ROBUST BLIND EXTRACTION OF A SIGNAL WITH THE BEST MATCH TO A PRESCRIBED AUTOCORRELATION

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ABSTRACT

Several blind extraction algorithms have been proposed that extract some signal of interest from a mixture of signals. We propose a novel blind extraction algorithm that extracts the signal that has an autocorrelation closest to a prescribed autocorrelation that serves as a mold. Based on the mold we perform a linear transformation of sensor correlation matrices. This transformation allows for the construction of a matrix with a specific eigenstructure. Each eigenvalue is related to the Euclidean distance between the mold and the actual autocorrelation of one of the source signals. The extraction filter that extracts the source signal with an autocorrelation closest to the mold is identified as the eigenvector that corresponds to the smallest eigenvalue. We show that this approach is more robust to noise than methods from literature, while it exploits comparable a priori information. The results are validated by means of simulations.

1. INTRODUCTION

Blind Signal Processing (BSP) has become a major research area during the last few years [1, 2]. A hot topic in this field is the Blind Source Separation (BSS) problem. In general, a BSS algorithm separates all signals from a mixture of source signals blindly. The separation in BSS can be performed blindly up to an unknown scaling and permutation. When only one of the signals is desired a classifier has to select the desired signal. A less addressed, but practically more interesting problem is the closely related Blind Signal Extraction (BSE) problem. In BSE the classifier is efficiently incorporated in the algorithm such that no undesired signals are extracted and not all signals have to be separated.

Several signal properties have been exploited by classifiers to distinguish between sources. Examples of these properties are sparseness, non-Gaussianity, smoothness and linear predictability. In [3], a class of BSE algorithms have been proposed that extract a signal of interest based on linear prediction. The signal that has the smallest normalized mean square prediction error is extracted. By utilizing a prescribed autocorrelation one is able to design the linear prediction filter. A disadvantage of this approach is that it assumes noise free measurements, which is not realistic in practice. In [4, 5] an attempt is made to perform BSE in case of noisy measurements, however the correlation of the noise is assumed to be very simple and measurable. In these approaches the contribution of the noise to the cost function is compensated in such a way that the noise-free cost function from [3] is obtained again. Although this compensation method is valid, it is very sensitive for false assumptions on the temporal and mixing properties of the noise as well as mismatches in the estimation of the noise statistics. When a mismatch is made an undesired signal may be extracted and the performance of the extraction filter decreases.

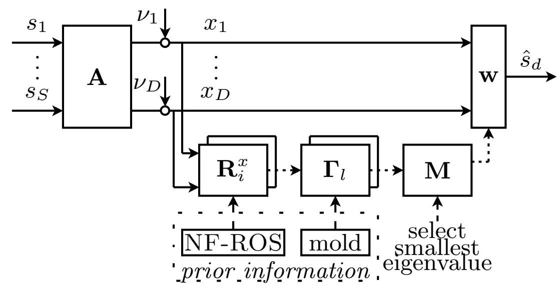

Figure 1: Novel BSE algorithm that extracts the source signal with the minimum Euclidean distance between a mold and the actual autocorrelation of the extracted source signal.

In [6] a BSE approach has been introduced to extract randomly one of the source signals. This method exploits a so-called Noise-Free Region Of Support (NF-ROS), which consists of a specifically chosen set of lags of correlation data. These lags are chosen such that the noise correlation is sufficiently small, which results in noise free sensor correlation data. The strength of this approach is that noisy correlation data is simply ignored instead of compensated. By performing a generalized eigenvalue decomposition of sensor correlation matrices, which are taken from this NF-ROS, the extraction filters are identified as generalized eigenvectors.

In the current paper we extend the work from [6]. Structure in the noise free sensor correlation data allows for the exploitation of prior information to distinguish between sources. We choose a different correlation matrix structure than is used in [6] and incorporate a prescribed autocorrelation, i.e. a mold. By performing a linear transformation of these noise free correlation matrices, based on the mold, we are able to construct a matrix from which the eigenvectors are the extraction filters. The desired extraction filter is the eigenvector that corresponds to the smallest eigenvalue, which is related to the Euclidean distance between the actual autocorrelation of the source signal and the mold.

The outline of this paper is as follows. In Section 2 we discuss the mixing model, notation and the mathematical objective of BSE. In Section 3 the second order statistics and the assumptions for a NF-ROS are introduced. Subsequently, the desired extraction filter is identified in Section 4 and the performance of the extraction filter is discussed and validated in Section 5. Finally, in Section 6 we conclude this work and give recommendations for future work.

2. BSE MODEL AND NOTATION

A model of the BSE scenario is depicted in Fig. 1. The D sensor signals $x_1[n], \cdots, x_D[n]$ with $n \in \mathbb{Z}$ are discrete samples of the continuous time signals $x_1(t), \cdots, x_D(t)$, where

$t = nT_s$ and T_s is the sampling time such that no aliasing occurs. These discrete sensor signals are assumed to be an instantaneous mixture of S source signals $s_1[n], \cdots, s_S[n]$, corrupted by D additive noise signals $\nu_1[n], \cdots, \nu_D[n]$. Mathematically, the mixing system is represented as a real valued, full rank mixing matrix $\mathbf{A} \in \mathbb{R}^{D\times S}$ and the sensor, source and noise signals are represented as column vectors $\mathbf{x}[n] \in \mathbb{R}^{D\times 1}, \mathbf{s}[n] \in \mathbb{R}^{S\times 1}$ and $\boldsymbol{\nu}[n] \in \mathbb{R}^{D\times 1}$, respectively. This vector-matrix representation allows for the following mathematical description of the sensor signals:

$$\mathbf{x}[n] = \sum_{j=1}^{S} \mathbf{a}^j s_j[n] + \boldsymbol{\nu}[n] = \mathbf{A}\mathbf{s}[n] + \boldsymbol{\nu}[n], \tag{1}$$

where $\mathbf{a}^j \in \mathbb{R}^{D\times 1}$, $\mathbf{A} = \begin{bmatrix}\mathbf{a}^1 \cdots \mathbf{a}^S\end{bmatrix}$ and

$$\mathbf{x}[n] \triangleq \begin{bmatrix} x_1[n] \\ \vdots \\ x_D[n] \end{bmatrix}, \mathbf{s}[n] \triangleq \begin{bmatrix} s_1[n] \\ \vdots \\ s_S[n] \end{bmatrix} \text{ and } \boldsymbol{\nu}[n] \triangleq \begin{bmatrix} \nu_1[n] \\ \vdots \\ \nu_D[n] \end{bmatrix}. \tag{2}$$

Row elements of a column vector are denoted by their row number as a subscript index, while column elements of a row vector are denoted with their column number as a superscript index. Matrix elements are represented with both their column and row numbers as superscript and subscript indices respectively. This notation allows for the following description of the individual sensor signals:

$$x_i[n] = \sum_{j=1}^{S} a_i^j s_j[n] + \nu_i[n] \quad \forall\, i \in \{1, \cdots, D\}. \tag{3}$$

The objective in a BSE scenario is to extract the desired signal $s_d[n]$, with $d \in \{1, \cdots, S\}$, from the sensor signals. In order to perform this extraction the sensor signals are filtered by a linear extraction filter that produces an output signal $y[n]$, as is depicted in Fig. 1. The extraction filter is represented as a row vector $\mathbf{w} \in \mathbb{R}^{1\times D}$, which results in the following description of the output signal:

$$y[n] = \sum_{i=1}^{D} w^i x_i[n] = \mathbf{w}\mathbf{x}[\mathbf{n}] = \mathbf{w}\mathbf{A}\mathbf{s}[n] + \mathbf{w}\boldsymbol{\nu}[n]. \tag{4}$$

In the current paper the extraction filter is designed to maximize the signal to noise ratio subject to suppression of all undesired source signals. From linear algebra it is known that the undesired signals are suppressed when this filter is taken from the (pseudo-)inverse of the mixing system, which only exists if there are at least the same amount of sensors as there are sources ($D \geq S$). If more sensors than sources are available the extra degrees of freedom may be utilized to improve the output signal to noise ratio. Here the focus is on the suppression of undesired source signals. Therefore we assume to have the same amount of sensors as sources ($D = S$), however in order to remain general in notation we keep using both symbols D and S where they belong.

With blind signal processing techniques it is widely known that the desired extraction filter can be determined up to an unknown scaling only, which is solved by normalizing the extraction filter. Furthermore, it is known that extra information is required to select a signal of interest. In this paper we assume to know the shape of the autocorrelation a priori, which we call a mold as is indicated in Fig. 1.

3. SECOND ORDER STATISTICS

The structure of the Second order Statistics (SOS) in the sensor signals is exploited, thus auto- and crosscorrelation functions. The correlation functions of the sensor, source and noise signals are defined as follows:

Definition 3.1. *The correlation function value of a signal pair (p_{i_1}, q_{i_2}) for all available i_1, i_2 at a given time $n \in \mathbb{Z}$ and with a certain lag $k \in \mathbb{Z}$ is defined as follows:*

$$r^{pq}_{i_1 i_2}[n,k] \triangleq \mathbb{E}\{p_{i_1}[n] q_{i_2}[n-k]\}, \tag{5}$$

where $\mathbb{E}\{\cdot\}$ is the mathematical expectation operator.

By replacing the signal pair (p_{i_1}, q_{i_2}) in Def. 3.1 by the sensor, source and noise signal pairs $(x_{i_1}, x_{i_2}), (s_{i_1}, s_{i_2})$ and (ν_{i_1}, ν_{i_2}) we obtain the corresponding correlation functions:

$$\begin{aligned} &r^x_{i_1 i_2}[n,k] \quad \forall\, 1 \leq i_1, i_2 \leq D, \\ &r^s_{i_1 i_2}[n,k] \quad \forall\, 1 \leq i_1, i_2 \leq S, \\ &r^\nu_{i_1 i_2}[n,k] \quad \forall\, 1 \leq i_1, i_2 \leq D, \end{aligned}$$

respectively. Finally we need to define the correlation functions $r^{s\nu}_{i_1 i_2}[n,k]$, which belong to the signal pairs (s_{i_1}, ν_{i_2}) for $1 \leq i_1 \leq S$ and $1 \leq i_2 \leq D$.

Most conventional methods utilize only the lags k from correlation functions, which restricts these methods to exploit only the non-whiteness property of the signals. As a result, non-stationary signals introduce a quality reduction. With the current definition of the correlation functions in Def. 3.1 for a time-lag pair (n,k), we are able to combine temporal signal properties, e.g. non-whiteness and non-stationarity. We assume that the SOS of non-stationary signals can be estimated by averaging over a number of samples close to the indicated time-lag pair.

Before we describe the structure in the sensor correlation functions we introduce some assumptions on the SOS of the source and noise signals such that we are able to define a Noise-Free Region Of Support (NF-ROS).

Definition 3.2. *The Noise-Free Region Of Support (NF-ROS), also denoted by Ω, is a set of time lag pairs (n,k) for which the noise correlation functions $r^\nu_{i_1 i_2}[n,k]$ and $r^{s\nu}_{i_1 i_2}[n,k]$ and the source crosscorrelation functions $r^s_{i_1 i_2}[n,k]$ for $i_1 \neq i_2$ equal zero. The total number of time-lag pairs in the NF-ROS is denoted by N, thus: $\Omega \triangleq \{\Omega_1, \cdots, \Omega_N\}$, where $\Omega_i = (n,k)_i$ and $|\Omega|$=N. Finally, the source autocorrelation functions $r^s_{ii}[n,k]$ are assumed sufficiently unequal in the NF-ROS such that they are linearly independent.*

Example 3.1. *Suppose that D sensors measure a mixture of S stationary, differently colored source signals that are each contaminated by additive, temporally white noise with variance σ_i^2 varying per sensor. In that case, the time index n can be ignored because the signals are stationary signals. Furthermore, lag $k = 0$ should not be taken into account because the noise contributes to the SOS of the sensor signals for that lag. The NF-ROS may be chosen as the first N lags larger than 0, thus $\Omega = \{(n,1), \cdots, (n,N)\}$ for any $n \in \mathbb{Z}$. Note: when the noisy correlation data is compensated in this scenario then the noise variances have to be measured, estimated or known a priori for each separate sensor. This requires more a priori information and is sensitive for errors. Therefore, the use of a NF-ROS is more robust.*

For time-lag pairs in the NF-ROS, $(n,k) \in \Omega$, the sensor correlation functions have the following structure:

$$r^x_{i_1 i_2}[\Omega] = \sum_{j=1}^{S} a^j_{i_1} a^j_{i_2} r^s_{jj}[\Omega] \quad \forall\, 1 \leq i_1, i_2 \leq D. \tag{6}$$

This structure can be visualized by defining the following sensor and source correlation matrices respectively:

$$\begin{aligned} \mathbf{R}^x_i &\triangleq \mathbb{E}\{\mathbf{x}[n]\mathbf{x}^T[n-k]\} \quad \forall\, (n,k)_i \in \Omega, \\ \mathbf{R}^s_i &\triangleq \mathbb{E}\{\mathbf{s}[n]\mathbf{s}^T[n-k]\} \quad \forall\, (n,k)_i \in \Omega, \end{aligned}$$

where $\mathbf{R}_i^x \in \mathbb{R}^{D\times D}$ and from the assumptions in the NF-ROS: $\mathbf{R}_i^s = \mathrm{diag}(r_{11}^s[\Omega_i], \cdots, r_{SS}^s[\Omega_i]) \in \mathbb{R}^{S\times S}$. The mutual relation between these correlation matrices follows from (6):

$$\mathbf{R}_i^x = \mathbf{A}\mathbf{R}_i^s(\mathbf{A})^T \quad \forall 1 \le i \le N. \tag{7}$$

The structure in (7) allows us to identify extraction filters.

4. FILTER IDENTIFICATION

The rationale behind our method is that extraction filters are identified as the eigenvectors of a Generalized Eigenvalue Decomposition (GEVD) of sensor correlation matrices, as was already introduced in [6].

Definition 4.1. *The GEVD of two sensor correlation matrices $\mathbf{R}_{i_1}^x$ and $\mathbf{R}_{i_2}^x$ is denoted by:*

$$\{\mathbf{w}, \lambda\} = \mathrm{gevd}\left(\mathbf{R}_{i_1}^x, \mathbf{R}_{i_2}^x\right), \tag{8}$$

where $\{\mathbf{w}, \lambda\}$ is the set of all eigenvectors and eigenvalues that solve the system: $\lambda\mathbf{w}\mathbf{R}_{i_1}^x = \mathbf{w}\mathbf{R}_{i_2}^x$.

Theorem 4.1. *Each eigenvector of a GEVD of two correlation matrices $\mathbf{R}_{i_1}^x$ and $\mathbf{R}_{i_2}^x$ for all $i_1 \neq i_2$ is the extraction filter of one of the source signals.*

Proof. The proof follows directly when we substitute (7) into Def. 4.1 and choose for the eigenvector $\mathbf{w}$ one row from the inverse of the mixing matrix $\mathbf{A}$, i.e. $\mathbf{w} = \mathbf{e}_j(\mathbf{A})^{-1}$ where $\mathbf{e}_j \in \mathbb{R}^{1\times S}$ is a vector with a one at the j'th column and zeros elsewhere. □

Although this filter identification problem is solved rather easily, we do not know which source signal is extracted when randomly an eigenvector is selected. Therefore, we exploit the structure in the generalized eigenvalues. We combine the eigenvalues with the mold in order to select the desired extraction filter.

The eigenvalues of the GEVD in (8) are given by:

$$\lambda^j = \frac{r_{jj}^s[\Omega_{i_2}]}{r_{jj}^s[\Omega_{i_1}]} \in \mathbb{R} \quad \forall j \in \{1, \cdots, S\}. \tag{9}$$

The mold gives us an a priori estimation of the two required correlation function values. Thus based on an a priori expected value of the eigenvalue we are able to identify the desired extraction filter.

In Section 4.1 we generalize these results such that we are able to search for the (absolute) smallest eigenvalue, which can help in order to develop more efficient algorithms. Furthermore, we generalize the results such that we can utilize the mold for more than two time-lag pairs only, which increases robustness.

4.1 Filter identification procedure

Suppose that the mold is given as an a priori available estimation $\mathbf{r}_e^s \in \mathbb{R}^{1\times N}$ of the autocorrelation of the desired source in the NF-ROS:

$$\mathbf{r}_e^s \triangleq [r_{ee}^s[\Omega_1] \quad \cdots \quad r_{ee}^s[\Omega_N]] \quad \forall \Omega_i \in \Omega. \tag{10}$$

We assume that for this estimate it holds that:

$$\frac{|\langle \mathbf{r}_e^s, \mathbf{r}_d^s\rangle|}{||\mathbf{r}_d^s||} > \frac{|\langle \mathbf{r}_e^s, \mathbf{r}_i^s\rangle|}{||\mathbf{r}_i^s||} \quad \forall i \neq d, \tag{11}$$

where $|\cdot|$ is the absolute value, $\langle\cdot,\cdot\rangle$ is the Euclidean inner product, $||\cdot||$ is the Euclidean norm and:

$$\mathbf{r}_i^s \triangleq [r_{ii}^s[\Omega_1] \quad \cdots \quad r_{ii}^s[\Omega_N]] \in \mathbb{R}^{1\times N} \; \forall i \in \{1, \cdots, S\}. \tag{12}$$

This assumption implies that the mold is closer to the actual autocorrelation of the desired source $\mathbf{r}_d^s$ than to any of the autocorrelations of the other sources.

In order to identify the desired extraction filter based on the mold we define the following linear combinations of correlation matrices:

$$\mathbf{\Gamma}_l = \sum_{i=1}^{N} \xi_l^i \mathbf{R}_i^x = \mathbf{A}\left(\sum_{i=1}^{D} \xi_l^i \mathbf{R}_i^s\right)(\mathbf{A})^T, \tag{13}$$

and $\boldsymbol{\xi}_l \triangleq \left[\xi_l^1, \cdots, \xi_l^N\right] \in \mathbb{R}^{1\times N}$. The linear combinations of source correlation matrices have the following structure:

$$\sum_{i=1}^{N} \xi_l^i \mathbf{R}_i^s = \mathrm{diag}\left\{\alpha_l^1, \cdots, \alpha_l^S\right\} \tag{14}$$

where $\alpha_l^i \triangleq \langle \boldsymbol{\xi}_l, \mathbf{r}_i^s\rangle$, for each vector $\boldsymbol{\xi}_l$.

Linear combinations of correlation matrices possess a similar structure as in (7) such that the extraction filters can be identified by the following GEVD:

$$\{\mathbf{w}, \lambda\} = \mathrm{gevd}\left(\mathbf{\Gamma}_{l_1}, \mathbf{\Gamma}_{l_2}\right). \tag{15}$$

In this case, the eigenvectors are again the extraction filters, but the eigenvalues obtain a new structure:

$$\lambda_{l_1 l_2}^i = \frac{\alpha_{l_2}^i}{\alpha_{l_1}^i} = \frac{\langle \boldsymbol{\xi}_{l_2}, \mathbf{r}_i^s\rangle}{\langle \boldsymbol{\xi}_{l_1}, \mathbf{r}_i^s\rangle} \quad \forall i \in \{1, \cdots, S\}. \tag{16}$$

Observe that each eigenvalue depends on the correlation vector $\mathbf{r}_i^s$ of one of the source signals, which is indicated in the superscript index, and the vectors $\boldsymbol{\xi}_{l_1}$ and $\boldsymbol{\xi}_{l_2}$ that form the linear combinations as is indicated by the subscript indices.

Theorem 4.2. *Suppose that we choose two linear combination vectors $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ as orthonormal vectors, then the eigenvalues are a measure for the angle φ_{12}^i between the vector $\boldsymbol{\xi}_1$ and the vector $\mathbf{r}_i^s$ projected on the two dimensional space spanned by $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$:*

$$\lambda_{12}^i = \frac{\langle \boldsymbol{\xi}_2, \mathbf{r}_i^s\rangle}{\langle \boldsymbol{\xi}_1, \mathbf{r}_i^s\rangle} = \tan\varphi_{12}^i \quad \forall i \in \{1, \cdots, S\}. \tag{17}$$

Proof. We find an orthonormal basis for the space $\mathbb{R}^{1\times N}$ by choosing the N vectors $\boldsymbol{\xi}_l \in \mathbb{R}^{1\times N}$ for $l \in \{1, \cdots, N\}$ orthonormal with respect to each other. Using this orthonormal basis we decompose the correlation vector $\mathbf{r}_i^s$ as follows:

$$\mathbf{r}_i^s = \sum_{l=1}^{N} r_i^l \boldsymbol{\xi}_l \quad \forall i \in \{1, \cdots, S\}, \tag{18}$$

where $r_i^l \in \mathbb{R}$. Given this decomposition, it follows that: $\langle \boldsymbol{\xi}_l, \mathbf{r}_i^s\rangle = r_i^l$, which results in the following eigenvalues:

$$\lambda_{12}^i = \frac{\langle \boldsymbol{\xi}_2, \mathbf{r}_i^s\rangle}{\langle \boldsymbol{\xi}_1, \mathbf{r}_i^s\rangle} = \frac{r_i^2}{r_i^1}. \tag{19}$$

On the other hand, the projection $\hat{\mathbf{r}}_i^s$ of the vector $\mathbf{r}_i^s$ onto the space spanned by $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ is given by: $\hat{\mathbf{r}}_i^s = r_i^1\boldsymbol{\xi}_1 + r_i^2\boldsymbol{\xi}_2$. It follows from geometry that the angle φ_{12}^i between the vector $\boldsymbol{\xi}_1$ and $\hat{\mathbf{r}}_i^s$ is characterized by $\tan(\varphi_{12}^i) = r_i^2/r_i^1$. □

If the mold is two dimensional, i.e. $N = 2$, then the problem is completely determined. If we choose $\boldsymbol{\xi}_1$ as: $\boldsymbol{\xi}_1 = \mathbf{r}_e^s/||\mathbf{r}_e^s||$, then the absolute smallest eigenvalue corresponds to the desired source according to the assumption in

(11). Otherwise, if $N > 2$, some dimensions are ignored and the projected autocorrelation vector of an undesired source may have a smaller angle with respect to the mold than the autocorrelation vector of the desired source. We solve this problem by using the following property: the generalized eigenvectors for multiple GEVD of different linear combinations of correlation matrices are the same.

Theorem 4.3. *Suppose we take the following summation of squared eigenvalues that correspond to source $s_i[n]$:*

$$m^i = \sqrt{\sum_{l=2}^{N} (\lambda_{1l}^i)^2} \quad \forall\, i \in \{1, \cdots, S\}, \tag{20}$$

where the eigenvalue structure is as in (16) and the vectors $\boldsymbol{\xi}_l$ form an orthonormal basis for $\mathbb{R}^{1\times N}$ with $\boldsymbol{\xi}_1 = \mathbf{r}_e^s / \|\mathbf{r}_e^s\|$. If (11) holds, then the value of m^i is minimal for that series of eigenvalues that correspond to the desired source signal.

Proof. By using the decomposition of a correlation vector from the proof of Thm. 4.2 it follows that:

$$m^i = \sqrt{\sum_{l=2}^{N} (\lambda_{1l}^i)^2} = \sqrt{\frac{\sum_{l=2}^{N} (r_i^l)^2}{(r_i^1)^2}} = \frac{\sqrt{\|\mathbf{r}_i^s\|^2 - (r_i^1)^2}}{|r_i^1|}. \tag{21}$$

Notice that by our assumption in (11) the value of $|r_i^1| / \|\mathbf{r}_i^s\| \in [0, 1]$ is maximal for the desired source, thus for $i = d$. It follows that $0 \le m^d < m^i$ for all $i \ne d$. $\square$

From Thm. 4.3 it follows that the desired extraction filter is identified as the eigenvector that corresponds to the smallest m^i for $i \in \{1, \cdots, S\}$.

4.2 Filter identification algorithm

The GEVD of two matrices $\boldsymbol{\Gamma}_{l_1}$ and $\boldsymbol{\Gamma}_{l_2}$ can be written as a conventional eigenvalue decomposition of the following matrix $\boldsymbol{\Gamma}_{l_2}(\boldsymbol{\Gamma}_{l_1})^{-1}$, if $\boldsymbol{\Gamma}_{l_1}$ is invertible. Thus, given that $\boldsymbol{\Gamma}_{l_1}$ is invertible:

$$\{\mathbf{w}, \lambda\} = \text{gevd}\left(\boldsymbol{\Gamma}_{l_1}, \boldsymbol{\Gamma}_{l_2}\right) = \text{eig}\left(\boldsymbol{\Gamma}_{l_2}(\boldsymbol{\Gamma}_{l_1})^{-1}\right), \tag{22}$$

where $\{\mathbf{w}, \lambda\} = \text{eig}(\boldsymbol{\Gamma})$ are the solutions of: $\lambda \mathbf{w} = \mathbf{w}\boldsymbol{\Gamma}$. Furthermore, if a matrix is multiplied by itself, i.e. $(\boldsymbol{\Gamma}_{l_1})^2 \triangleq \boldsymbol{\Gamma}_{l_1}\boldsymbol{\Gamma}_{l_1}$, then the eigenvalues of that matrix are squared while the eigenvectors remain the same.

From these properties it follows that the eigenvalues of the following eigenvalue problem correspond to the squared values of m^i in (20):

$$\{\mathbf{w}, \lambda\} = \text{eig}\left(\sum_{l=2}^{N} \left(\boldsymbol{\Gamma}_l(\boldsymbol{\Gamma}_1)^{-1}\right)^2\right), \tag{23}$$

where $\lambda^i = (m^i)^2$. If we compute the eigenvector that corresponds to the smallest eigenvalue of (23), then we have the desired extraction filter.

We summarize this procedure in the following algorithm:

- Find the mold $\mathbf{r}_e^s \in \mathbb{R}^{1\times N}$ in the NF-ROS for which (11) holds;
- Calculate or estimate the sensor correlation matrices $\mathbf{R}_i^x$ for $i \in \{1, \cdots N\}$;
- Find a complete set of orthonormal vectors $\boldsymbol{\xi}_1, \cdots, \boldsymbol{\xi}_N$ for $\mathbb{R}^{1\times N}$, where $\boldsymbol{\xi}_1 = \mathbf{r}_e^s / \|\mathbf{r}_e^s\|$.
- Calculate N linear combinations $\boldsymbol{\Gamma}_l$ of the sensor correlation matrices;
- Combine the matrices $\boldsymbol{\Gamma}_l$ as in (23);
- Compute the eigenvector that corresponds to the smallest eigenvalue of (23).

This procedure is validated by means of simulations.

5. SIMULATION RESULTS AND DISCUSSION

We validate the novel BSE algorithm by showing that it outperforms the Linear Prediction based BSE (LP-BSE) methods for noisy measurements from [4, 5]. In order to make a fair comparison we first give a description of the LP-BSE problem as a GEVD problem.

5.1 Linear prediction based BSE as a GEVD

In our simulations the linear prediction filter $\mathbf{b}$, with filter coefficients b_p for $p \in \{1, \cdots, N\}$, was chosen as the optimal Wiener filter based on the prescribed autocorrelation. By utilizing this filter we computed prediction error signals $\mathbf{e}[n] \in \mathbb{R}^{D\times 1}$ from the measurements $\mathbf{x}[n] \in \mathbb{R}^{D\times 1}$ as follows:

$$e_i[n] = x_i[n] - \sum_{p=1}^{N} b_p x_i[n-p] \quad \forall\, i \in \{1, \cdots, D\}. \tag{24}$$

The extraction filter from [3] was then identified from the following GEVD of the following two correlation matrices:

$$\{\mathbf{w}, \lambda\} = \text{gevd}\left(\mathbf{R}^x, \mathbf{R}^e\right), \tag{25}$$

where $\mathbf{R}^e \triangleq \mathbb{E}\{\mathbf{e}[n](\mathbf{e}[n])^T\}$ and $\mathbf{R}^x \triangleq \mathbb{E}\{\mathbf{x}[n](\mathbf{x}[n])^T\}$. Each eigenvalue corresponds to the normalized mean square prediction error in [3] and the respective eigenvectors are the filters that extract the corresponding source signal. Therefore, the eigenvector that corresponds to the smallest eigenvalue can be selected as the desired extraction filter. In case of sensor noise both correlation matrices $\mathbf{R}^x$ and $\mathbf{R}^e$ are compensated to find the extraction filter, as is proposed in [4,5].

5.2 Simulation setup

We simulated an instantaneous mixing system with three different noise scenarios. The sources consisted of three stationary Auto Regressive Moving Average (ARMA) signals s_1, s_2 and s_3, from which s_1 was the desired signal. The source signals were created by filtering zero mean white Gaussian signals. The pole pairs of these filters were complex conjugates: $p_1 = -0.7 \pm 0.7i$, $p_2 = 0.1 \pm 0.9i$ and $p_3 = 0.9 \pm 0.15i$, and the zeros were: $z_1 = 0.98$, $z_2 = 0.86$ and $z_3 = 0.92$, which corresponded to source s_1, s_2 and s_3 respectively. The corresponding source signal variances were: $(\sigma_1^s)^2 = 0.88, (\sigma_2^s)^2 = 1.1$ and $(\sigma_3^s)^2 = 0.93$.

The sensor signals were computed according to the relation in (1), with the following instantaneous mixing system:

$$\mathbf{A} = \begin{bmatrix} 0.5488 & -0.0086 & -0.0805 \\ 0.0965 & -0.4677 & -0.3520 \\ -0.3117 & -1.0405 & -0.4808 \end{bmatrix}, \tag{26}$$

and with different noise $\boldsymbol{\nu}[n]$ for each mixing scenario. In the first scenario no noise was assumed, i.e. $\boldsymbol{\nu}[n] = \mathbf{0}$. The LP-BSE method for noise free measurements (NF-LP) and the novel BSE method with a NF-ROS of the lags 0 until 4 (NF-BSE) were used to find the extraction filter. In the second scenario, we contaminated each sensor with white Gaussian noise. The noise power distribution was given by: $(\sigma_1^\nu)^2 = 0.75, (\sigma_2^\nu)^2 = 0.65$ and $(\sigma_3^\nu)^2 = 0.72$. The LP-BSE method with white noise compensation (WN-LP) and the novel BSE method with a NF-ROS of lag 1 until 5 (WN-BSE) were used to find the extraction filter for this scenario. In the third mixing scenario the temporal structure of the noise was changed into a Moving Average 1 (MA1) structure. The zeros of the MA1 filters were given by: $z_1 = 0.15, z_2 = 0.81, z_3 = -0.70$ for the respective noise signals. For this scenario the LP-BSE method with colored noise compensation (MA1-LP) and the novel BSE method with a NF-ROS of lag 2 until 6 (MA1-BSE) were used to

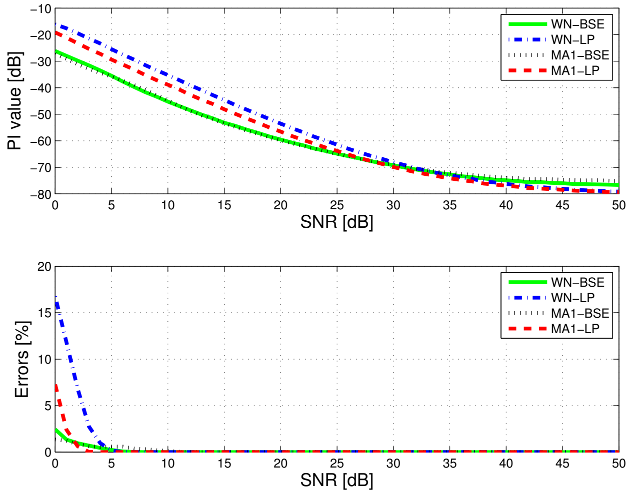

Figure 2: Simulation results for noisy mixtures.

find the extraction filter. The compensation is performed with noise correlation matrices that were calculated from theory, thus extra prior information is used for the LP-BSE methods.

For each scenario our mold was the exact autocorrelation function of the desired source signal s_1, which was also used for the calculation of the prediction filter. We measured the performance of the methods by the Performance Index (PI) from [1], which is defined as follows:

$$\mathrm{PI} \triangleq 10\log_{10}\left(\frac{1}{S-1}\left(\sum_{s=1}^{S}\frac{(g_s)^2}{\max\{(g_1)^2,\cdots,(g_S)^2\}}-1\right)\right).$$

This PI measures the suppression of sources from the following extraction performance vector: $\mathbf{g} \triangleq \mathbf{w}\mathbf{A}/\,||\mathbf{w}\mathbf{A}|| \in \mathbb{R}^{1\times S}$. If the performance vector $\mathbf{g}$ is close to a vector with a one at one of the columns and zeros elsewhere then the PI value is low. If the output signal contains still a mixture of several sources then the PI value is high. A PI value of 0dB corresponds to a uniform mixing of the sources. Furthermore, we determine if the desired source s_1 is extracted.

We performed a Monte Carlo simulation in which we simulated each mixing scenario 5000 times for a range of Signal to Noise Ratios (SNR) from 0 dB until 50 dB, where:

$$\mathrm{SNR} \triangleq 10\cdot\log_{10}\frac{\mathbb{E}\{||\mathbf{A}\mathbf{s}||^2\}}{\mathbb{E}\{||\boldsymbol{\nu}||^2\}}\quad [\mathrm{dB}]. \tag{27}$$

Each simulation a new realization of the 3×50000 source and noise signal samples was taken and each time the PI value was calculated. The results for both methods in case of the noisy scenarios are depicted in Fig. 2. For each method the mean PI value, thus the mean interference of non-extracted sources, over the 5000 simulations per SNR is depicted in the upper graph, while in the lower graph the error percentage per method and SNR is depicted. The error percentage consists of the relative number of extraction filters that extract another source than the desired source. The mean PI values of the noise free mixing scenarios were -77.3 dB and -79.8 dB for the NF-BSE and NF-LP method, respectively. Both noise free methods found the extraction filter without errors.

5.3 Discussion

From the simulations we observe that both the LP-BSE method and the novel BSE method have a similar, excellent performance in case of a noise free measurement. These noise free simulations introduced a lower bound for the expected PI value of approximately -80 dB. The simulations with noisy measurements, for which the results are depicted in Fig. 2, show three interesting regions. First, when the SNR reaches above 30 dB then the PI value reaches towards the lower bound and is excellent and free of errors for both methods and noise scenarios. Second, for SNR values between 5 dB and 30 dB the desired source is extracted without errors, but the PI values, i.e. the suppression of the undesired sources, of both methods increases for lower SNR values. However, the novel BSE method has similar PI values for both noisy scenarios and above that better PI values than the LP-BSE methods. Finally, when the SNR becomes lower than 5 dB the novel BSE method outperforms the LP-BSE method both on the PI value and the error rate.

From these observations we conclude that the novel BSE method is more robust to noise than the LP-BSE method. This robustness is obtained because we deal with the noise in a very simplified manner. When correlation data is corrupted by noise then we simply ignore that correlation data instead of compensating for the noise contribution. In our simulations the noise characteristics were assumed to be known exactly for the LP-BSE methods. In practice, this will not be the case and mismatches in the noise compensation will lead to a performance reduction, while the novel BSE method is insensitive for these mismatches.

6. CONCLUSIONS AND FUTURE RESEARCH

In this paper we proposed a novel approach to BSE. This novel method extracts the desired source signal by the exploitation of a priori information in the form of a NF-ROS and a mold of the autocorrelation that belongs to the desired source signal. We have shown by means of simulations that the exploitation of a NF-ROS makes the method more robust to noise than a linear predication based BSE method and it exploits less a priori information.

Future research topics are as follows. If extra sensors are available then they should be utilized for noise reduction. Furthermore, mismatches in the a priori determination of the mold should be accounted for in the algorithm and the method should be extended such that the NF-ROS can be determined blindly. Finally, BSE should be performed for more complex, non instantaneous, mixing systems.

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