Rate-optimized power allocation for OFDM transmission with multiple DF relays and individual power constraints

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***Abstract*—We[1] consider an OFDM (orthogonal frequency division multiplexing) point to point transmission scheme improved by means of multiple relays. For each carrier, symbols sent by the source during a first time slot may be retransmitted during a second time slot by the relays, which are assumed to be of the DF (Decode-and-Forward) type. For each relayed carrier the destination implements maximum ratio combining between what is received over the direct link and what is received from the relay(s). Perfect CSI (channel state information) knowledge is assumed. The paper investigates the power allocation problem in order to maximize the rate offered by the scheme. The source is allowed to transmit a new symbol during the second time slot when none of the relays is assisting. The constraints of decodability at the relays are properly handled. The optimization is conducted for individual constraints on the powers at the source and at the relays. The theoretical analysis is illustrated by numerical results.**

I. Introduction

In previous contributions we have considered OFDM (orthogonal frequency division multiplexing) transmission schemes improved by means of a single relay operating in the decode and forward (DF) mode. Two protocols have been considered, differing by the behavior of the source during the second time slot. In a first case, the source is always idle even when the relay is non assisting. In an improved protocol, the source sends during the second time slot a new symbol on each carrier for which the relay is non assisting. The power allocation problem has been solved for an objective function which is the rate of the system. The optimization has been achieved with a proper handling of the decodability constraint at the relay, and for both types of constraints on the power: a sum power constraint or individual power constraints at the source and at the relay. These results have been reported in [1]–[3]. In the current paper we consider a similar setup where now the transmission is helped by means of multiple DF relays. In a recent contribution [4], the power allocation problem has been tackled for a constraint on the sum of the powers at the source and at the relays. This appears as a natural step to solve the problem for individual power constraints, which is topic of this paper. The main steps of the sum power case are however repeated here for the sake of clarity.

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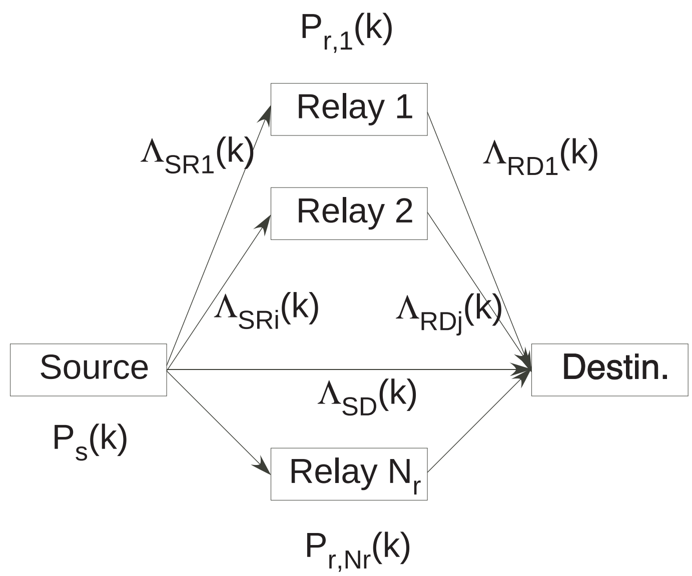

Fig. 1. Structure of the system for carrier k.

Setups with multiples relays have also been considered in the literature. In [5], the authors have considered OFDM with multiple decode and forward relays. The objective of their work is to minimize the total transmission power by allocating bits and power to the individual subchannels. A selective relaying strategy is chosen. In the current paper, the objective function chosen is the maximization of the rate. Besides that, the truly optimum allocation is obtained and shown to use several relays.

II. System description

Assuming the cyclic prefix technique works properly everywhere, the OFDM transmission system can be described by looking at each individual carrier. The block diagram associated with the system for one particular carrier is depicted in figure 1.

During the first signalling period, a symbol is sent by the source on each carrier. The relays then decode and possibly relay some of the symbols during the second time slot. When assisting, the relays are constrained to use the same carrier as that used by the source. Based on the two signalling intervals, the destination implements maximum ratio combining for the carriers with relaying.

Let us denote by $\sqrt{P_s(k)}$ (resp. $\sqrt{P_{r,i}(k)}$) the amplitude of the symbol at the source (resp. ith relay) for carrier k, and by $\lambda_{sd}(k)$ (resp. $\lambda_{r_id}(k)$) the complex channel gain for tone k between source (resp. relay i) and destination. The noise sample observed by the destination at tone k during the first period is $n_1(k)$, and $n_2(k)$ during the second period. These two noise samples are zero-mean circular Gaussian, white and uncorrelated with the same variance σ_n^2. Denoting by $s(k)$ the unit energy symbol transmitted over tone k, the destination gets at the end of the first time slot,

$$y_{sd}(k) = \sqrt{P_s(k)}\,\lambda_{sd}(k)\,s(k) + n_1(k). \quad (1)$$

During the second time slot, for coherence issues at the receiver side, each assisting relay $1 \le i \le N_r$ (N_r is the number of relays) sends $\sqrt{P_{r,i}(k)}\exp^{-j\arg\lambda_{r_id}(k)}\ s(k)$. This means that the phase of the channel is precompensated, which requires the relays to know their respective gains $\lambda_{r_id}(k)$. Hence the destination receives during the second time slot

$$y_{rd}(k) = \sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|\,s(k) + n_2(k). \quad (2)$$

After proper maximum ratio combining over the two time slots at the destination, the decision variable $r(k)$ obtained at the k-th output of the N_t-FFT (Fast Fourier transform of size N_t, N_t being the number of carriers) and the related signal to noise ratio $\gamma(k)$ are given by

$$\begin{aligned} r(k) &= \sqrt{P_s(k)}\,\lambda_{sd}^*(k)\,y_{sd}(k) \\ &+ \Big(\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|\Big)^*\,y_{rd}(k) \quad (3) \\ \gamma(k) &= \frac{P_s(k)\,|\lambda_{sd}(k)|^2 + (\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|)^2}{\sigma_n^2}. \quad (4) \end{aligned}$$

III. Rate optimization for a sum power constraint

The achievable rate of the system for a duration of 2 OFDM symbols is defined by [6]:

$$\begin{aligned} R &= 2\sum_{k\in S_s}\log\left(1+\frac{P_s(k)\,|\lambda_{sd}(k)|^2}{\sigma_n^2}\right) \\ &+ \sum_{k\in S_r}\log\left(1+\frac{P_s(k)\,|\lambda_{sd}(k)|^2}{\sigma_n^2}\right. \\ &+ \left.\frac{(\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|)^2}{\sigma_n^2}\right) \quad (5) \end{aligned}$$

where S_s is the set of carriers (or tones) receiving power at the source only, and S_r the complementary set, that is, carriers receiving power at the source and at one relay at least. At this point the sets S_s and S_r are not known. Their definitions (meaning which carriers are allocated to each) is an outcome of the optimization procedure. For a relayed carrier, one has to remember the assumption on the decode and forward operating mode of the helping relays. For any relay j assisting in the relaying phase for carrier k, one must have that

$$\begin{aligned} P_s(k)\,|\lambda_{sr_j}(k)|^2 &\ge \\ P_s(k)\,|\lambda_{sd}(k)|^2 &+ \Big(\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|\Big)^2 \quad (6) \end{aligned}$$

where $\lambda_{sr_j}(k)$ is the channel gain between the source and relay j for carrier k. This constraint means that the global rate of the system between source and destination cannot be above the rate achievable on any of the links between the source and the assisting relays, otherwise some relays would not be able to decode which is in contradiction with the fact that the relay is able to decode. Note that for each relayed carrier, the set of relays assisting for that carrier has to be determined.

The constraint on the sum power is given by

$$\left[\sum_{k\in S_s} 2P_s(k) + \sum_{k\in S_r}\Big[P_s(k) + \sum_i P_{r,i}(k)\Big]\right] \le P_t \quad (7)$$

where P_t is the total power budget. The objective function together with the constraints leads to the following Lagrangian:

$$\begin{aligned} \mathcal{L}_1 &= 2\sum_{k\in S_s}\log\left(1+\frac{P_s(k)|\lambda_{sd}(k)|^2}{\sigma_n^2}\right) \\ &+ \sum_{k\in S_r}\log\left(1+\frac{P_s(k)\,|\lambda_{sd}(k)|^2}{\sigma_n^2}\right. \\ &+ \left.\frac{(\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|)^2}{\sigma_n^2}\right) \\ &- \mu\left[\sum_{k\in S_s} 2P_s(k) + \sum_{k\in S_r}\Big[P_s(k) + \sum_i P_{r,i}(k)\Big] - P_t\right] \\ &- \sum_{k\in S_r}\sum_j \rho_{kj}\,\Big[P_s(k)\,|\lambda_{sd}(k)|^2 \\ &+ \Big(\sum_i \sqrt{P_{r,i}(k)}|\lambda_{r_id}(k)|\Big)^2 - P_s(k)\,|\lambda_{sr_j}(k)|^2\Big]. \quad (8) \end{aligned}$$

The Lagrange multipliers are denoted by μ for the power constraint and by ρ_{kj} for the decodability constraints. As explained in [4], relaying for carrier k should only be considered when we have at least one j such that $|\lambda_{sr_j}(k)|^2 > |\lambda_{sd}(k)|^2$.

In [4] it has been shown that for a relayed carrier k, one relay will be saturated and other ones may assist and they have a nonsaturated decodability constraint. For any two assisting relays j and j', we obtain from the KKT conditions that

$$\frac{|\lambda_{r_jd}(q)|}{\sqrt{P_{r,j}(q)}} = \frac{|\lambda_{r_{j'}d}(q)|}{\sqrt{P_{r,j'}(q)}}. \quad (9)$$

This, with the saturation or active constraint for relay j_q, leads to the following values for the relay powers:

$$
\begin{aligned}
P_{r,j}(q) &= P_s(q)\left[|\lambda_{sr_{j_q}}(q)|^2 - |\lambda_{sd}(q)|^2\right] \\
&\times \frac{|\lambda_{r_jd}(q)|^2}{\left(\sum_i |\lambda_{r_id}(q)|^2\right)^2}.
\end{aligned} \tag{10}
$$

The power allocated to carrier q is given by $P(q)$ which is obtained by

$$
\begin{aligned}
P(q) &= P_s(q) + \sum_i P_{r,i}(q) \\
&= P_s(q)\left[1 + \frac{|\lambda_{sr_{j_q}}(q)|^2 - |\lambda_{sd}(q)|^2}{\sum_i |\lambda_{r_id}(q)|^2}\right]
\end{aligned} \tag{11}
$$

which shows the link between $P(q)$ and $P_s(q)$. In view of all these, we can conclude that for a carrier with one assisting relay fulfilling the constraint, and several other assisting relays, power $P(q)$ leads to a contribution to the rate given by

$$
R_r(q) = \log\left(1 + \frac{P(q)\,|\lambda_{sr_{j_q}}(q)|^2\,\beta(q)}{\sigma_n^2}\right) \tag{12}
$$

with

$$
\beta(q) = \frac{\sum_i |\lambda_{r_id}(q)|^2}{|\lambda_{sr_{j_q}}(q)|^2 - |\lambda_{sd}(q)|^2 + \sum_i |\lambda_{r_id}(q)|^2}. \tag{13}
$$

It appears that the impact of $P(q)$ on the rate is increased with the product $|\lambda_{sr_{j_q}}(q)|^2\,\beta(q)$. As $|\lambda_{sr_{j_q}}(q)|^2 \geq |\lambda_{sd}(q)|^2$, $\beta(q)$ increases with the value of $\sum_i |\lambda_{r_id}(q)|^2$. Therefore, for a given choice of the saturated relay j_q, all possible additional values of $|\lambda_{r_id}(q)|^2$ should be retained. In other words, all relays i for which decoding is possible, which means $|\lambda_{sr_i}(q)|^2 > |\lambda_{sr_{j_q}}(q)|^2$, should be used. The choice of the saturated relay j_q is a compromise between a high value of $|\lambda_{sr_{j_q}}(q)|^2$ that is directly beneficial to the rate, and a lower value that allows to select more relays $|\lambda_{r_id}(q)|^2$ fulfilling the decoding constraint. Hence all the values $|\lambda_{sr_j}(q)|^2 > |\lambda_{sd}(q)|^2$ should be ranked in increasing order. For each one considered as candidate for the relay with saturated constraint j_q, the value of $|\lambda_{sr_{j_q}}(q)|^2\,\beta(q)$ should be computed. The maximum is then kept.

About a non relayed carrier, we have by setting the derivative of the Lagrangian (8) to 0 that

$$
\begin{aligned}
\frac{\partial R}{\partial P_s(q)} &= 2\left(1 + \frac{P_s(q)\,|\lambda_{sd}(q)|^2}{\sigma_n^2}\right)^{-1}\frac{|\lambda_{sd}(q)|^2}{\sigma_n^2} \\
&= 2\mu.
\end{aligned} \tag{14}
$$

For a total power $P(q)$ allocated to carrier q (over the two instants), the rate evolves as

$$
R_s(q) = \log\left[\left(1 + \frac{P(q)\,|\lambda_{sd}(q)|^2}{2\,\sigma_n^2}\right)^2\right] \tag{15}
$$

where the 2 in front of the $\log$ has been moved as an exponent inside the $\log$. Let us denote by $|\lambda_\beta(q)|^2$ the maximum value that can be found for $|\lambda_{sr_{j_q}}(q)|^2\,\beta(q)$. When $|\lambda_{sd}(q)|^2 > |\lambda_\beta(q)|^2$ we have for any value of $P(q)$ that $R_s(q) > R_r(q)$. On the contrary, when $|\lambda_{sd}(q)|^2 < |\lambda_\beta(q)|^2$ we have $R_s(q) < R_r(q)$. However this is valid only for $P(q) \leq \lambda_t(q)$ where

$$
\lambda_t(q) = 4\,\sigma_n^2\,\frac{|\lambda_\beta(q)|^2 - |\lambda_{sd}(q)|^2}{|\lambda_{sd}(q)|^4}. \tag{16}
$$

Based on this a new form can obtained for a Lagrangian:

$$
\begin{aligned}
\mathcal{L}_3 &= 2\sum_{k\in S_s}\log\left(1 + \frac{P(k)\,|\lambda_{sd}(k)|^2}{2\,\sigma_n^2}\right) \\
&+ \sum_{k\in S_r}\log\left(1 + \frac{P(k)\,|\lambda_\beta(k)|^2}{\sigma_n^2}\right) \\
&- \mu\left[\sum_{k\in S_s} P(k) - P_t\right]
\end{aligned} \tag{17}
$$

with $P(k) = P_s(k) + \sum_i P_{r,i}(k)$ for a relayed carrier where the relay powers can be computed from (10). For a non relayed carrier, $P(k) = 2P_s(k)$.

Equating to 0 the derivatives of this Lagrangian with respect to the power, we get for $k \in S_s$,

$$
P(k) = 2\left[\frac{1}{\mu} - \frac{\sigma_n^2}{|\lambda_{sd}(k)|^2}\right]_+ \tag{18}
$$

where $[.]_+$ stands for $\max[0,.]$. Similarly, for $k \in S_r$,

$$
P(k) = \left[\frac{1}{\mu} - \frac{\sigma_n^2}{|\lambda_\beta(k)|^2}\right]_+. \tag{19}
$$

At the end of the power allocation one checks if any of the relayed carriers receives an amount of power above the threshold given by (16). If this happens, the relayed carrier fulfilling this condition and having the largest value of $|\lambda_{sd}(.)|^2$ is moved from the set S_r to the set S_s. The power allocation is computed again. This procedure is iterated till none of the relayed carrier receives an amount of power larger than its associated threshold. This procedure is referred below as the reallocation step. Different reallocation strategies have been experimented and all lead to the same result. Yet the optimality still remains to be proven.

IV. Individual power constraints

The methodology followed here is similar to that used in [1]. The individual power constraints are given by

$$
\sum_{k=1}^{N_t} P_s(k) \leq P_s \text{ and } \sum_{k=1}^{N_t} P_{r,i}(k) \leq P_{r,i} \tag{20}
$$

for all relays $1 \leq i \leq N_r$, rather than constraint (7). These $N_r + 1$ constraints lead to the use of $N_r + 1$ Lagrange multipliers. A first point to be noted is the fact that it may happen that the power constraint on some of the relays will not be saturated at the optimum, depending on the channel parameters. This case will not be investigated further. N_r+1 Lagrange multipliers, μ_s and $\mu_{r,i}$, now have to be used for the power constraints. One element in the direction of the solution lies in the observation [7] that the rate only depends on the products of powers and (possibly modified) channel gains. Hence allocating power P to a carrier with gain $|\lambda|^2$ provides the same rate as allocating power μP to a carrier with gain $|\lambda|^2/\mu$. Let us define the following modified gains: $|\lambda^\mu_{sd}|^2 = |\lambda_{sd}|^2/\mu_s$; $|\lambda^\mu_{sr_i}|^2 = |\lambda_{sr_i}|^2/\mu_s$; $|\lambda^\mu_{r_id}|^2 = |\lambda_{r_id}|^2/\mu_{r_i}$. The equivalent powers under consideration are now $P^\mu_s(q) = \mu_s\, P_s(q)$ and $P^\mu_{r_i}(q) = \mu_{r,i}\, P_{r,i}(q)$. The reasoning has to be adapted to the fact that now the power constraints are separate and do no longer concern a single power budget. Let us again denote by S_s the set of carriers receiving source power only, and by set S_r the set of other carriers using at least one relay. Let us first assume that the appropriate multipliers μ_s and $\mu_{r,i}$ have been found. The objective function together with the constraints on decoding at the different relays lead to the following Lagrangian:

$$\begin{aligned}
\mathcal{L}^\mu_1 &= 2\sum_{k\in S_s} \log\left(1+\frac{P^\mu_s(k)|\lambda^\mu_{sd}(k)|^2}{\sigma_n^2}\right)\\
&+ \sum_{k\in S_r} \log\left(1+\frac{P^\mu_s(k)\,|\lambda^\mu_{sd}(k)|^2}{\sigma_n^2}\right.\\
&+ \left.\frac{(\sum_i \sqrt{P^\mu_{r,i}(k)}|\lambda^\mu_{r_id}(k)|)^2}{\sigma_n^2}\right)\\
&- \left[\sum_{k\in S_s} 2P^\mu_s(k) + \sum_{k\in S_r} [P^\mu_s(k) + \sum_i P^\mu_{r,i}(k)]\right.\\
&- \left.\mu_s\, P_s - \sum_i \mu_{r,i}\, P_{r,i}\right]\\
&- \sum_{k\in S_r}\sum_j \rho_{kj}\left[P^\mu_s(k)\,|\lambda^\mu_{sd}(k)|^2\right.\\
&+ \left.(\sum_i \sqrt{P^\mu_{r,i}(k)}|\lambda^\mu_{r_id}(k)|)^2 - P^\mu_s(k)\,|\lambda^\mu_{sr_j}(k)|^2\right]
\end{aligned} \tag{21}$$

It is interesting to compare this Lagrangian with the one given by (8). Actually they both have the same structure. The first difference is that (8) is based on P's and λ's while (21) is based on P^μ's and λ^μ's. Assuming that μ_s and the $\mu_{r,i}$ are known, and thanks to the use of the modified gains and powers, the individual power constraints give rise to a single sum power constraint. The associated Lagrange multiplier now has to be equal to 1 which is the other difference. Based on these observations, it turns out that under the assumption of known μ_s and $\mu_{r,i}$ all the results derived in section III apply to our problem with individual power constraints, and to the powers and the gains that have been properly normalized. In particular, for a relayed carrier, out of all the contributing relays, a single one has the constraint active. The power allocated to carrier q is given by $P^\mu(q)$ which is obtained by

$$\begin{aligned}
P^\mu(q) &= P^\mu_s(q) + \sum_i P^\mu_{r,i}(q)\\
&= P^\mu_s(q)\left[1+\frac{|\lambda^\mu_{sr_{j_q}}(q)|^2 - |\lambda^\mu_{sd}(q)|^2}{\sum_i |\lambda^\mu_{r_id}(q)|^2}\right].
\end{aligned} \tag{22}$$

Adapting what has been found above for the sum power case, the routing of carrier q to set S_s or set S_r is based on the comparison of $|\lambda^\mu_{sd}(q)|^2$ with $|\lambda^\mu_\beta(q)|^2$, the maximum value that can be found for $|\lambda^\mu_{sr_{j_q}}(q)|^2\,\beta^\mu(q)$ with

$$\beta^\mu(q) = \frac{\sum_i |\lambda^\mu_{r_id}(q)|^2}{|\lambda^\mu_{sr_{j_q}}(q)|^2 - |\lambda^\mu_{sd}(q)|^2 + \sum_i |\lambda^\mu_{r_id}(q)|^2}. \tag{23}$$

When $|\lambda^\mu_{sd}(q)|^2 > |\lambda^\mu_\beta(q)|^2$ carrier q is allocated to set S_s. When $|\lambda^\mu_{sd}(q)|^2 < |\lambda^\mu_\beta(q)|^2$, carrier q is allocated to set S_r if $P^\mu(q) \le \lambda^\mu_t(q)$ where

$$\lambda^\mu_t(q) = 4\,\sigma_n^2\,\frac{|\lambda^\mu_\beta(q)|^2 - |\lambda^\mu_{sd}(q)|^2}{|\lambda^\mu_{sd}(q)|^4} \tag{24}$$

and to set S_s if $P^\mu(q) \ge \lambda^\mu_t(q)$. This is implemented by means of the waterfilling and reallocation step described above.

Up to now it was assumed that the $\mu_{r,i}$ and μ_s were known. In fact there are single values $\mu_{r,i}$ and μ_s for which the power constraints are simultaneously fulfilled. As in [1] a Newton Raphson iterative approach is proposed to find the correct values. At iteration l, the power prices $\mu_{r,i}$ and μ_s are updated according to

$$\begin{bmatrix} \mu_s^{l+1}\\ \mu_{r,1}^{l+1}\\ \vdots\\ \mu_{r,N_r}^{l+1}\end{bmatrix} = \begin{bmatrix} \mu_s^{l}\\ \mu_{r,1}^{l}\\ \vdots\\ \mu_{r,N_r}^{l}\end{bmatrix} - \lambda\, J^{-1}\begin{bmatrix} \sum_q P_s(q) - P_s\\ \sum_q P_{r,1}(q) - P_{r,1}\\ \vdots\\ \sum_q P_{r,N_r}(q) - P_{r,N_r}\end{bmatrix}$$

with

$$J = \begin{bmatrix} \dfrac{\partial \sum_q P_s(q)}{\partial \mu_s} & \dots & \dfrac{\partial \sum_q P_s(q)}{\partial \mu_{r,N_r}}\\ \vdots & \ddots & \vdots\\ \dfrac{\partial \sum_q P_{r,1}(q)}{\partial \mu_s} & \dots & \dfrac{\partial \sum_q P_{r,N_r}(q)}{\partial \mu_{r,N_r}}\end{bmatrix}. \tag{25}$$

V. Results

In order to illustrate the theoretical analysis, numerical results are provided and discussed for situations with $N_r = 2$ and $N_r = 3$ possible relays. The number of carriers is set to $N_t = 128$. Channel impulse responses (CIR) of length 32 are generated. The taps are i. i. d. zero mean circular complex gaussian, and of unit variance for all links except the links λ_{sr_i} which are 10 dBs higher. From these CIRs, FFTs are computed to provide the corresponding λ_{xy}. We set $\sigma_n^2 = 1$. The individual power constraints are $P_s = 200$ and $P_{r,i} = 50$ for all relays. The sum power constraint is chosen to be $P_t = 200 + N_r \times 50$. Figure 2 provides the gains $|\lambda_{sr_i}(k)|^2$ (solid curve), $|\lambda_{sd}(k)|^2$ (dash-dotted), $|\lambda_{r_id}(k)|^2$ (dashed) in dB for the realization under consideration in the case $N_r = 2$. Figures 3 shows the power allocation ($\circ$) obtained for the optimized method and the individual power constraints. The possible further split where appropriate among source power (solid line) and powers of the relays (dashed) is shown. The $\times$s indicate that at least one relay is assisting ($\times$ at the top of the figure) or none ($\times$ in 0). When no relay is assisting it has to be remembered that the corresponding amount of power ($\circ$)

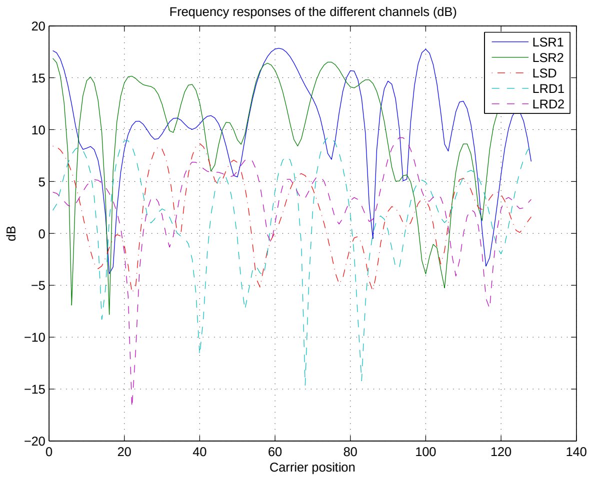

Fig. 2. Gains $|\lambda_{sr}(k)|^2, |\lambda_{sd}(k)|^2, |\lambda_{rd}(k)|^2$ in dB.

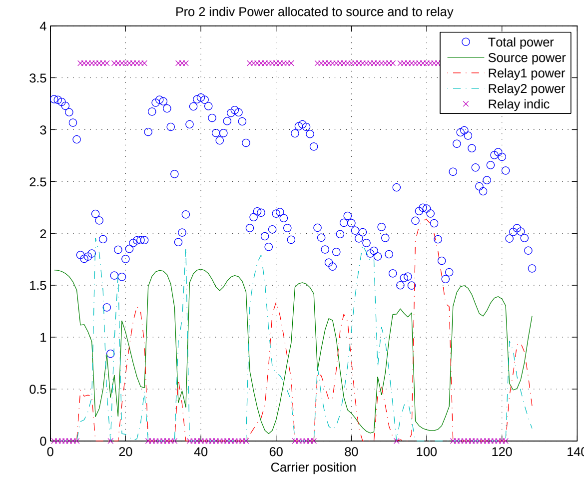

Fig. 3. $N_r = 2$. Final power allocation to source and relay in the case of individual power constraints for the optimum approach.

has to be shared over two successive time slots. The bit rate achieved in this case is 492 bits per 2 OFDM symbols duration. For the same channel realization and a sum power constraint, the total rate is 498 bits per 2 OFDM symbols duration. To show the advantage of power allocation, the rate obtained with uniform power allocation has also been computed. The power of each node (source or relays) is equally distributed across all carriers. The best combination of relays fulfilling the decodability constraint is searched for each carrier. For the not relayed mode, the power allocated by the source to the carrier under consideration is further split between the two time slots. A decision is made about the set S_s or S_r to which to allocate the carrier by comparing the rates obtained with the two modes. For the realization under consideration the bit rate achieved is 387 bits per 2 OFDM symbols duration.

Figures 4 shows the power allocation obtained for the optimized method and the individual power constraints in the case of $N_r = 3$. The channel realization is different and not reported here for the sake of concision. The bit rate achieved in this case is 567 bits per 2 OFDM symbols duration for individual constraints, and 572 bits per 2 OFDM symbols duration for the sum power constraint. Uniform power allocation leads to a rate of 414 bits per 2 OFDM symbols duration.

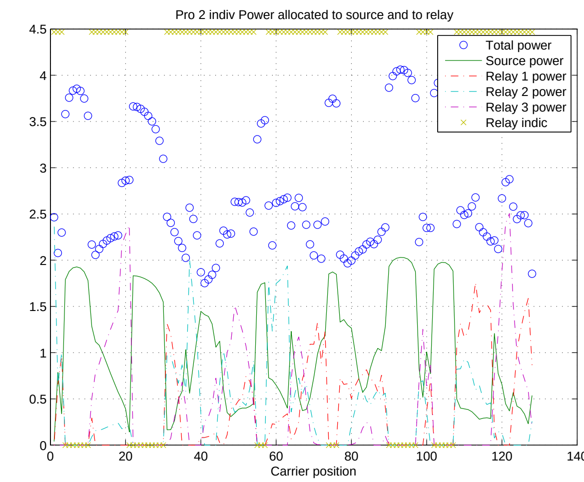

Fig. 4. $N_r = 3$. Final power allocation to source and relay in the case of individual power constraints for the optimum approach.

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