3D SOUNDFIELD REPRODUCTION USING NON-SPHERICAL LOUDSPEAKER ARRAYS

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ABSTRACT

Three dimensional (3D) spatial soundfield reconstruction techniques typically need multiple loudspeakers placed on the surface of a sphere covering a desired region of reconstruction. Such array geometries have certain limitations that make them impractical in most of the intended applications. In this paper, we show how to reconstruct 3D soundfields using a set of circular arrays. The proposed method uses circular harmonics alongwith the underlying theory of wavefield propagation. Specifically, we use the properties of the associated Legendre functions and the spherical Hankel functions for loudspeaker placement. As an illustration, we design a third order spherical harmonic reconstruction system using 16 loudspeakers.

1. INTRODUCTION

Spatial soundfield reconstruction is an important problem whereby a listener is able to be immersed in a realistic, yet virtual, sound environment. Possible applications include complex supervisory control systems such as telecommunications and air traffic control systems, teleconferencing and telepresence applications, gaming industry, and auditory displays. Whilst there has been a deal of progress in designing practical two dimensional (2D) spatial soundfield reconstruction systems, however, extending them to operate well in 3D requires unrealistic loudspeaker array configurations. In this paper, we present a systematic way of designing a realistic loudspeaker array geometry and associated signal processing methodology to reconstruct a desired spatial soundfield over 3 dimensional space.

There are three main approaches for 3D soundfield reconstruction: (i) Wave field synthesis (WFS) approach [1–3], (ii) Inverse or Least squares approach where a loudspeaker response of a given geometry is compared to a desired field over a set of points [4]; and (iii) Spherical harmonic expansion based approach [5–8]. Both WFS and spherical harmonic based approaches have provided elegant solutions for 3D soundfield reconstruction and their error analysis. However, to recreate a 3D sound environment, the loudspeakers generally need to be located in all directions in 3D resulting in geometries (such as spherical) which are impractical in most of the intended applications.

This paper considers 3D soundfield reconstruction using spherical harmonic expansion. We make use of the characteristics of associated Legendre functions together with the concept of continuous aperture functions to develop non-spherical loudspeaker array geometry for 3D soundfield reproduction.

2. 3D SOUNDFIELD ANALYSIS

2.1 Spherical Harmonic Expansion

An arbitrary soundfield within a source free region can be expressed [6] using spherical harmonics as

$$S(r,\theta,\phi;k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \alpha_{nm}(k) j_n(kr)\mathscr{P}_{nm}(\cos\theta)E_m(\phi) \quad (1)$$

where m and n (≥ 0) are integers, $\alpha_{nm}(k)$ are the spherical harmonic coefficients of the soundfield, $k = 2\pi f/c$ is the wavenumber, f is the frequency, c is the speed of sound, $j_n(\cdot)$ are the spherical Bessel functions of order n, $E_m(\phi) = (1/\sqrt{2\pi})e^{jm\phi}$ are the normalized exponential functions and

$$\mathscr{P}_{nm}(\cos\theta) = \sqrt{\frac{2n+1}{2}}\sqrt{\frac{(n-|m|)!}{(n+|m|)!}}P_{n|m|}(\cos\theta) \quad (2)$$

are the normalized associated Legendre functions. Note that the normalized exponential functions and the normalized associated Legendre functions form orthonormal basis sets in azimuth $\phi \in [0,2\pi)$ and elevation $\theta \in [0,\pi]$, respectively.

2.2 Desired Soundfield Coefficients

Let the region of interest Ω be a sphere of radius R. Since we are interested in the limited region Ω, the soundfield here can be expressed by a finite set of coefficients, i.e a finite number of terms of (1). Thus, for a soundfield within Ω, the infinite summation in (1) can be truncated [9] to $N = \lceil keR/2 \rceil$. Hence, giving the total number of coefficients required to describe the desired sound field by $(N+1)^2$. Table 1 depicts the growth of the number of coefficients as order n grows with modes m ranging from $-n$ to n.

Given the desired soundfield by $\{\alpha^{\mathrm{d}}_{nm}(k)\}$ for $n = 0,\dots,N$, $m = -n,\dots,n$, the goal of this work is to design a practically realizable loudspeaker array configuration to recreate the desired soundfield in the region of interest Ω.

3. SOUNDFIELD USING MULTIPLE LOUDSPEAKERS

In this section, we briefly outline the theory of 3 dimensional spherical harmonic based soundfield reconstruction as reported in the literature.

3.1 Least Squares Solution

Let there be $Q > (N+1)^2$ loudspeakers randomly placed outside the region Ω at locations $\mathbf{y}_q \equiv (r_q,\theta_q,\phi_q)$, $q = 1,\dots,Q$.

$m \diagdown n$	0	1	2	…	N
N					α_{NN}
⋮					
2			α_{22}		⋮
1		α_{11}	α_{21}		
$m=0$	α_{00}	α_{10}	α_{20}	…	α_{N0}
-1		$\alpha_{1(-1)}$	$\alpha_{2(-1)}$		
-2			$\alpha_{2(-2)}$		⋮
⋮					⋮
$-N$					$\alpha_{N(-N)}$

Table 1: Soundfield coefficients arranged with order n and degree m.

Then the soundfield at a point $\mathbf{x} \in \Omega$ due to these loudspeakers is given by

$$\widetilde{S}(r,\theta,\phi;k) = \sum_{q=1}^{Q} w_q(k) \frac{e^{ik\|\mathbf{y}-\mathbf{x}\|}}{4\pi\|\mathbf{y}-\mathbf{x}\|}, \tag{3}$$

where $\mathbf{x} \equiv (r,\theta,\phi)$, and $w_q(k)$ are the loudspeaker weights. The goal here is to determine the loudspeaker weights which produce the desired soundfield.

We substitute the Jacobi-Anger expansion [10]

$$\frac{e^{ik\|\mathbf{y}_q-\mathbf{x}\|}}{\|\mathbf{y}_q-\mathbf{x}\|} = 4\pi ik \sum_{n=0}^{\infty}\sum_{m=-n}^{n} h_n^{(1)}(ky_q)\mathscr{P}_{nm}(\cos\theta_q)E_{-m}(\phi_q) \times j_n(kr)\mathscr{P}_{nm}(\cos\theta)E_m(\phi) \tag{4}$$

into (3), and equate with (1) to obtain

$$\alpha_{nm}(k) = \sum_{q=1}^{Q} ikw_q(k)h_n^{(1)}(kr_q)\mathscr{P}_{nm}(\cos\theta_q)E_{-m}(\phi_q). \tag{5}$$

The loudspeaker weights $w_q(k)$ can then be obtained by evaluating (5) for $n = 0,\ldots,N$, and $m = -n,\ldots,n$ and setting a system of simultaneous equations [6]. Such a system of equations could be solved using the Least Squares method.

3.2 Mode-matching on a sphere

Mode-matching is a more elegant but practically complex method where the concept of a continuous loudspeaker aperture [7] is used. In this solution for soundfield reconstruction, a large number of loudspeakers are placed on a sphere with radius $\tilde{R} > R$. The weights of the loudspeakers are given by a theoretical *continuous aperture function*[1] $\rho(\theta,\phi;k)$ at the loudspeaker position. The corresponding soundfield due to the continuous spherical loudspeaker is given by

$$\widetilde{S}(r,\theta,\phi;k) = \int_0^{2\pi}\int_0^{\pi} \rho(\widetilde{\theta},\widetilde{\phi};k)\frac{\exp(k\|\widetilde{\mathbf{y}}-\mathbf{x}\|)}{4\pi\|\widetilde{\mathbf{y}}-\mathbf{x}\|} \times \sin\widetilde{\theta}\, d\widetilde{\phi}\, d\widetilde{\theta}. \tag{6}$$

[1]A continuous aperture function is a limiting case of a closely packed set discrete loudspeakers.

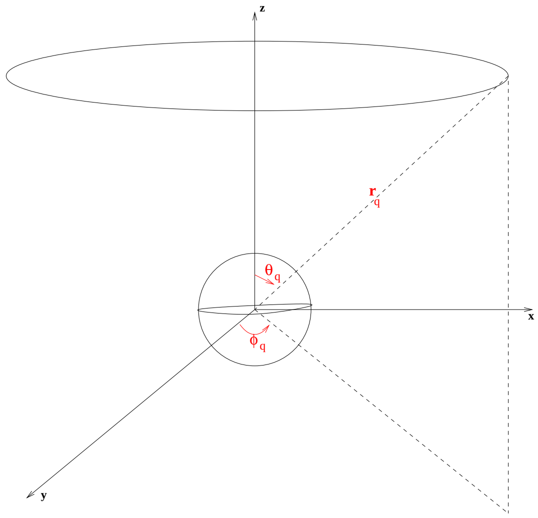

Figure 1: Circular continuous aperture located at (r_q,θ_q) outside the region of interest Ω

An arbitrary spherical aperture function can be written using spherical harmonics as

$$\rho(\theta,\phi;k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \gamma_{nm}(k)\mathscr{P}_{nm}(\cos\theta)E_m(\phi). \tag{7}$$

where $\gamma_{nm}(k)$ are the spherical harmonic coefficients of the aperture function. By substituting (4) and (7) into (6) and evaluating the integration, we have

$$\widetilde{S}(r,\theta,\phi;k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} ik\gamma_{nm}(k)h_n^{(1)}(k\widetilde{R}) \times j_n(kr)\mathscr{P}_{nm}(\cos\theta)E_m(\phi) \tag{8}$$

By equating the loudspeaker soundfield (8) with the desired field, we can obtain the unknown aperture function

$$\alpha_{nm}^{\mathrm{d}}(k) = ik\,\gamma_{nm}(k)\,h_n^{(1)}(k\widetilde{R}). \tag{9}$$

This is called mode-matching since a mode of the desired soundfield is matched to the corresponding mode of the aperture function. The continuous spherical aperture $\rho(\theta,\phi;k)$ needs to be sampled to find an equivalent array of loudspeakers. However, placing loudspeakers equidistantly on a sphere is not straightforward. Similarly, it is hard to imagine having a practical spherical shell loudspeaker array where the desired region of interest is in the middle of the spherical array.

4. CIRCULAR MULTISPEAKER CONFIGURATION

In this section, we consider an alternative array structure for soundfield reconstruction.

4.1 Circular Aperture

Consider a theoretical horizontal continuous circular loudspeaker aperture located at an elevation angle of θ_q and a distance r_q from the origin of the desired region of interest as shown in Figure 1. Let the aperture function[2] of the continuous loudspeaker be $\rho_q(\phi;k)$. Since the aperture function is a periodic function of azimuth angle ϕ, we can use the Fourier series to write [11]

$$\rho_q(\phi;k) = \sum_{m=-\infty}^{m=\infty} \beta_m^{(q)}(k) E_m(\phi) \tag{10}$$

where $\beta_m^{(q)}(k)$ are the frequency dependent Fourier coefficients. Let us examine the resulting soundfield within the region of interest due to this circular loudspeaker aperture in the following theorem.

Theorem 4.1 *The spherical harmonic coefficients of the soundfield in the region of interest Ω due to a horizontal continuous circular loudspeaker aperture at (r_q, θ_q) with a aperture function $\rho_q(\phi;k)$ is given by*

$$\alpha_{nm}^{(q)}(k) = ik\, h_n^{(1)}(kr_q)\mathscr{P}_{nm}(\cos\theta_q)\beta_m^{(q)}(k) \tag{11}$$

where

$$\beta_m^{(q)}(k) = \int_0^{2\pi} \rho_q(\phi;k) E_{-m}(\phi)\, d\phi \tag{12}$$

are the Fourier series coefficients of the aperture function.

Proof The soundfield due to the circular aperture can be written as

$$S_q(r,\theta,\phi;k) = \int_0^{2\pi} \rho_q(\hat{\phi},k) \frac{\exp(ik\|\mathbf{y}_q - \mathbf{x}\|)}{4\pi\|\mathbf{y}_q - \mathbf{x}\|}\, d\hat{\phi} \tag{13}$$

where $\mathbf{y}_q \equiv (r_q, \theta_q, \hat{\phi})$ is a point on the circular aperture and $\mathbf{x} \equiv (r,\theta,\phi)$ is a point within the region of interest. By substituting (10) and (4) into (13) and integrating, we can express (13) in the spherical harmonic expansion form (1) and obtain the coefficients $\alpha_{nm}(k)$ as in (11).

We have following comments on Theorem 4.1:

- The circular aperture function is completely described by the Fourier coefficients $\{\beta_m^{(q)}(k)\}$.
- Note from (11) that each $\beta_m^{(q)}(k)$ for a specific m, would only induce soundfield coefficients of fixed degree m and orders $n = |m|, \ldots, \infty$. Hence, we can control the soundfield of a given degree m (along a row of Table 1) by choosing appropriate values for $\beta_m^{(q)}(k)$.
- The normalized associated Legendre functions $\mathscr{P}_{nm}(\cdot)$ have a number of zeros (see Figures 2 and 3). Thus, for some values of n, m and θ_q, the induced soundfield coefficient $\alpha_{nm}(k)$ in (11) is equal to zero irrespective of the value of $\beta_m^{(q)}(k)$. We exploit this fact later in the paper to create a new loudspeaker layout for spatial sound reproduction.

A single circular aperture (10) can only control the soundfield coefficients along degrees m but not on orders n. Thus, we consider multiple circular continuous loudspeakers in the next section.

[2]Sometimes this is referred as the *driving function.*

4.2 Multiple circles

Suppose there is a set of Q circles of horizontal continuous loudspeakers located at (r_q, θ_q) for $q = 1, \ldots, Q$, with corresponding aperture functions $\rho_q(\phi;k)$ given by (10). Then the coefficients of the resulting soundfield are given by

$$\alpha_{nm}(k) = \sum_{q=1}^{Q} ik\, h_n^{(1)}(kr_q)\mathscr{P}_{nm}(\cos\theta_q)\beta_m^{(q)}(k). \tag{14}$$

Note that for a specific m, the aperture function coefficients $\beta_m^{(q)}(k)$ from all circles contribute to the soundfield coefficients of degree m.

For a finite dimensional spherical region of interest with radius R, we only need to control soundfield coefficients up to order $N = \lceil keR/2 \rceil$. Thus, by having a sufficient number of circular loudspeakers, we can control the required soundfield coefficients to reconstruct a desired given soundfield with the region of interest. In the next section, we show how to calculate the aperture function coefficients when given a desired soundfield.

4.3 Matrix Formulation

Suppose the desired soundfield is given by $(N+1)^2$ coefficients $\alpha_{nm}^{\mathrm{d}}(k)$. To find the required aperture coefficients $\beta_m^{(q)}$, we equate the left hand side of (14) to $\alpha_{nm}^{\mathrm{d}}(k)$ for a specific m and $n = |m|, |m|+1, \cdots, N$. We write the resulting set of simultaneous equations in matrix form, as

$$\mathbf{A}_m = \mathbf{H}_m \mathbf{B}_m \tag{15}$$

where $\mathbf{A}_m = [\alpha_{|m|m}^{\mathrm{d}}(k), \alpha_{(|m|+1)m}^{\mathrm{d}}(k), \ldots, \alpha_{Nm}^{\mathrm{d}}(k)]^T$,

$$\mathbf{H}_m = ik \times \tag{16}$$

$$\begin{bmatrix} h_1^{(1)}(kr_{|m|})\mathscr{P}_{mm}(\cos\theta_1) & \cdots & h_{|m|}^{(1)}(kr_Q)\mathscr{P}_{mm}(\cos\theta_Q) \\ \vdots & \ddots & \vdots \\ h_N^{(1)}(kr_1)\mathscr{P}_{Nm}(\cos\theta_1) & \cdots & h_N^{(1)}(kr_Q)\mathscr{P}_{Nm}(\cos\theta_Q) \end{bmatrix},$$

and $\mathbf{B} = [\beta_m^{(1)}, \ldots, \beta_m^{(Q)}]^T$.

Equation (15) could be solved for $\mathbf{B}$ using the Least Squares method provided that $\mathbf{H}_m$ is non singular. Such a solution may or may not exist if an arbitrary set of circles are used. In a practical set up, we need to avoid certain angles. In the following section we develop a systematic procedure to set up a circular loudspeaker array system.

5. IMPLEMENTATION

5.1 Location of Circles

We suggest the following procedure to determine the location of circular apertures and further calculate the relevant aperture coefficients.

Step 1 ($m = N$ series): If the desired region of interest is order limited to N, then the only applicable soundfield coefficient for this series is $\alpha_{NN}^{\mathrm{d}}(k)$ as there are no lower order coefficients and the higher order components will have negligible effect on this region. Thus, we have

$$\alpha_{NN}(k) = \sum_{q=1}^{Q} 4\pi ik\, h_N^{(1)}(kr_q)\mathscr{P}_{NN}(\cos\theta_q)\beta_N^{(q)}(k). \tag{17}$$

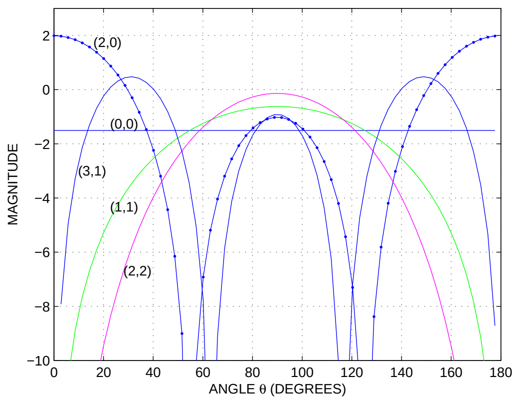

Figure 2: Magnitude of the normalized associate Legendre functions $\mathscr{P}_{nm}(\cos\theta)$ in dB, where the addition of order n and mode m are even: $(n,|m|) = (0,0);(2,0);(1,1);(2,2);(3,1)$

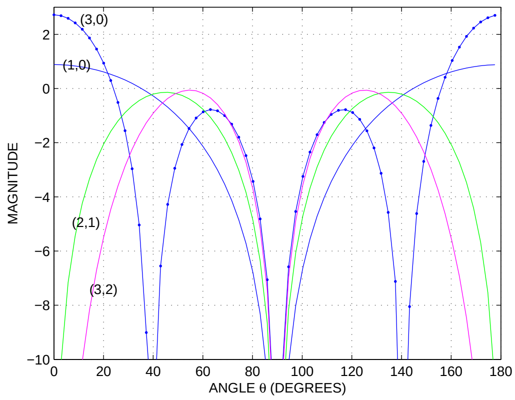

Figure 3: Magnitude of the normalized associate Legendre functions $\mathscr{P}_{nm}(\cos\theta)$ in dB, where the addition of order n and mode m are odd: $(n,|m|) = (1,0);(2,1);(3,0);(3,2)$

In this case, there is only one coefficient to be controlled, and hence, we only need a single circle. In order to choose θ_q for this circle such that $\mathscr{P}_{NN}(\cos\theta_q) \neq 0$, we select $\theta_q = \pi/2$, as all *even*[3] associated Legendre functions have significantly large and stable values (see Figure 2). Thus, we choose the *x-y* plane to place the first circle. We also set $\beta_N^{(q)}(k) = 0$ for all other circles (which we will add to the system in the subsequent steps). Thus, (17) reduces to

$$\alpha_{NN}(k) = 4\pi ik\, h_N^{(1)}(kr_1)\mathscr{P}_{NN}(\cos\theta_1)\beta_N^{(1)}(k), \qquad (18)$$

which can be used to determine $\beta_N^{(1)}(k)$.

Step 2 ($m = N-1$ series): For this series, $n = N-1$ and $n = N$ coefficients are applicable, and the corresponding Legendre functions are $\mathscr{P}_{(N-1)(N-1)}(\cdot)$ and $\mathscr{P}_{N(N-1)}(\cdot)$. Since there are two soundfield coefficients, the system of simultaneous equations (15) has only two equations. Thus, $\beta_{N-1}^{(q)}(k)$, from two circles will suffice to realize the desired soundfield coefficients. As one of the soundfield coefficients is even, we can reuse the first circle ($q = 1$), with the aperture coefficient $\beta_{N-1}^{(1)}(k)$. The second circle needs to be located at a particular θ_2 where $\mathscr{P}_{N(N-1)}(\theta_2) \neq 0$. We can use Figure 3 to determine an appropriate value[4] for θ_2. We also set $\beta_{N-1}^{(q)}(k) = 0$ for $q > 2$, i.e., for other circles. The matrix equation (15) becomes $\mathbf{A}_{N-1} = \mathbf{H}_{N-1}\mathbf{B}_{N-1}$ where $\mathbf{A}_{N-1} = [\alpha^{\mathrm{d}}_{(N-1)(N-1)}(k)\alpha^{\mathrm{d}}_{N(N-1)}(k)]^T$, $\mathbf{H}_{N-1} = ik\times$

$$\begin{bmatrix} h_{N-1}^{(1)}(kr_1)\mathscr{P}_{N-1N-1}(\cos\theta_1) & h_{N-1}^{(1)}(kr_2)\mathscr{P}_{N-1N-1}(\cos\theta_2) \\ h_N^{(1)}(kr_1)\mathscr{P}_{NN-1}(\cos\theta_1) & h_N^{(1)}(kr_2)\mathscr{P}_{NN-1}(\cos\theta_2) \end{bmatrix},$$

and $\mathbf{B}_{N-1} = [\beta_{N-1}^{(1)}\beta_{N-1}^{(2)}]^T$. Therefore, we obtain $\mathbf{B}_{N-1} = \mathbf{H}_{N-1}^{-1}\mathbf{A}_{N-1}$.

Step 3 ($m = N-2$ series): In this case, we need to control three soundfield coefficients $(N-2, N-2), (N-1, N-2), (N, N-2)$. To solve this system, we reuse the first two circles and introduce a third circle with appropriate θ_q where $\mathscr{P}_{(N-2)(N-2)}(\theta_3) \neq 0$. As before, we set $\beta_{N-2}^{(q)}(k) = 0$ for $q > 3$. Now we use (15) to determine $\beta_{N-2}^{(q)}(k)$ for $q = 1,2,3$.

Step N+1 ($m = 0$ series): There are $N+1$ coefficients in this series. Hence, we can reuse all previously established circles together with a new circle. Since the final circle is needed for a single coefficient, it can be a single point at $\theta_{N+1} = 0$. As before, $\beta_0^{(q)}(k)$ for $q = 1,\ldots,N+1$ can be calculated from (15).

Note that the same set of circles could be reused for negative values of m, i.e., starting with $m = -N$ from the first circle.

5.2 Discretization

For practical implementation, we need to discretize the continuous aperture functions at each circle [11]. Since there are only a finite number of Fourier coefficients in the aperture functions $\rho_q(\phi;k)$ they can be implemented by a finite number of loudspeakers.

6. SIMULATION

To illustrate the technique, we simulate a third order system ($N = 3$) in this paper. For a third order system the region of interest is a sphere of radius 3cm and frequency of operation is 3500 Hz. Hence, we can accurately reproduce a given soundfield within this spherical region. For a larger reproduction region, we need a higher order system.

As outlined in Section 5, we locate four circles and determine the aperture function coefficients for each circle. The design information is tabulated in Table 2 together with the desired spherical harmonic coefficients for each circle and the respective $\beta_m^{(q)}$ values.

The desired soundfield for the simulation is a plane wave arriving at an angle of $(\theta,\phi) = (90^o, 90^o)$, which gives the

[3] *Even* and *odd* are defined when the sum order n and degree m are even and odd, respectively.

[4] A complete guideline to choosing elevation angles for even and odd associated Legendre functions are given in [12].

circle no.	r_q, θ_q	α_{nm}	$\beta_m^{(q)}$
1	1.8m, 90^o	$\alpha_{33},\alpha_{22},\alpha_{11},\alpha_{00}$	$\beta_3^1,\beta_2^1,\beta_1^1,\beta_0^1$
2	1.7m, 65^o	$\alpha_{32},\alpha_{21},\alpha_{10}$	$\beta_2^2,\beta_1^2,\beta_0^2$
3	2.4m, 25^o	α_{31},α_{20}	β_1^3,β_0^3
4	2.1, 0^o	α_{30}	β_0^4

Table 2: Design parameters for a third order system using four circles to determine $\beta_m^{(q)}$ for the desired $\alpha_{nm}(k)$.

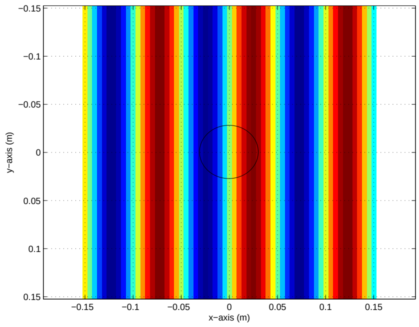

Figure 4: Imaginary part of the desired soundfield: A plane wave arriving at an angle of $(\theta,\phi) = 90^o, 90^o$ at an operating frequency of 3500 Hz

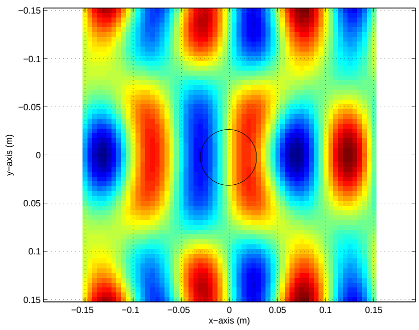

Figure 5: Imaginary part of the reproduced soundfield with the desired region of interest encircled at an operating frequency of 3500 Hz

desired $\{\alpha_{nm}^{\mathrm{d}}(k)\}$. Figure 4 depicts a cross section of the desired plane wave soundfield along the x-y plane where the desired region is marked by a circle. We then calculate the required aperture function coefficients for the third order system. The corresponding loudspeaker weights are calculated by sampling the aperture functions of each circle. The reproduce soundfield is plotted in Fig 5.

7. CONCLUSION

Practical implementation of 3D soundfield reconstruction systems are difficult due to complex loudspeaker array configurations. In this paper, we have showed a technique to design a practically realizable and robust loudspeaker array system by strategically placing circles of loudspeaker arrays. A third order system using 16 loudspeakers was implemented showing results that reconstructed field resembled the desired plane wave field within the desired region of interest. We plan to extend the simulation of a higher order system, thus increasing the size of reproduction region, and detailed error analysis in a future publication.

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