DESIGN OF STABLE TWO-DIMENSIONAL IIR NOTCH FILTER USING ROOT MAP

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ABSTRACT

In this paper, the two-dimensional (2-D) IIR notch filter design problem is presented. First, the linear fractional transformation (LFT) in complex analysis is used to construct the desired root map of 2-D polynomial. Then, the numerator and denominator of transfer function of 2-D IIR notch filter can be obtained from the LFT used in the construction of root map. This design not only has closed-form transfer function but also satisfies the bounded input/bounded output (BIBO) stability condition. Finally, design example and image application are demonstrated to show the effectiveness of the proposed approach.

1. INTRODUCTION

In many signal processing applications, there is a need for a notch filter which is characterized by a unit gain at all frequencies except at the sinusoidal frequencies where their gain is zero. These applications include communication, control, image processing and biomedical engineering etc. In one-dimensional (1-D) case, a typical example is to cancel 50 or 60 Hz power line interference in the recording of electrocardiograms (ECGS) [1]. In two-dimensional (2-D) case, two examples are to eliminate a 2-D sinusoidal interference pattern superimposed on an image [2] and to reduce blocking artifact from DCT coded image [3]. Thus, it is interesting to design notch filter to remove the sinusoidal interferences corrupted on a desired signal.

The ideal frequency response of 2-D notch filter is given by

$$D(\omega_1,\omega_2)=\begin{cases}0 & (\omega_1,\omega_2)=\pm(\omega_{1N},\omega_{2N})\\ 1 & otherwise\end{cases} \tag{1}$$

where $(\omega_{1N},\omega_{2N})$ is the prescribed notch frequency. The problem is how to design 2-D filter $H(z_1,z_2)$ to approximate $D(\omega_1,\omega_2)$ as well as possible. So far, the design of 2-D notch filter can be classified into two categories. One is FIR filter design, the other is IIR filter design. In the FIR case, the design methods include least squares approach [4], transformation techniques [5], singular value decomposition approach [6] and constrained nonlinear optimization method [7]. In the IIR case, the design approaches contain decomposition method [8] and outer product expansion [9]. So far, the 2-D adaptive IIR notch filters based on outer product expansion have been developed for removing sinusoidal interference with unknown or time-varying frequencies [10][11].

On the other hand, the root map of 2-D polynomial has been a useful tool for checking the stability of 2-D IIR digital filters [12]-[14]. Due to the success of this tool in stability checking, we will use the root map to design 2-D notch filter in this paper. The proposed design not only has closed-form transfer function but also satisfies the BIBO stability condition. Compared with conventional methods in [8][9], our method does not need to decompose the 2-D notch filter design into several sub-filter designs, so the proposed approach is easier to use than conventional methods. Now, the design details are described in next sections.

2. ROOT MAP AND LINEAR FRACTIONAL TRANSFORM

In this section, the root map of 2-D polynomial is first defined. Then, we study the relation between root map and stability of 2-D IIR filter. Finally, the linear fractional transformation is described.

2.1 Definition of Root Map

Consider the 2-D polynomial with order (N_1,N_2) below:

$$P(z_1,z_2)=\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_2}p(k_1,k_2)z_1^{-k_1}z_2^{-k_2} \tag{2}$$

If $P(z_1,z_2)=0$ is regarded as a mapping between the z_1 plane and z_2 plane, the root map is defined as the image of the unit circle of one variable in the plane of other variable under this implicit mapping $P(z_1,z_2)=0$. Now, let us describe the details of these two maps below: First, the 2-D polynomial $P(z_1,z_2)$ can be rearranged as the following form:

$$\begin{aligned}P(z_1,z_2)&=\sum_{k_1=0}^{N_1}\left(\sum_{k_2=0}^{N_2}p(k_1,k_2)z_2^{-k_2}\right)z_1^{-k_1}\\ &=P[z_2](z_1)\end{aligned} \tag{3}$$

The above expression $P[z_2](z_1)$ can be interpreted as 1-D polynomial in variable z_1 whose coefficients are polynomial in variable z_2. When z_2 is fixed, the $P[z_2](z_1)$ can be factored to yield N_1 roots. Thus, the first root map Γ_1 is defined as the loci of N_1 roots of $P[z_2](z_1)$ when parameter z_2 traverses the unit circle $z_2=e^{j\omega_2}$. Second, the 2-D polynomial $P(z_1, z_2)$ can be rearranged as the following form:

$$P(z_1, z_2) = \sum_{k_2=0}^{N_2}\left(\sum_{k_1=0}^{N_1} p(k_1, k_2) z_1^{-k_1}\right) z_2^{-k_2} \quad (4)$$
$$= P[z_1](z_2)$$

where the expression $P[z_1](z_2)$ can be interpreted as 1-D polynomial in variable z_2 whose coefficients are polynomial in variable z_1. When z_1 is fixed, the $P[z_1](z_2)$ can be factored to yield N_2 roots. Thus, the second root map Γ_2 is defined as the loci of N_2 roots of $P[z_1](z_2)$ when parameter z_1 traverses the unit circle $z_1 = e^{j\omega_1}$.

2.2 Stability and Root Map

For the 2-D first quadrant IIR filter with order (M_1, M_2, N_1, N_2), its transfer function is given by

$$H(z_1, z_2) = \frac{\sum_{k_1=0}^{M_1}\sum_{k_2=0}^{M_2} q(k_1, k_2) z_1^{-k_1} z_2^{-k_2}}{\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_2} p(k_1, k_2) z_1^{-k_1} z_2^{-k_2}} \quad (5)$$
$$= \frac{Q(z_1, z_2)}{P(z_1, z_2)}$$

In [12], Shanks has shown that $H(z_1, z_2)$ is stable if and only if the following conditions are true:

$$P(z_1, z_2) \neq 0 \quad if \quad |z_1| \geq 1, \ |z_2| = 1 \quad (6a)$$
$$P(z_1, z_2) \neq 0 \quad if \quad |z_1| = 1, \ |z_2| \geq 1 \quad (6b)$$

It is obvious that the condition in Eq.(6a) implies that the root map Γ_1 of the unit circle of variable z_2 lies inside the unit circle in the z_1 plane. And, the condition in Eq.(6b) means that the root map Γ_2 of the unit circle of z_1 lies within the unit circle in the z_2 plane. Thus, stability condition of Shank's theorem will be satisfied if two root maps lie within unit circle. So, we can control the stability of 2-D IIR filter by controlling two root maps to lie inside the unit circle.

2.3 Linear Fractional Transform

In [15], the linear fractional transformation (LFT) from complex variable z_1 to z_2 is defined by

$$z_2 = \frac{c_1 z_1 + c_2}{c_3 z_1 + c_4} \quad (7)$$

where c_1, c_2, c_3 and c_4 are constants. It can be shown that the LFT transforms circles and lines into circles and lines. Moreover, the Eq.(7) can be rewritten in the form

$$c_3 + c_4 z_1^{-1} - c_1 z_2^{-1} - c_2 z_1^{-1} z_2^{-1} = 0 \quad (8)$$

This expression is the same as $P(z_1, z_2) = 0$ with order $(1,1)$ if we choose $p(0,0) = c_3$, $p(1,0) = c_4$, $p(0,1) = -c_1$, and $p(1,1) = -c_2$. Thus, we can use the properties of LFT in complex analysis to construct the desired root maps of $P(z_1, z_2) = 0$ with order $(1,1)$. In next section, the details that design 2-D IIR notch filter using root map and LFT will be described.

3. DESIGN OF 2-D IIR NOTCH FILTER

In this section, we will summarize some important root maps. Then, these root maps are used to design 2-D IIR notch filter. The first important root map is described by the following fact:

Fact 1: Choosing parameters

$$a = \frac{\rho^2 - r^2 - r}{r+1}, \quad b = \frac{-\rho}{r+1} \quad (9)$$

and let 2-D polynomial with order $(1,1)$ be

$$B(z_1, z_2) = a z_1^{-1} z_2^{-1} + b z_1^{-1} + b z_2^{-1} + 1 \quad (10)$$

then we have the results:

(1) The two root maps Γ_1 and Γ_2 are the same.

(2) The two root maps are both circles with center at ρ and radius r.

(3) The equalities $B(1, \rho + r) = 0$ and $B(\rho + r, 1) = 0$ hold.

Proof: Because $B(z_1, z_2) = 0$ is a special LFT in complex variable textbook in [15], this fact can be proved by using techniques in [15]. The details are described below. Since the symmetry property $B(z_1, z_2) = B(z_2, z_1)$ is valid, two root maps are the same. So, we only need to find the root map Γ_2 because Γ_1 is the same as Γ_2. To find Γ_2, let complex variables z_1 and z_2 be represented by

$$z_1 = x + jy$$
$$z_2 = u + jv \quad (11)$$

Substituting Eq.(11) into $B(z_1, z_2) = 0$, it yields

$$(b + u)x - vy = -bu - a$$
$$vx + (b + u)y = -bv \quad (12)$$

Since z_1 is on the unit circle, we have the constraint $x^2 + y^2 = 1$. Based on this constraint and Eq.(12), we get

$$\left(u - \frac{b(a-1)}{1-b^2}\right)^2 + v^2 = \left(\frac{a-b^2}{1-b^2}\right)^2 \quad (13)$$

Substituting Eq.(9) into Eq.(13), we have

$$(u - \rho)^2 + v^2 = r^2 \quad (14)$$

This means that the root map Γ_2 is a circle with center at ρ and radius r. Finally, it is easy to show that the equalities $B(1, \rho + r) = 0$ and $B(\rho + r, 1) = 0$ hold by using direct substitution.

Based on the results in Fact 1, we have the further results of root map below:

Fact 2: Let 2-D polynomial with order $(2,2)$ be

$$F(z_1, z_2, \rho, r, \theta_1, \theta_2) = [1, \ z_1^{-1}, \ z_1^{-2}]\Phi \begin{bmatrix} 1 \\ z_2^{-1} \\ z_2^{-2} \end{bmatrix} \quad (15)$$

where matrix Φ is

$$\Phi = \begin{bmatrix} 1 & 2b\cos(\theta_2) & b^2 \\ 2b\cos(\theta_1) & \begin{matrix}(2a\cos(\theta_1+\theta_2) \\ +2b^2\cos(\theta_1-\theta_2))\end{matrix} & 2ab\cos(\theta_1) \\ b^2 & 2ab\cos(\theta_2) & a^2 \end{bmatrix} \quad (16)$$

with parameters $a = \frac{\rho^2 - r^2 - r}{r+1}$ and $b = \frac{-\rho}{r+1}$, then we have the results:

(1) The root map Γ_1 in z_1 plane consists of two circles with center at $\rho e^{\pm j\theta_1}$ and radii r.

(2) The root map Γ_2 in z_2 plane consists of two circles with center at $\rho e^{\pm j\theta_2}$ and radii r.

(3) The equalities $F(e^{j\theta_1}, (\rho+r)e^{j\theta_2}, \rho, r, \theta_1, \theta_2) = 0$ and $F(e^{-j\theta_1}, (\rho+r)e^{-j\theta_2}, \rho, r, \theta_1, \theta_2) = 0$ hold.

(4) The equalities $F((\rho+r)e^{j\theta_1}, e^{j\theta_2}, \rho, r, \theta_1, \theta_2) = 0$ and $F((\rho+r)e^{-j\theta_1}, e^{-j\theta_2}, \rho, r, \theta_1, \theta_2) = 0$ hold.

Proof: The 2-D polynomial in Eq.(15) can be factorized as

$$F(z_1, z_2, \rho, r, \theta_1, \theta_2) = B_1(z_1, z_2) B_2(z_1, z_2) \quad (17)$$

where

$$B_1(z_1, z_2) = a(z_1 e^{-j\theta_1})^{-1}(z_2 e^{-j\theta_2})^{-1} + b(z_1 e^{-j\theta_1})^{-1} + b(z_2 e^{-j\theta_2})^{-1} + 1 \quad (18a)$$

$$B_2(z_1, z_2) = a(z_1 e^{j\theta_1})^{-1}(z_2 e^{j\theta_2})^{-1} + b(z_1 e^{j\theta_1})^{-1} + b(z_2 e^{j\theta_2})^{-1} + 1 \quad (18b)$$

Thus, $F(z_1, z_2, \rho, r, \theta_1, \theta_2) = 0$ implies that $B_1(z_1, z_2) = 0$ or $B_2(z_1, z_2) = 0$. Using this factorization and the results in Fact 1, the proof can be shown easily.

Now, let us use the Fact 2 to construct the transfer function of 2-D IIR notch filter with order $(2, 2, 2, 2)$. The design problem is how determine the filter coefficients $p(k_1, k_2)$ and $q(k_1, k_2)$ in Eq.(5) such that the frequency response $H(e^{j\omega_1}, e^{j\omega_2})$ approximates the ideal response $D(\omega_1, \omega_2)$ in Eq.(1) as well as possible if the notch frequency $(\omega_{1N}, \omega_{2N})$ is given. In this paper, the key step to design IIR notch filter $H(z_1, z_2)$ is to choose numerator polynomial $Q(z_1, z_2)$ and denominator polynomial $P(z_1, z_2)$ in Eq.(5) as the form:

$$Q(z_1, z_2) = F(z_1, z_2, \rho_1, r_1, \omega_{1N}, \omega_{2N}) \quad (19a)$$

$$P(z_1, z_2) = F(z_1, z_2, \rho_2, r_2, \omega_{1N}, \omega_{2N}) \quad (19b)$$

where parameters ρ_1, r_1, ρ_2, r_2 need to be prescribed. In this paper, we choose these four parameters below:

$$\rho_1 = 1 - \alpha, \quad r_1 = \alpha \quad (20a)$$

$$\rho_2 = 1 - \alpha, \quad r_2 = \alpha - \varepsilon \quad (20b)$$

where α is a number in the interval $(0,1)$, and ε is a very small positive number. Based on this choice, the root maps of numerator polynomial $Q(z_1, z_2)$ and denominator polynomial $P(z_1, z_2)$ are shown in Fig.1. Now, several remarks are made below:

(1) From Eq.(20a), we have $\rho_1 = 1 - r_1$. This equality makes the two root map circles of numerator polynomial $Q(z_1, z_2)$ be tangent to the unit circle. Using Fact 2 and $\rho_1 = 1 - r_1$, it can be shown that

$$Q(e^{j\omega_{1N}}, e^{j\omega_{2N}}) = Q(e^{-j\omega_{1N}}, e^{-j\omega_{2N}}) = 0 \quad (21)$$

Thus, there is zero gain at the notch frequency for the designed IIR notch filter.

(2) The reason to choose ε as a positive number is to make the root map circles of denominator polynomial $P(z_1, z_2)$ lie within unit circle. Thus, the designed IIR notch filter is guaranteed stable.

(3) In order to obtain unit gain at all frequencies except at the notch frequency, the ε must be chosen as small as possible. When ε is very small, the root maps of $P(z_1, z_2)$ will be very close to ones of $Q(z_1, z_2)$.

(4) Because we choose $\rho_2 = \rho_1$, the root map circles of $P(z_1, z_2)$ and $Q(z_1, z_2)$ have the same center. Thus, we may refer to this design as "concentric circle design".

Finally, substituting Eq.(19) and Eq.(20) into Eq.(5), the transfer function of the designed 2-D IIR notch filter is given by

$$H(z_1, z_2) = \frac{Q(z_1, z_2)}{P(z_1, z_2)} = \frac{F(z_1, z_2, 1-\alpha, \alpha, \omega_{1N}, \omega_{2N})}{F(z_1, z_2, 1-\alpha, \alpha-\varepsilon, \omega_{1N}, \omega_{2N})} \quad (22)$$

Using Eq.(15), the above transfer function can be computed easily without requiring optimization and complicated calculation. In next section, numerical examples will be used to illustrate the effectiveness of the proposed design.

4. DESIGN EXAMPLE AND APPLICATION

In this section, one design example of proposed 2-D IIR notch filter is first illustrated. Then, we compare the proposed design method with conventional outer product expansion method in [9]. Finally, we use the 2-D IIR notch filter to remove sinusoidal interference superimposed on an image.

Example 1: 2-D IIR Notch Filter Design

In this example, we will study the performance of the designed 2-D IIR notch filter. The notch frequency is chosen as $(\omega_{1N}, \omega_{2N}) = (0.5\pi, 0.5\pi)$. And, the design parameters are $\alpha = 0.2$ and $\varepsilon = 0.001$. Fig.2(a) show the resultant magnitude response. It is clear that $|H(e^{j\omega_1}, e^{j\omega_2})|$ has unit gain at all frequencies except at notch frequency $(0.5\pi, 0.5\pi)$ where gain is zero. So, the specification is fitted well. However, the details of the notch are underneath the unit gain plane. In order to show the performance of the designed filter better, Fig.2(b) plots the loss $1 - |H(e^{j\omega_1}, e^{j\omega_2})|$. From this result, we observe that the magnitude response has small ripples in the vicinity of the notch. Fortunately, this ripple can be reduced by reducing the parameter ε. To illustrate this fact, Fig.2(c)(d) show the magnitude response and loss of notch filter when ε is reduced to 0.0001 and other design parameters are not changed. From this result, it is clear that error ripple near notch frequency has been reduced. Moreover, it is interesting to study the shape of 3-dB contour of notch in which the magnitude $|H(e^{j\omega_1}, e^{j\omega_2})|$ is equal to $1/\sqrt{2}$. Fig.3 shows the 3-dB contours of the designed 2-D IIR notch filters $H(z_1, z_2)$ for various parameters α and ε when notch frequency is chosen as $(\omega_{1N}, \omega_{2N}) = (0.5\pi, 0.5\pi)$. It is clear that the 3-dB contour is symmetric about the notch frequency $(0.5\pi, 0.5\pi)$. And, the size of 3-dB contour is reduced if parameters α and ε are reduced. Thus, we can use parameters α and ε to control the shape of 3-dB contour of notch.

Example 2: Comparison with Conventional 2-D IIR Notch Filter Design

In this example, we will compare the proposed design method with conventional outer product expansion method in [9]. Given the notch frequency $(\omega_{1N}, \omega_{2N})$ and parameter BW, the transfer function of conventional 2-D IIR notch filter in [9] is given by

$$H(z_1, z_2) = 1 - \frac{1}{2} H_{b1}(z_1) H_{b2}(z_2) \left(1 - H_{a1}(z_1) H_{a2}(z_2)\right) \quad (23)$$

where filters

$$H_{bi}(z_i) = \frac{1}{2}\left(1 - \frac{a_{i2} - a_{i1} z_i^{-1} + z_i^{-2}}{1 - a_{i1} z_i^{-1} + a_{i2} z_i^{-2}}\right) \quad i=1,2 \quad (24)$$

$$H_{ai}(z_i) = \frac{b_i + z_i^{-1}}{1 + b_i z_i^{-1}} \quad i=1,2 \quad (25)$$

with the coefficients

$$a_{i1} = \frac{2\cos(\omega_{iN})}{1 + \tan\left(\frac{BW}{2}\right)}, \quad a_{i2} = \frac{1 - \tan\left(\frac{BW}{2}\right)}{1 + \tan\left(\frac{BW}{2}\right)} \quad i=1,2 \quad (26)$$

$$b_i = \frac{\sin\left(\frac{\omega_{iN}}{2} - \frac{\pi}{4}\right)}{\sin\left(\frac{\omega_{iN}}{2} + \frac{\pi}{4}\right)} \quad i=1,2 \quad (27)$$

Fig.4 shows the magnitude response $|H(e^{j\omega_1}, e^{j\omega_2})|$ and the loss $1 - |H(e^{j\omega_1}, e^{j\omega_2})|$ of this conventional IIR notch filter for $(\omega_{1N}, \omega_{2N}) = (0.5\pi, 0.5\pi)$ and $BW = 0.005\pi$. Comparing Fig.2 with Fig.4, it can be seen that the performance of the proposed design method is similar to one of the conventional method in [9]. However, the order of the proposed filter $H(z_1, z_2)$ in Eq.(22) is $(2, 2, 2, 2)$ and the order of conventional filter $H(z_1, z_2)$ in Eq.(23) is $(3, 3, 3, 3)$. So, the implementation complexity of proposed filter will be less than the complexity of conventional filter if the direct-form realization is chosen.

Example 3: Sinusoidal Interference Removal on Image

In this example, we will use the proposed IIR notch filter to remove the sinusoidal interference superimposed on an image with size 512×512. The image shown in Fig.5(a) is the Lake image corrupted by a sinusoidal interference below:

$$50 \sin(0.1\pi m + 0.2\pi n) \quad (28)$$

Now, let us design a 2-D IIR notch filter with notch frequency $(\omega_{1N}, \omega_{2N}) = (0.1\pi, 0.2\pi)$, $\alpha = 0.1$ and $\varepsilon = 0.01$ to remove this sinusoidal interference in spatial domain. The filtered image is shown in Fig.5(b). It is clear that the interference has been removed by the proposed 2-D IIR notch filter. In the above, only the case of single sinusoid is studied. For the case of multiple sinusoidal interferences, the cascade notch filter can be used to remove the multiple interferences if the notch frequencies are chosen as the frequencies of sinusoids.

5. CONCLUSIONS

In this paper, the root map has been presented to design a stable 2-D IIR notch filter. First, the linear fractional transformation (LFT) in complex analysis is used to construct the desired root map of 2-D polynomial. Then, the numerator and denominator of transfer function of 2-D IIR notch filter can be obtained from the LFT used in the construction of root map. Finally, design example and image application are demonstrated to show the effectiveness of the proposed approach. However, only IIR notch filter design is considered here. Thus, it is interesting to design other 2-D IIR filters by using root map in the future.

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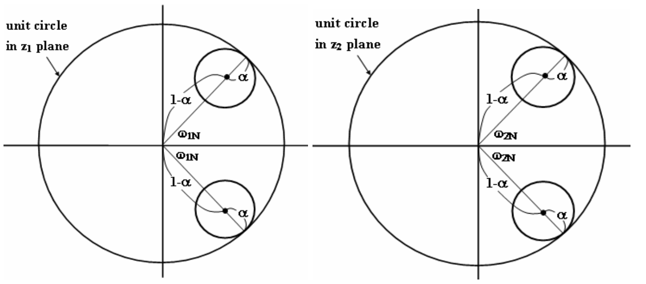
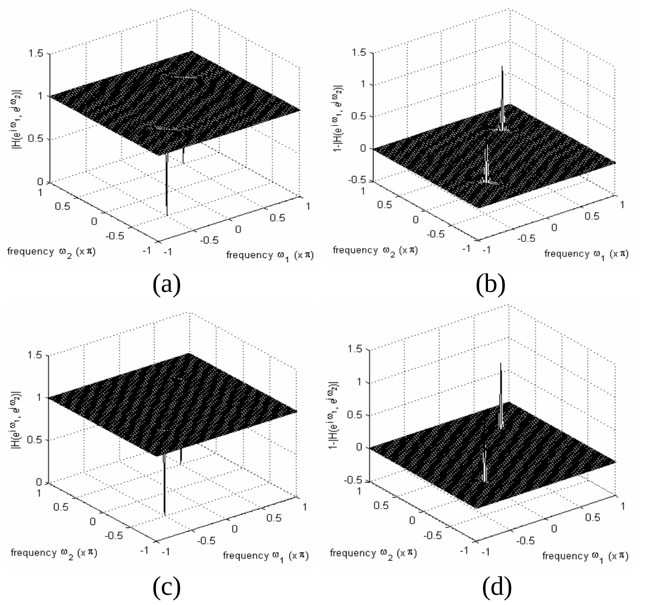


(a) (b) (c) (d)

Fig.1 The root maps of numerator polynomial $Q(z_1, z_2)$ and denominator polynomial $P(z_1,z_2)$ of 2-D IIR notch filter (a) Root map Γ_1 of $Q(z_1,z_2)$. (b) Root map Γ_2 of $Q(z_1,z_2)$. (c) Root map Γ_1 of $P(z_1,z_2)$. (d) Root map Γ_2 of $P(z_1,z_2)$.



(a) (b) (c) (d)

Fig.2 The magnitude response $|H(e^{j\omega_1},e^{j\omega_2})|$ and loss $1-|H(e^{j\omega_1},e^{j\omega_2})|$ of the designed IIR notch filter (a)(b) The results with $\alpha = 0.2$ and $\varepsilon = 0.001$. (c)(d) The results with $\alpha = 0.2$ and $\varepsilon = 0.0001$.

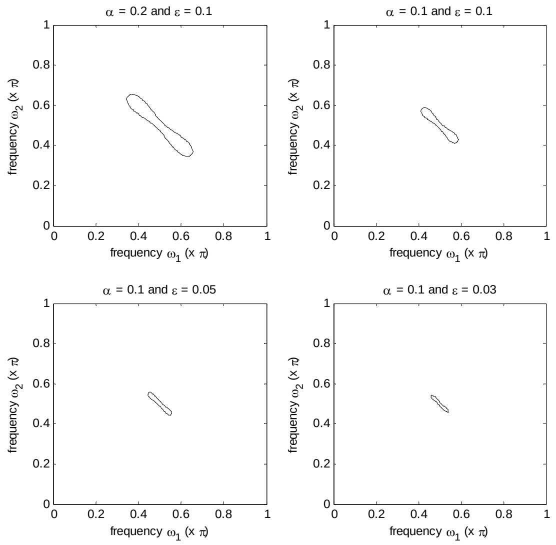
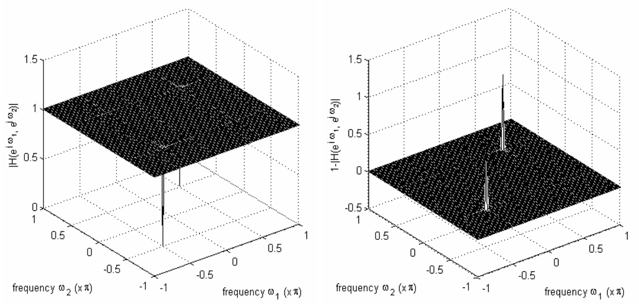
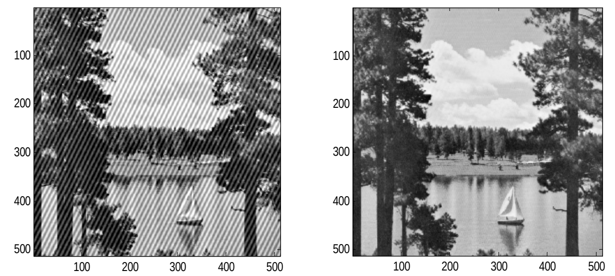


Fig.3 The 3-dB contours of the designed 2-D IIR notch filters $H(z_1, z_2)$ for various parameters α and ε when notch frequency is chosen as $(\omega_{1N}, \omega_{2N}) = (0.5\pi, 0.5\pi)$.



(a) (b)

Fig.4 The magnitude response $|H(e^{j\omega_1},e^{j\omega_2})|$ and loss $1-|H(e^{j\omega_1},e^{j\omega_2})|$ of the conventional 2-D IIR notch filter for $(\omega_{1N}, \omega_{2N}) = (0.5\pi, 0.5\pi)$ and $BW = 0.005\pi$.



(a) (b)

Fig.5 Example of sinusoidal interference removal on an image (a) The corrupted image (b) The image restored by using 2-D IIR notch filter.