MULTIPLE DESCRIPTION SCALAR QUANTIZATION WITH SUCCESSIVE REFINEMENT

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ABSTRACT

This work presents a framework for multiple description coding (MDC) based scalable image coding. The paper starts with defining the conditions for deviating from the conventional multiple description scalar quantization (MDSQ) by extending the MDSQ into multiple-channel (more than 2) unbalanced descriptions. These two conditions are extended to define the new framework for MDSQ with successive refinement. We propose that the split or refinement factor for each of the refinement side quantizers should be greater than the number of diagonals filled in the index assignment matrix of the side quantizer design and should not be integer multiples of each other, in order to reduce the distortion in joint decoding when the number of descriptions included in joint decoding is increased. Proofs and verification by simulations of these propositions are are shown in the paper. Experimental results show high robustness of the proposed scheme compared to single description scalable image coding.

1. INTRODUCTION

Recent years have seen a great improvement in multimedia content coding in terms of high coding gain, scalability, random accessibility, low complexity and high resilience to transmission errors. The error resilience is usually achieved by either using error control codes, error concealment mechanisms or using new paradigms, such as, joint source-channel coding and or multiple description source coding (MDC) [1]. The latter is a simple, yet a very effective solution. In MDC, the source is encoded into two different bit streams with similar rate-distortion performance, known as balanced description bit streams, and they can be decoded independently for a low quality version or jointly for a high quality version of the same content. If parts of individual descriptions are affected from transmission losses, then the joint decoding compensates for these errors and decodes the content as accurate as possible. MDC is useful for transmission along packet networks, where loss of packets occur due to various link bandwidths, buffer capacities and network congestion and transmission along wireless channels, where bit wise errors occur due to fading. More importantly, MDC is very applicable in distributed and scalable content storage.

The simplicity in the MDC concept is that it can be easily integrated into existing coding frameworks. The simplest way is to create different spatio-temporal versions of the content by downsampling followed by individual encoding of each of the description using an existing source coder. This has been widely used in multiple description video coding. The other way is to integrate creating multiple descriptions into usual coding modules, such as, the decorrelating transform, quantization and entropy coding [2]-[5].

The most commonly used MDC method is modifying the quantization process in a source coder and famously known as the Multiple Description Scalar Quantization (MDSQ) [5]. In MDSQ, first a central quantizer is designed and then side quantizers containing balance rate-distortion performance is obtained by alternating merging of the bins in the central quantizer. The design problem of MDSQ and quanitzation bin index assignment conditions for a memoryless Gaussian source has been addressed in [2]. For multiple description image coding, most MDSQ solutions have been used in the wavelet transform domain [5]-[9].

The emergence of using the wavelet transform in image coding has resulted in incorporating extra features, such as scalable decoding into image coding algorithms. As scalable coding usually uses hierarchical representations of spatial-quality coding layers with progressive interdependencies, any error in lower layers, for example in low frequency sub bands, can propagate into the higher layers. Therefore, in scalable coding, low spatial-quality layers need to be highly protected for channel errors. In addition to hierarchical channel coding strategies, MDC can also be used to make scalable coded bit stream robust. One such example includes Embedded MDSQ (EMDSQ), where a set of embedded side quantizers generating two descriptions are derived from an embedded central quantizer [8, 10].

Early MDSQ algorithms focussed on obtaining descriptions with balanced rate-distortion performance. In recent work [11] we derived the conditions for obtaining unbalanced descriptions and their joint decoding. We also extended these conditions for creating more than two descriptions for MDC. In this paper, we use our results on unbalanced descriptions to formulate the conditions for successive refinement of side quantizers of the multiple description scalar quantizers. We demonstrate how successive refinements is used for highly robust scalable image coding.

The rest of the paper is organized as follows: An overview of the MDSQ scheme and the conditions for creating and joint decoding of unbalanced descriptions are presented in Section 2 and Section 3, respectively. The conditions for MDSQ with successive refinement are presented in Section 4. Simulations and experimental results using the proposed coding in scalable image coding are shown in Section 5 followed by conclusion in Section 6.

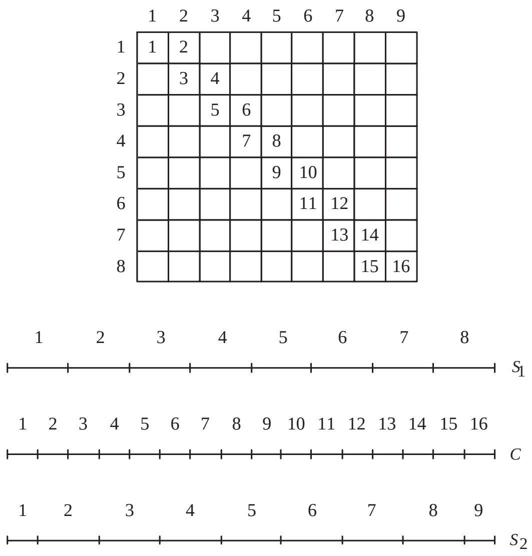


Figure 1: Index assignment matrix with central and side quantizers for ($n = 16$ and $k = 2$).

2. AN OVERVIEW OF MULTIPLE DESCRIPTION SCALAR QUANTIZATION (MDSQ)

In wavelet-based MDC, firstly, the input source is decorrelated by using the wavelet transform. Then MDSQ is used for generating two (or more) balanced descriptions. Let d_1 and d_2 be the two descriptions having rate-distortion properties of (R_1, D_1) and (R_2, D_2). The two descriptions are created using the side quantizers derived from a central quantizer having rate-distortion properties of (R_0, D_0). The two descriptions are transmitted along two channels separately. Individual decoding of descriptions results in either D_1 or D_2 distortion levels, depending on which of the description is received. Joint decoding of both bit streams can achieve a distortion level of D_0. For balanced descriptions, we can define the relationship between these parameters as

$$\begin{aligned} D_1 &\approx D_2, \\ R_1 &\approx R_2, \\ D_0 &\leq D_1, D_2, \\ R_0 &= R_1 + R_2, \end{aligned} \tag{1}$$

where R_0 is the effective rate when both descriptions are received.

An MDSQ consists of two parts: A scalar quantizer that maps a set of random variables to another countable set, commonly called a central quantizer and an index assignment matrix that splits the indexes of central quantizer into two side quantizers. The description generated from MDSQ contains redundancy that can be controlled by the number of diagonals (k) filled in the index assignment matrix. The design problem of MDSQ for a memoryless gaussian source has been addressed in detail in [2]. Figure 1 shows such an example of the index assignment matrix and its corresponding central and side quantizers for $k = 2$ and $n = 16$, where n represents the number of bins in the central quantizer.

3. UNBALANCED MULTI-CHANNEL MDSQ

The MDC model can be generalized into N number of descriptions considering N channels and $(2^N - 1)$ separate decoders (both individual and joint). When 2 or more descriptions are received, they are decoded jointly to achieve lower distortion. In current MDC approaches, the $N > 2$ case is realized by the sub-sampling based packetization of the two descriptions [5] or by using different number of refinement passes as used in embedded image coding [6]. In both schemes, the creation of N descriptions is independent of MDSQ. Our previous work [11] presented an efficient way of using multiple MDSQs to obtain multi-channel ($N > 2$) descriptions. Since a single MDSQ, which consists of a single central quantizer and 2 side quantizers, usually considers balanced descriptions as specified in Eq. (1), it is more efficient to design each of the central quantizer with various central rate-distortion performances (R_0, D_0) to get unbalanced descriptions for the $N > 2$ case. Such a scenario enables joint decoding of two or more unbalanced descriptions. Therefore, it requires careful consideration of the mutual overlaps of each of the side quantizer bins, in order to improve the quality (*i.e.,* to reduce the distortion) when the number of jointly decoded descriptions are increased. We formulate the conditions for this scenario as follows:

Let d_i^j be the descriptions created from side quantizer S_i^j using MDSQ number j with distortion D_i^j and rate R_i^j, where $i = 1, 2$ and $1 < j \leq J$. J is the total number of MDSQs. Rate and distortion metrics of two consecutive MDSQs when both descriptions are received is related as,

$$\begin{aligned} D_0^j &\leq D_0^{j-1}, \\ R_0^j &\geq R_0^{j-1}. \end{aligned} \tag{2}$$

For satisfying the above relationship we should know the corresponding relationship between the number of quantizer bins between the MDSQs $j-1$ and j.

Let n_0^{j-1} and n_0^j be the number of quantizer bins of the central quantizers of MDSQs $j-1$ and j, respectively. In order to satisfy the above constraints, they should be related as,

$$n_i^j = a n_i^{j-1}, \tag{3}$$

where a is an integer and $a > 1$. In other words, δ_{j-1} and δ_j, the quantizer bin sizes of the MDSQs $j-1$ and j, respectively, are related as

$$\delta_j = \frac{\delta_{j-1}}{a}. \tag{4}$$

J multiple MDSQs designed using the condition in Eq. (3) (or Eq. (4)), generate $2J$ number of descriptions. When N number of descriptions, where $N < 2J$, are jointly decoded the rate distortion performances constrained by the following relationships are desired.

$$\begin{aligned} D_0^{1,2,\cdots,N} &< D_0^J, \\ D_0^{1,2,\cdots,N} &< D_i^j, \\ R_0^1 \leq \cdots \leq R_0^N &\leq R_0^J, \end{aligned} \tag{5}$$

where $D_0^{1,2,\cdots,N}$ is the distortion of combination of N descriptions from $2J$ descriptions.

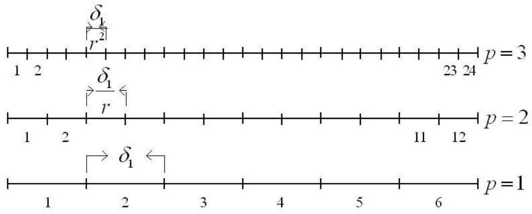


Figure 2: Embedded quantizer for three levels $P = 3$.

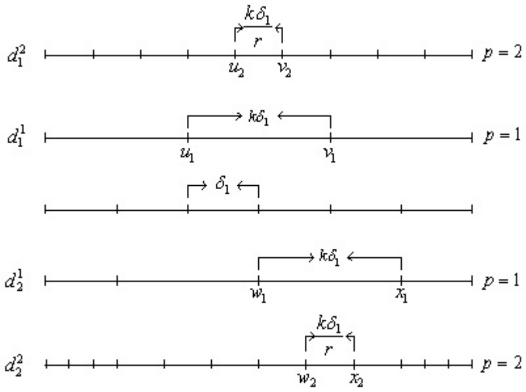


Figure 3: Embedded side quantizers for $P = 2$.

4. MDSQ WITH SUCCESSIVE REFINEMENT (MDSQ-SR)

We extend the multi-channel unbalanced description MDSQ result to formulate a framework for MDSQ with successive refinement and propose a scalable multiple description coding scheme using the new MDSQ-SR scheme. In embedded quantization used in scalable image coding, the quantizer bins at higher data rates are embedded within the quantizer bins of lower data rates.

Let P be the number of levels of embedded quantizers, *i.e.,* $C^0, C^1, \cdots, C^P$. Then the quantizer bins of C^p are embedded within the quantizer bins of C^{p-1}, where p is the embedded level index. In other words, each quantizer bin of the quantizer C^{P-1} is split into more number of bins to form the quantizer C^p. If n is the number of quantizer bins at quantizer C^{p-1} and each quantizer bin is split into r number of bins then the total number of bins at quantizer C^p is rn. Figure 2 shows the embedded quantizer having 3 levels with $r = 2$.

In order to adapt the concept of embedded quantization in the side quantizers within the MDC scheme to obtain MDSQ with successive refinement we formulate the new constraints and derive the conditions as follows. For the first quantization level (*i.e.,* $p = 1$), the initial MDSQ design approach in Section 3 is considered and the rate distortion constraint remains the same as in Eq. (1). Let d_1^p and d_2^p be the descriptions at quantization level p. The distortion of the side and the implicit central descriptions at quantization level p are D_1^p, D_2^p and D_0^p, respectively. The quantizer bins of the two side quantizers generated by the MDSQ are split into further smaller bins for the refinement with refinement or split factor (r). Figure 3 shows the side and central quantizers for 2 levels of embedded quantization. Let δ_1 be the quantizer bin width of the central quantizer at $p = 1$ leading to the side quantizer bin width of $k\delta_1$ at level $p = 1$. Let u_p, v_p and w_p, x_p be the minimum and maximum value of any quantizer bin of the side quantizers 1 and 2, at level p respectively. For $p = 1$, the values of u_1, v_1 and w_1, x_1 are related as,

$$\begin{aligned} v_1 &= u_1 + k\delta_1, \\ x_1 &= w_1 + k\delta_1. \end{aligned} \tag{6}$$

Since the two side quantizers bins are either leading or lagging each other, w_1 and u_1 are related as

$$w_1 = u_1 \pm (k-1)\delta_1. \tag{7}$$

As an example, the values of u_p,v_p,w_p and x_p for $p = 2$ are related as,

$$\begin{aligned} u_2 &= u_1 + \frac{ik\delta_1}{r}, \\ v_2 &= u_2 + \frac{k\delta_1}{r} = u_1 + (i+1)\frac{k\delta_1}{r}, \end{aligned} \tag{8}$$

where $i = 0, 1, \cdots, r-1$, and

$$\begin{aligned} w_2 &= w_1 + \frac{jk\delta_1}{r}, \\ x_2 &= w_2 + \frac{k\delta_1}{r} = w_1 + (j+1)\frac{k\delta_1}{r}, \end{aligned} \tag{9}$$

where $j = 0, 1, \cdots, r-1$.

As we know, that for any MDC scheme the distortion of the combined description has to be less than the distortion of the individual description at required rate. Similarly for the successive refinement quantizers-based MDC scheme the distortion of the individual description at quantization level p should be less than the distortion at quantization level $p-1$. On the other hand, the distortion of combined descriptions at level p is not only less than the distortion of both description at level $p-1$ but also less than the distortion of individual description at level p. We summarize these constraints as follows:

$$D_i^p < D_i^{p-1}, \tag{10}$$

$$D_0^p < D_0^{p-1}, \tag{11}$$

$$D_0^p < D_i^p, \tag{12}$$

provided that $R_0^p > R_0^{p-1}$, where $i = 1, 2$ and $p = 1, 2, \cdots, P$. For satisfying these constraints for MDC using embedded side quantizers we propose the following quantizer conditions on the values of k and r.

Proposition 1 *In order to satisfy the side quantizer distortion constraints between successive refinements when two descriptions are joint decoded at* $p > 1$ *the split factor should be greater than one,* i.e., $r > 1$.

Proof: With reference to Figure 3 and for satisfying Eq. (10), we need

$$(x_2 - w_2) < (x_1 - w_1),$$
$$(v_2 - u_2) < (v_1 - u_1).$$

Since the quantizer bin size of the side quantizers is k times that of the central quantizer at level $p = 1$, we can rewrite the above as

$$\frac{k\delta_1}{r} < k\delta_1,$$

which is simplified to $r > 1$. ■.

Proposition 2 *In order to satisfy the distortion constraints between the successive refinement and the corresponding implicit central quantizers of the current and the previous levels when joint decoded, r and k should not be integer multiples of each other and $r > k$.*

Proof: With reference to Figure 3 and for satisfying Eq. (11) and Eq. (12), we need

$$min(x_2, v_2) - max(u_2, w_2) < min(x_1, v_1) - max(u_1, w_1),$$

which can be simplified as follows:

$$min(x_2, v_2) \neq min(x_1, v_1), \quad (13)$$
$$max(u_2, w_2) \neq max(u_1, w_1). \quad (14)$$

As shown in Eq. (7), w_1 and u_1 are related to each other by two forms: w_1 is either leading or lagging u_1 by the factor $(k-1)\delta$. Here we first consider $w_1 = u_1 - (k-1)\delta_1$ relationship. For this case, the other values are

$$\begin{aligned} w_2 &= w_1 + \frac{jk\delta_1}{r} = u_1 - (k-1)\delta_1 + j\frac{k\delta_1}{r}, \\ x_1 &= u_1 - (k-1)\delta_1 + k\delta_1 = u_1 + \delta_1, \\ x_2 &= u_1 - (k-1)\delta_1 + (j+1)\frac{k\delta_1}{r}. \end{aligned}$$

Since $min(x_1, v_1) = u_1 + \delta_1$, Eq. (13) can be rewritten as

$$min(x_2, v_2) \neq u_1 + \delta_1.$$

But, $min(x_2, v_2) = u_1 + \delta_1$ when the following two conditions are satisfied: Firstly,

$$\begin{aligned} u_1 - (k-1)\delta_1 + (j+1)\frac{k\delta_1}{r} &= u_1 + \delta_1, \\ j &= \frac{r}{k}(k - \frac{k}{r}), \end{aligned}$$

which is simplified to $j = r - 1$.
Secondly,

$$u_1 + (i+1)\frac{k\delta_1}{r} = u_1 + \delta_1,$$

which is only possible if $i = 0$ and $k = r$.

On the other hand, since

$$max(u_1, w_1) = u_1,$$

Eq. (14) can be rewritten as

$$max(u_2, w_2) \neq u_1.$$

But $max(u_2, w_2) = u_1$ when the following two conditions are satisfied: Firstly,

$$u_1 + \frac{ik\delta_1}{r} = u_1,$$

which is only possible if $i = 0$.
Secondly,

$$\begin{aligned} u_1 - (k-1)\delta_1 + \frac{jk\delta_1}{r} &= u_1, \\ j &= \frac{r}{k}(k-1), \end{aligned}$$

which is only possible if k and r are integer multiples of each other and $r > k$. ■.

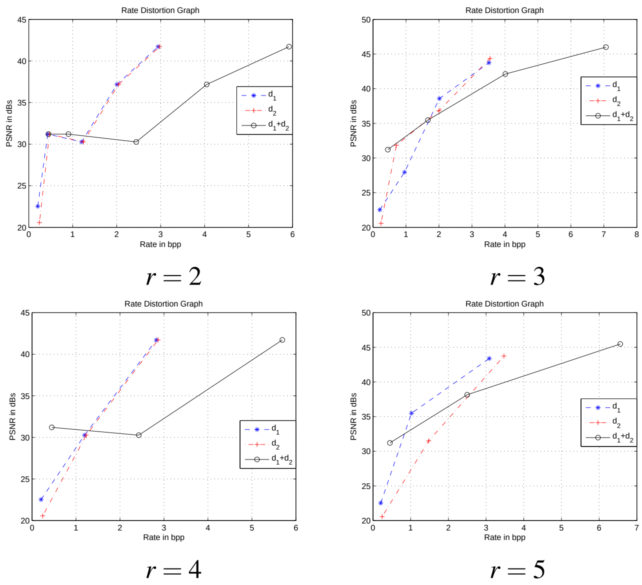


Figure 4: Row 1: Rate Distortion plots of Gold Hill image for $N = 2$ descriptions when $k = 2$ for different r.

These propositions and proves show that if $k = r$ and k and r are multiple of each other, the constraints in Eq. (11) and Eq. (12) are not satisfied. The same can be shown for the second condition, *i.e.,* $w_1 = u_1 + (k-1)\delta_1$. Thereby, we design a series of side quantizers that allow successive refinement for reducing the distortion of the joint decoding when the number of descriptions decoded are increased.

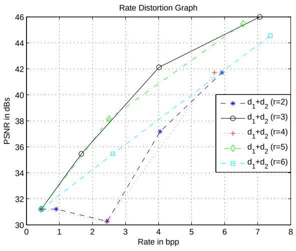
5. SIMULATIONS AND RESULTS

We evaluate the proposed scheme in two steps: Firstly, considering transmission along a lossless channel (in order to study the rate distortion performance for different values of r) and secondly considering transmission along a lossy (packet erasure) channel.

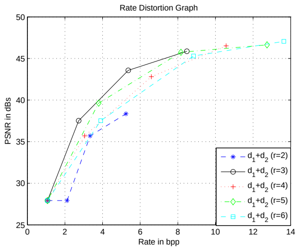
Figure 4 shows the rate distortion plots of the side and central description for $r = 2, 3, 4, 5$ and $k = 2$ for Gold Hill image. It is evident from figures, that when $r = 3$ and $r = 5$ the rate distortion performance is better when joint decoded than when individually decoded. On the other hand, for $r = 2$ and $r = 4$, the rate distortion plot when joint decoded is the same as if they are individually decoded. Simulation results verify the conditions proposed in this paper, *i.e.,* the split or refinement factor r should be greater than the number of diagonals k filled in the index assignment matrix and are not integer multiples of each other.

For the comparison of the rate distortion performance when both descriptions are received for different values of r when $k = 2$ is shown in Figure 5. It is evident that the rate distortion plots for odd values of r when $k = 2$ are better than those for even values of r.

The second set of experiments we performed was to evaluate the designed coder for transmission over a packet erasure channel. Wavelet tree based packetization is used for each description. Let M be the total number of packets and p be the number of lost packets. There is a total of ${}^{M}C_p$ number of combinations to loose p packets from M packets. The average PSNR at particular number of packet loss is then cal-

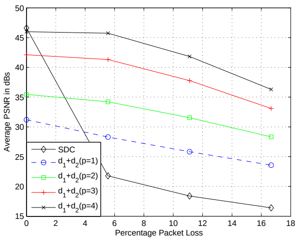


(a)

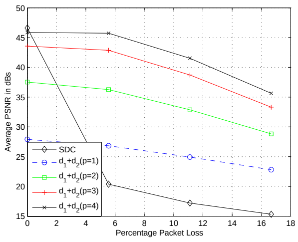


(b)

Figure 5: Rate Distortion Graph when both descriptions are received for different values of r when $k = 2$ for (a) Gold Hill and (b) Barbara images.



(a)



(b)

Figure 6: Effect of packet loss on the PSNR for all loss patterns for a number of 18 transmitted packets and probability of loss for (a) Gold Hill and (b) Barbara images.

culated by measuring and averaging the PSNR of all possible combinations. The average PSNR value of the decoded image for different quantization level P and single embedded description is shown in Figure 6 for Gold Hill and Barbara images.

Decoded Gold Hill images at the same quantization level but different splitting factor r is shown in Figure 7. It is clear from images that the MDC based on the embedded side quantizers outperforms the embedded single description coding (SDC) case. Furthermore, the decoded image quality is better for the higher values of r provided that the conditions proposed for r and k are satisfied.

6. CONCLUSIONS

We have extended the conditions for unbalanced multi channel MDC to define a new framework for MDSQ with successive refinement. We proposed that the split or refinement factor r for each of the refinement side quantizers should be greater than the number of diagonals k filled in the index assignment matrix of the side quantizer design and should not be integer multiples of each other, in order to improve the fidelity (to reduce the distortion) in joint decoding when the number of descriptions included in joint decoding is increased. We demonstrated the use of the proposed method to integrate quality scalable decoding in the MDSQ-based multiple description image coding. The experimental results show high robustness of the proposed scheme compared to single description scalable image coding. Our future work includes integration of resolution scalability in to this framework in order to obtain highly robust full scalable image bit streams.

SDC

$P = 2, r = 2$

$P = 2, r = 3$ $P = 2, r = 4$

Figure 7: Portion of the Decoded Gold Hill image after 18% packet loss.

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