Gold codes, MOC Gold codes, M-ary MOC Gold codes, and Complementary Golay codes, in the presence of additive white Gaussian noise (AWGN) and Rayleigh flat fading. The main conclusions are gathered in section IV.

2. MUTUALLY ORTHOGONAL COMPLEMENTARY SEQUENCES

By definition, a pair of mutually orthogonal complementary sequences exists when the sum of the two periodic autocorrelation functions is equal to $\delta(n)$ and the periodic cross-correlation function is zero for any cyclic shift [17]. We consider the same MOC definition for both periodic and aperiodic cases.

Let $x(n)$, with $n = 0,1,2...,N-1$, be one of the *N* points of a periodic sequence x. Its discrete Fourier transform (DFT) and its inverse discrete Fourier transform (IDFT) [27] are defined as:

$$DFT\left[x(n)\right] = \sum_{n=0}^{N-1} x(n) W_N^{kn} , \quad (1)$$

$$IDFT\left[X(k)\right] = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-kn} . \quad (2)$$

For convenience of notation, W_N is defined as $W_N = \exp(-j2\pi/N)$, where $j = \sqrt{-1}$.

Using the DFT and IDFT, the periodic cross-correlation between sequences *x* and *y* may be defined as [14], [27]:

$$\begin{aligned} R_{xy}(n) &= \sum_{k=0}^{N-1} x(k)\, y^{*}\left[\mathrm{mod}(k+n,N)\right] \\ &= IDFT\left[DFT(x)\, DFT^{*}(y)\right] \end{aligned} \quad (3)$$

where n is an integer, the superscript * stands for the complex conjugate, and mod(*a,b*) is the remainder of *a* divided by *b*. A complex value $x(n)$ is equal to $x\left[\mathrm{mod}(n,N)\right]$ when *x* is a periodic sequence with period *N*.

When *x* = *y*, (3) is defined as the periodic autocorrelation function. A sequence *x* is called a perfect sequence if it has an ideal periodic autocorrelation function:

$$R_{xx}(n) = N\delta(n) = \begin{cases} N, & \mathrm{mod}(n,N)=0 \\ 0, & \mathrm{mod}(n,N)\neq 0 \end{cases} . \quad (4)$$

As it is well known, any constant amplitude sequence defined in the frequency domain corresponds to a perfect sequence in the time domain. In other words, it is possible to say that the sequence:

$$x_p(n) = IDFT\left[x(n)\right], \quad (5)$$

for $0 \le n \le N-1$, where n is an integer, is a perfect sequence if $|x(n)|$ is a constant value for all *N* discrete values of *n*.

Finally, we define a pair $\left\{\mathrm{Re}\left[x_{p,i}\right], \mathrm{Im}\left[x_{p,i}\right]\right\}$ of real sequences identified by the subscript *i*, with $0 < i \le Q$, where Q is an integer. An interesting pair *i* is when $\mathrm{Re}\left[x_{p,i}\right]$ and $\mathrm{Im}\left[x_{p,i}\right]$ are the real and imaginary parts of a complex perfect sequence $x_{p,i} = IDFT\left[x_i\right]$, with $x_i = \mathrm{Re}\left[x_i\right]$ and $|x_i|^2 = N$ for all *N* elements.

Property: The IDFT of each sequence $x_i \in \left\{x_0, x_1, ..., x_i, ..., x_{Q-2}, x_{Q-1}\right\}$ is a perfect periodic conjugate symmetric sequence if x_i is a real sequence and $|x_i(n)|$ is a constant value (*n, i* and *Q* are integers and $0 \le n \le N-1$). All complex sequences $IDFT\left[x_i(n)\right]$, of length *N*, can be decomposed into *Q* pairs of real sequences:

$$\mathrm{Re}\left[x_{p,i}(n)\right] = \mathrm{Re}\left\{IDFT\left[x_i(n)\right]\right\} \quad (6)$$

and

$$\mathrm{Im}\left[x_{p,i}(n)\right] = \mathrm{Im}\left\{IDFT\left[x_i(n)\right]\right\}, \quad (7)$$

which are orthogonal for any cyclic shift.

By using (4) and (5), it is possible to generate any perfect sequence x_p with length N whenever sequence *x* lies on a constant magnitude circle, for example $|x(n)| = \sqrt{N}$, for $0 \le n \le N-1$.

Using the property of a periodic conjugate symmetric sequence [26], it is easy to find that:

$$DFT\left[x_p(n)\right] = \mathrm{Re}\left[x(n)\right], \quad (8)$$

where $x_p(n) = IDFT\left\{\mathrm{Re}\left[x(n)\right]\right\}$, as given in (6) and (7).

The orthogonality of each pair of sequences (6) and (7) is confirmed by a null periodic cross correlation for any value of *n*:

$$R_{\mathrm{Re}\left[x_{p,i}\right],\mathrm{Im}\left[x_{p,i}\right]}(n) = 0, \quad 0 \le n \le N-1 . \quad (9)$$

When (4) is verified, the periodic autocorrelation $R_{x_p x_p}(n)$ is a real sequence, because:

$$\mathrm{Re}\left[R_{x_p,x_p}(n)\right] = R_{\mathrm{Re}\left[x_p\right],\mathrm{Re}\left[x_p\right]}(n) + R_{\mathrm{Im}\left[x_p\right],\mathrm{Im}\left[x_p\right]}(n) = N.\delta(n), \quad (10)$$

$$\mathrm{Im}\left[R_{x_p,x_p}(n)\right] = R_{\mathrm{Im}\left[x_p\right],\mathrm{Re}\left[x_p\right]}(n) - R_{\mathrm{Re}\left[x_p\right],\mathrm{Im}\left[x_p\right]}(n) = 0 . \quad (11)$$

All perfect sequences generated using the *Property* above are periodic conjugate symmetric sequences. This means, by definition, that $\mathrm{Re}\left[x_p(n)\right] = \mathrm{Re}\left[x_p(-n)\right]$ and $\mathrm{Im}\left[x_p(n)\right] = -\mathrm{Im}\left[x_p(-n)\right]$. Using these two expressions, it is possible to rewrite $R_{\mathrm{Re}\left[x_p\right],\mathrm{Im}\left[x_p\right]}(n)$ and find that:

$$R_{\mathrm{Re}\left[x_p\right],\mathrm{Im}\left[x_p\right]}(n) = -R_{\mathrm{Im}\left[x_p\right],\mathrm{Re}\left[x_p\right]}(n) . \quad (12)$$

Replacing (12) into (11), $R_{\mathrm{Re}\left[x_{p,i}\right],\mathrm{Im}\left[x_{p,i}\right]}(n) = 0$ is obtained and it is concluded that $\mathrm{Re}\left[x_{p,i}\right]$ and $\mathrm{Im}\left[x_{p,i}\right]$ (pairs of no null sequences) are orthogonal for any cyclic shift (all time-shift values $0 \le n \le N-1$).

All these good correlation properties have been confirmed by simulation, and some of the results obtained are discussed in the next section.

3. SIMULATION RESULTS

In order to evaluate their applicability in CDMA communication systems, mutually orthogonal complementary sets generated by applying an IDFT to $N+1$ bipolar Gold sequences, with length *N,* have been analyzed. Notice that only one of the $N+2$ sequences of the Gold set (one of the two *m*-sequences of the preferred pair) has been excluded. Good periodic and aperiodic autocorrelation properties (when $N = 127$) can be observed in figure 1 a) and b).

As expected, the maximum out-of-phase periodic autocorrelation value of 128 complex perfect sequences of the set {IDFT[128_Gold_Seq]} is a null value (figure 1 - a) when the *Time-shift* is not zero). However, the maximum out-of-phase aperiodic autocorrelation value of 128 complex perfect sequences of the set {IDFT[128_Gold_Seq]} is less than 18% of its maximum autocorrelation value (figure 1 - b) when the *Time-shift* is not zero). This means that the new MOC sequences derived from a subset of Gold codes (MOC Gold codes) may be a good choice for asynchronous DS-CDMA scenarios, when a periodic or aperiodic autocorrelation function is used for detection.

For electronic implementation in synchronous or asynchronous transmission scenarios, the MOC Gold codes should be converted into *M*-ary MOC Gold codes. The number of amplitude levels that should be used has been investigated, with the results presented in figure 2. This figure shows the MaxCC values normalized by the maximum autocorrelation value, for each pair of real sequences (6) and (7). M-ary MOC codes can be recorded in a read only memory (ROM), in order to simplify the electronic transmitter. The hardware of the CDMA receiver may also be simplified by using the same ROM and avoiding the use of a rake receiver (by using a single path receiver).

According to figure 2, an analog-to-digital converter (ADC) with a resolution higher than 1 bit must be used. For this reason, different *M*-ary MOC Gold codes have been simulated. In our following simulations, the bipolar MOC Gold codes (obtained with the signal function *Sgn*(.)) were discarded, based on the results included in Table I for set A.

For more than two amplitude levels, an ADC may be used to generate approximations of (6) and (7). Moreover, it has been observed that the aperiodic MaxCC value can be approximately four times lower than the values presented in figure 2, if only one pair is considered, i.e. if $Q = 1$.

Most of the DS-CDMA comparative performance studies reported in the literature are limited to scenarios with only two users or with an infinite number of users. For comparison purposes, we have chosen the same scenario considered in [28], where Gold codes (with $N=31$) were used in a multiuser interference environment, with AWGN and Rayleigh flat fading.

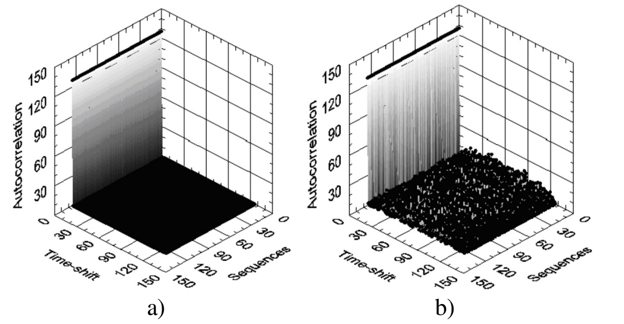

Figure 1 – a) Periodic autocorrelation functions of 128 complex perfect sequences of the set {IDFT[128_Gold_Seq]}, with null out-of-phase values. b) Aperiodic autocorrelation functions of 128 complex perfect sequences of the set {IDFT[128_Gold_Seq]}. The length of all sequences is 127.

Set	*Q*	In-phase MaxCC	Out-of-phase MaxCC
A	**256**	**40,2%**	**45,7%**
B	**256**	**16,5%**	**18,3%**
C	**128**	**0,787%**	**8,9%**
D	**128**	**$<10^{-13}$**	**$<10^{-13}$**

Table I - Simulation results for the normalized maximum absolute value of periodic cross-correlation (in phase "$n = 0$" and out-of-phase "$n \neq 0$") for 4 different sets of *Q* sequences with length $N = 127$ chips. The sets considered were:
A. {Sgn(Re[IDFT[Gold]]) ∪ Sgn(Im[IDFT[Gold]])};
B. {Re[IDFT[Gold]] ∪ Im[IDFT[Gold]]};
C. {IDFT[Gold]};
D. {Re[IDFT[Gold]], Im[IDFT[Gold]]} between the sequences of each pair defined by (6) and (7).

The received DS-CDMA signal, considering a Rayleigh flat fading channel and AWGN, may be expressed by:

$$y(t)=\sum_{r=1}^{Q} s_r(t).h(t)+n(t). \qquad (13)$$

where $s_r(t)$ is the signal of the user *r*, and $n(t)$ is the AWGN. As it is well known, Rayleigh fading [29], [30] is a multiplicative distortion. For a flat fading channel, $h(t)$ is modelled by a single tap with zero delay. Besides, $h(t)$ is a wide-sense stationary (WSS) complex Gaussian process, with zero-mean and unit variance, whose amplitude varies according to a Rayleigh probability density function. The complex Gaussian samples generated for $h(t)$ are shaped with a filter whose power spectral density function is given by:

$$S_{hh}(f)=\frac{1.5}{\pi f_d\sqrt{1-\left(\frac{f}{f_d}\right)^2}},\ |f|<f_d, \qquad (14)$$

where f_d is the Doppler frequency.

Our simulations were made with the best pair of each set. The code selection was made based on the maximum autocorrelation value of all codes (after the process of quantification of {IDFT[128_Gold_Seq]}). Figure 3 shows the Bit Error Rate (BER) results that we have obtained.

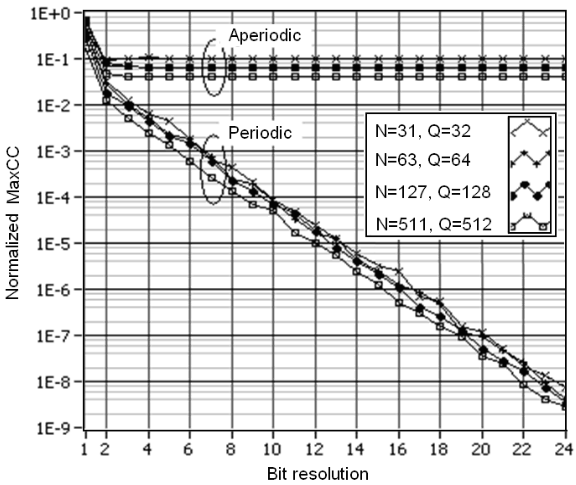

Figure 2 – Normalized periodic and aperiodic maximum absolute value of periodic cross-correlation between two sequences of each pair defined by (6) and (7).

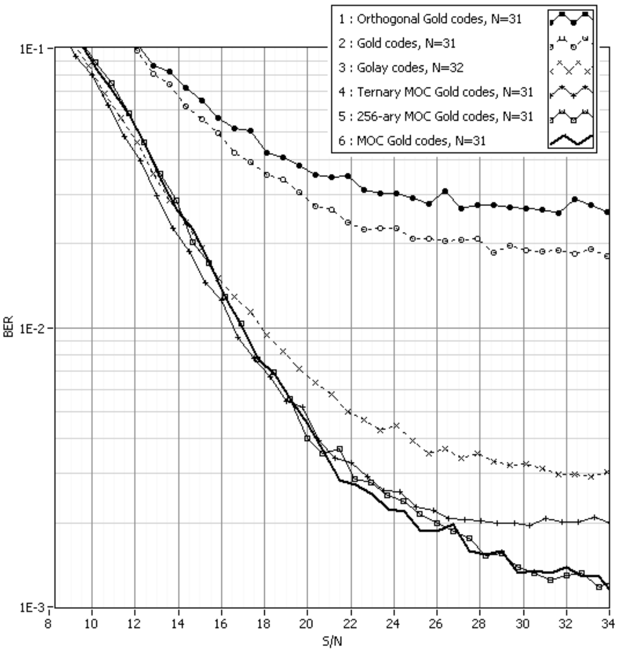

Figure 3 – Bit Error Rate (BER) versus Signal-to-Noise Ratio (*S/N*) with different sets of codes in a Rayleigh flat fading channel, for a two users asynchronous CDMA scenario. The detection was based on the maximum value of the periodic autocorrelation function.

A larger set of M-ary MOC Gold codes (union of different pairs) has been considered, for a DS-CDMA scenario with a Rayleigh flat fading channel. With this larger set, the BER performance of the M-ary MOC Gold codes is close to the performance with Gold codes. However, when only $N/2$ codes are considered, the M-ary MOC Gold codes are clearly better than Gold codes. Notice that M-ary MOC Gold codes have the same performance as MOC Gold codes, when $M \geq 2^8$ (ADC with 8 bits).

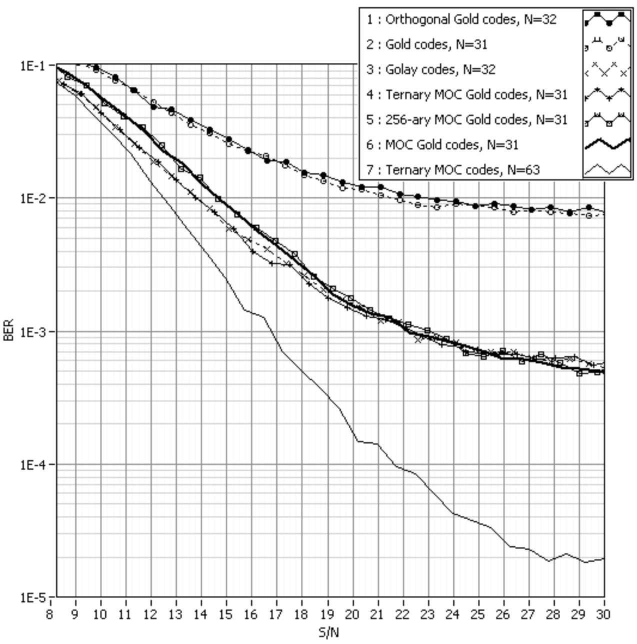

Figure 4 – Bit Error Rate (BER) versus Signal-to-Noise Ratio (*S/N*) with different sets of codes in a Rayleigh flat fading channel, for a two users asynchronous CDMA scenario. The detection was based on the maximum value of the aperiodic autocorrelation function.

All different pairs of codes (with equal average power) of figure 3 have been evaluated with a periodic autocorrelation function used for asynchronous detection (based on the maximum value). These new codes have also been evaluated with an aperiodic autocorrelation function [31], with clearly better BER results, as may be observed in figure 4. The new codes seem to be better than Orthogonal Gold codes, Gold codes, and Complementary Golay codes, when a periodic or aperiodic autocorrelation function is used for asynchronous detection. The worst BER performance results were obtained with Hadamard codes (not represented). All our selected codes are real codes. Therefore, we chose to ignore complex codes such as Chu, Frank, or Zadoff-Chu perfect sequences in our simulations (which were made only with real codes). We have also performed preliminary simulations of asynchronous CDMA using our ternary MOC codes and a 9-ary quadrature amplitude modulation (with coherent detection), when only MAI is considered. With this specific CDMA scenario (and considering $N = 31$), we found that our ternary MOC codes can be better than bipolar Gold codes (used with binary phase-shift keying and coherent detection).

4. CONCLUSION

We have shown that perfect periodic conjugate symmetric sequences can be obtained by applying an inverse discrete Fourier transform to any set of real bipolar sequences. Each of these perfect sequences can be transformed into pairs of mutually orthogonal real sequences (orthogonal for any cyclic shift value). Mutually orthogonal complementary sequences derived from bipolar Gold codes can be transformed into real *M*-ary codes that provide a greater immunity to multi-path interference than other real codes, such as Hadamard codes, Orthogonal Gold codes, Gold codes, and Complementary Golay codes. Good BER performance with *M*-ary MOC Gold codes ($M \geq 3$) has been found by simulation of asynchronous DS-CDMA scenarios with a Rayleigh flat fading channel and AWGN. Ternary MOC Gold codes should be a better choice than Complementary Golay codes, when the periodic or aperiodic autocorrelation function is used for detection.

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