Bias-Correction Method in Bearing-Only Passive Localization

Yiming Ji, Changbin Yu, Brian D.O. Anderson
Research School of Information Sciences and Engineering
The Australian National University
Canberra ACT 2601, Australia
{yiming.ji, brad.yu, brian.anderson}@anu.edu.au

Abstract— **In this paper a novel analytical approach to approximate and correct the bias in 2D localization problem is proposed. This new method mixes Taylor series and Jacobian matrices to determine the bias, and leads to an easily computed analytical bias expression. Importantly, we compare the proposed approach with a well-cited previous method using simulation data. Further we apply our method to bearing-only localization algorithms. Monte Carlo simulation results demonstrate that the proposed method performs satisfactorily when the underlying geometry makes the localization problem reasonable. Furthermore the proposed method performs better than the comparison method and also is effective over a larger area. Although the method is presented in detail for bearing-only localization algorithms, the analysis methodology is also valid for other kinds of localization algorithms.**

I. Introduction

Recently, there has been increasing interest in techniques for determining location of targets in different application fields. For instance, in environmental applications, such as forest fire detection and flood detection, sensing data without knowing the sensor location is meaningless. Again, accurate location of targets is also required in military operations, such as battlefield surveillance and monitoring friendly forces [1]. Therefore, many localization algorithms have been proposed in recent years, see e.g. [2-4, 14-15].

In most practical situations, noise in measurement data is inevitable. Hence the true position of the target cannot be obtained. And frequently if not generally, any position estimate will be biased. Therefore in order to obtain a better estimates of the target it is desirable to correct the bias, assuming it is computable, or approximately computable. However rather few works concentrate on the bias problem. Doğançay et al. [5] develop a bias compensation algorithm to reduce the position estimation bias. The simulation examples illustrate the significant bias reduction of the proposed algorithm. Nevertheless this bias compensation algorithm is not generic: the method is only applicable to TDOA localization.

In [6], an introduction to tensor algebra is given with a few examples in estimation theory. One of the applications of tensor algebra addressed in the paper treats the bias in non-linear systems with a noisy observable. The method expands the non-linear function which maps measurements to target positions to second order in the noise using a Taylor series. The expected value of the second order term is considered as the analytic expression of bias, and the concepts are illustrated to obtain the bias in the Cartesian coordinates of a target where noisy range and bearing measurements (from a single point) are given. However the main focus of [6] is how to use tensor algebra, rather than bias analysis. Therefore there is no systematic analysis and detailed simulation for the bias problem.

Gavish and Weiss [7] examine the performance of two well known bearing-only location algorithms, viz. the maximum likelihood and the Stansfield estimators. Analytical expressions are derived for the covariance matrix of the estimation error and the bias, which permit performance comparison for any case of the two algorithms. In order to obtain the analytical expressions for bias, the first derivative of the maximum likelihood cost function is expanded by a Taylor series. Three expansions of different orders were obtained separately. The final expression for the bias involves the variance of the measurement noise and the derivatives of the cost function. Additionally, the analytic expression of the bias is independent of the localization algorithm. However the derivation involves truncating three different Taylor series expansions which may lead to imprecise results.

In this paper, we present a general method to reduce the position estimation bias in 2D localization algorithms. To obtain an analytic expression for the bias, a Taylor series is used to expand the localization mapping g (which maps the measurements to position estimates) to a certain order. Though using more terms of Taylor series may lead to higher precision, it also will result in more complicated calculation. We conjecture that the expansions beyond second-order offer negligible improvement. In many situations the correction using terms to second-order is completely adequate. However more terms will be used in a future study. The expected value of the second-order term, which involves derivatives of g, is considered as the bias. Generally, however, to compute the derivatives of g analytically is very difficult. In contrast, to calculate the inverse mapping of g (call it f) and its derivatives is much easier. Therefore we substitute the derivatives of f for the derivatives of the localization mapping g by using the Jacobian matrix of f, leading to an easily calculated analytic expression for the bias. In this paper we will apply our method by way of example to bearing-only localization algorithms, though the proposed method in this paper is generic. To illustrate the performance of our method, we compare it with the GW (Gavish and Weiss [7]) method based on simulation. The main reason for selecting the GW approach as the comparative method is that, like our algorithm but unlike most other bias correction methods such as the approach proposed by Doğançay, the GW method is generic, i.e. in principle it can be used for many types of localization algorithm. Moreover, various simulation results on the GW method in [7] show that the analytical expression of bias can calculate the bias very well in certain situations. The Monte Carlo simulation results in our paper verify that the proposed method performs better than the GW approach.

The rest of the paper is organized as follows. We propose the new bias-correction approach in Section II. The results of Monte Carlo simulations are provided in Section III. Section

Y. Ji and B. D.O. Anderson are supported by National ICT Australia-NICTA. C. Yu and B. D.O. Anderson are supported by the Australian Research Council under DP-0877562.

IV summarizes the results and comments on future work.

II. Bias Analysis in Localization Algorithms

In this section we will first formulate the localization problem. Then a novel bias-correction method will be presented in subsection B. All analysis is done in two-dimensional space.

A. Problem Statement

In two-dimensional space, the bearing-only localization problem can be formulated as follows. Suppose there is an emitter or target whose coordinate vector is $\mathbf{x} = (x_1, x_2)^T = (x, y)^T$. Suppose further a set of bearing measurements $\Theta = (\theta_1, \theta_2, ..., \theta_N)^T$ (N denotes the number of anchors) can be obtained from a number of anchors at known positions. In the noiseless case we have

$$\Theta = \mathbf{f}(\mathbf{x}) \tag{1}$$

where $\mathbf{f} = (f_1, ..., f_N)$ denotes the mapping from the target to the measurements. The function $\mathbf{f}$ is assumed (as is reasonable) to be obtained analytically according to the geometric relationship between the target and anchors. Localization amounts to inverting f.

In practice, however, there will be noise in measurements. Therefore the mapping from target position to measurements can be described by a nonlinear equation as follows:

$$\Theta = \mathbf{f}(\mathbf{x}) + \delta\Theta \tag{2}$$

where $\delta\Theta = (\delta\theta_1, ..., \delta\theta_N)^T$ denotes the noise in measurements, which is assumed to be zero-mean Gaussian with $N \times N$ covariance matrix $S = \text{diag}(\sigma^2_{\theta_1}, ..., \sigma^2_{\theta_N})$.

When the number of anchors is more than or equal to three ($N \geq 3$), equations (1) and (2) will be overdetermined. In other words, there will generally be no solution to the equation (2) except in the noiseless case. In order to obtain an approximate position estimate, various methods have been presented such as maximum likelihood, least squares, etc [8, 12]. No matter what type of method is used, the main idea of these approaches is similar: transform the localization problem to be an optimization problem as follows.

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} F_{\text{cost-function}}(\mathbf{x}, \Theta) \tag{3}$$

By solving the above optimization problem (which is often computationally difficult) we obtain the estimated position $\hat{\mathbf{x}}$.

B. A Novel Method

1. Three Anchors Situation

A scenario with three anchors (N=3) and one target is analyzed in this subsection. The analysis will be restricted to Cartesian coordinates in this paper. However, the proposed approach is independent of the choice of coordinates.

Assume f_1, f_2 and f_3 (which together form a vector function $\mathbf{f}$ in II.A) are the mappings from target to measurement data. We can obtain the following equations according to the simple geometric relationships (shown in Fig. 1). Here we only take $f_1(\theta_1 = f_1)$ for example while $f_2(\theta_2 = f_2)$ and $f_3(\theta_3 = f_3)$ have the similar forms.

$$\theta_1 = f_1(x, y) = \pi + \text{actan}(\frac{x - x_1}{y - y_1})(\text{mod}2\pi) \tag{4}$$

where $(x,\ y)$ denotes the position of target, while $(x_1,\ y_1)$ denotes the known positions of anchor 1. Furthermore θ_1, θ_2 and θ_3 (together forming a measurement vector $\Theta = (\theta_1, \theta_2, \theta_3)^T$ as in II.A) are angle measurements from each anchor to the target, relative to a global direction (i.e. North).

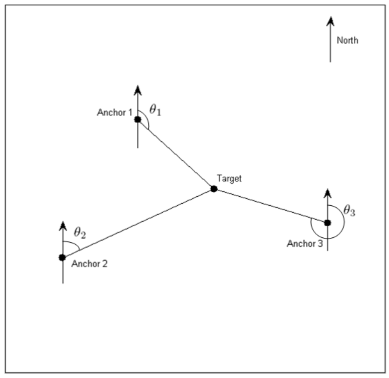

Fig. 1. Geometry of the three anchors situation

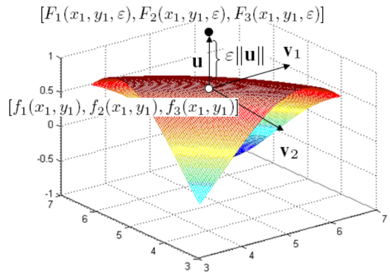

Fig. 2. Introduce One Variable

The method we are proposing will involves the inverse mapping of f_i and the Jacobian matrix of the inverse mapping. However in the noisy case which means the true values of the θ_i are replaced by noisy values, the equation set f_1, f_2 and f_3 will be generically unsolvable, since it is overdetermined. In other words the number of scalar measurements is larger than the number of unknowns. Therefore we do not have an inverse mapping of f_i and thus cannot use the Jacobian matrix of the inverse mapping. In order to solve the overdetermined problem, here we propose an approach based on least squares method to introduce an extra variable into the mapping set.

Consider a three dimensional space, with axes corresponding to the three bearing measurements. Assume a surface (shown in Fig. 2) consists of points which correspond to sets of noiseless measurements. According to the least squares method, the cost function has the form

$$F_{\text{cost-function}}(\mathbf{x}, \Theta) = \sum_{n=1}^{3} (f_n - \theta_n)^2 = \sum_{n=1}^{3} \delta\theta_n^2 \tag{5}$$

The least squares method, in fact, attempts to find a point $(f_1(x, y), f_2(x, y), f_3(x, y))$ on the surface corresponding to a set of noisy measurements (off the surface) to minimize the distance between the two points.

Assume, in Fig. 2, the black point denotes a set of noisy measurements, and the white point is the corresponding point on the surface[1]. The black point must be on the normal

[1]Sometimes the corresponding point is not unique. At that time we assume further information can be obtained to resolve this ambiguity.

vector to the surface passing through the white one. The distance between the two points can be denoted as $\varepsilon\|\mathbf{u}\| = \sqrt{\delta\theta_1^2 + \delta\theta_2^2 + \delta\theta_3^2}$, where $\mathbf{u}$ is a normal vector at the white point and ε is a coefficient to ensure that the length of $\varepsilon\|\mathbf{u}\|$ agrees with the distance between the white point and black point. The normal vector $\mathbf{u}$ can be calculated as follows.

At the white point we can obtain two tangent vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ as follows.

$$\mathbf{v}_1 = (\frac{\partial f_1}{\partial x}, \frac{\partial f_2}{\partial x}, \frac{\partial f_3}{\partial x})^T, \mathbf{v}_2 = (\frac{\partial f_1}{\partial y}, \frac{\partial f_2}{\partial y}, \frac{\partial f_3}{\partial y})^T \quad (6)$$

By cross multiplying the two vectors, we can obtain a normal vector $\mathbf{u} = \mathbf{v}_1 \times \mathbf{v}_2$.

Note that $f_1(x,y), f_2(x,y)$ and $f_3(x,y)$ can be readily written down according to simple geometric relationships. Therefore a new set of functions F_1, F_2, F_3 (which together form a vector function $\mathbf{F}$) parameterizing a noisy measurement vector can be obtained through moving from any point on the surface, defined by f_1, f_2 and f_3 along the normal vector for some distance $\varepsilon\|\mathbf{u}\|$. The new set of functions determine equations which are no longer overdetermined because an extra variable ε has been introduced. Different x, y and ε give different points.

To sum up we have the the new mapping $\mathbf{F} : R^3 \to R^3$ from target position to measurements as follows.

$$\Theta = \mathbf{F}(x, y, \varepsilon) = \mathbf{f}(x, y) + \varepsilon\mathbf{u} \quad (7)$$

Suppose the inverse mapping of $\mathbf{F} = (F_1, F_2, F_3)$ is $\mathbf{G} = (G_1, G_2, G_3)$, with the G_i localization mappings. Thus we have:

$$x = G_1(\theta_1, \theta_2, \theta_3) \quad (8)$$

$$y = G_2(\theta_1, \theta_2, \theta_3) \quad (9)$$

$$\varepsilon = G_3(\theta_1, \theta_2, \theta_3) \quad (10)$$

It can be verified that there are derivatives of any order of G_1, G_2 and G_3. If θ_1, θ_2 and θ_3 are noiseless, then ε will be zero. Else, suppose they represent noisy values $\tilde{\theta}_i$, due to a noise $\delta\theta_i$. Now we can expand G_1, G_2 and G_3 about the point $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3)$ by Taylor series. Suppose the Taylor series is truncated to second order. For example,

$$\begin{aligned} x + \delta x =& G_1(\tilde{\theta_1}, \tilde{\theta_2}, \tilde{\theta_3}) \\ =& G_1(\theta_1 + \delta\theta_1, \theta_2 + \delta\theta_2, \theta_3 + \delta\theta_3) \\ =& G_1(\theta_1, \theta_2, \theta_3) + [\frac{\partial G_1}{\partial \theta_1}\delta\theta_1 + \frac{\partial G_1}{\partial \theta_2}\delta\theta_2 + \frac{\partial G_1}{\partial \theta_3}\delta\theta_3] \\ &+ \frac{1}{2!}(\delta\theta_1 \frac{\partial}{\partial \theta_1} + \delta\theta_2 \frac{\partial}{\partial \theta_2} + \delta\theta_3 \frac{\partial}{\partial \theta_3})^2 G_1(\theta_1, \theta_2, \theta_3) \end{aligned}$$

By taking the expected value of the above equation we can obtain an approximation for the expected value of δx as in terms of derivatives of G_1 and the measurement noise variances:

$$E(\delta x) = \frac{1}{2}(\sigma_{\theta_1}^2 \frac{\partial^2}{\partial \theta_1^2} + \sigma_{\theta_2}^2 \frac{\partial^2}{\partial \theta_2^2} + \sigma_{\theta_3}^2 \frac{\partial^2}{\partial \theta_3^2}) G_1(\theta_1, \theta_2, \theta_3) \quad (11)$$

$E(\delta y)$ can be obtained in the same way. Here $E(\delta x)$ and $E(\delta y)$ are considered as the bias. The tensor form of the bias can be obtained in [6]. There is however a serious practical difficulty with this approach, as we now explain.

Note that while $\mathbf{F}$ can be analytically computed this is almost certainly difficult or impossible for $\mathbf{G}$. Furthermore, when one considers for example a three dimensional problem involving TDOA and angle data, the calculation of $\mathbf{G}$ and its derivatives would be much harder again. In fact, in almost all cases, it is much easier to obtain the derivatives of forward mappings (F_1, F_2 and F_3) and very difficult if not impossible to obtain analytically the derivatives of $\mathbf{G}$. Therefore we consider how to use F_1, F_2, F_3 and their derivatives to compute the second derivatives of $\mathbf{G}$, using a Jacobian matrix.

The following equations derive from one property of the Jacobian matrix.

$$G_J = \begin{pmatrix} \frac{\partial G_1}{\partial \theta_1} & \frac{\partial G_1}{\partial \theta_2} & \frac{\partial G_1}{\partial \theta_3} \\ \frac{\partial G_2}{\partial \theta_1} & \frac{\partial G_2}{\partial \theta_2} & \frac{\partial G_2}{\partial \theta_3} \\ \frac{\partial G_3}{\partial \theta_1} & \frac{\partial G_3}{\partial \theta_2} & \frac{\partial G_3}{\partial \theta_3} \end{pmatrix}, F_J = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial \varepsilon} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial \varepsilon} \\ \frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} & \frac{\partial F_3}{\partial \varepsilon} \end{pmatrix} \quad (12)$$

where

$$G_J F_J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (13)$$

Solving equation (13), we can obtain the analytical expressions of $\frac{\partial G_i}{\partial \theta_j}(i = 1,2,3; j = 1,2,3)$ in terms of $\frac{\partial F_i}{\partial x}$, $\frac{\partial F_i}{\partial y}$ and $\frac{\partial F_i}{\partial \epsilon}$ for $i = 1,2,3$, and thus as analytical expressions in terms of x, y and ϵ. For ease of exposition we use $G_{\theta_j}^i$ to denote the expressions of $\frac{\partial G_i}{\partial \theta_j}(i = 1,2,3; j = 1,2,3)$ as functions of x, y and ϵ. Here we take $\frac{\partial G_1}{\partial \theta_1}$ for example. We can obtain the following equation.

$$\frac{\partial G_1}{\partial \theta_1} = G_{\theta_1}^1 \quad (14)$$

Differentiating the equation (14) in respect to x, y and ε respectively we can obtain an equation set as follows.

$$\begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_2}{\partial x} & \frac{\partial F_3}{\partial x} \\ \frac{\partial F_1}{\partial y} & \frac{\partial F_2}{\partial y} & \frac{\partial F_3}{\partial y} \\ \frac{\partial F_1}{\partial \varepsilon} & \frac{\partial F_2}{\partial \varepsilon} & \frac{\partial F_3}{\partial \varepsilon} \end{pmatrix} \times \begin{pmatrix} \frac{\partial^2 G_1}{\partial {\theta_1}^2} \\ \frac{\partial^2 G_1}{\partial \theta_1 \theta_2} \\ \frac{\partial^2 G_1}{\partial \theta_1 \theta_3} \end{pmatrix} = \begin{pmatrix} \frac{\partial G_{\theta_1}^1}{\partial x} \\ \frac{\partial G_{\theta_1}^1}{\partial y} \\ \frac{\partial G_{\theta_1}^1}{\partial \varepsilon} \end{pmatrix} \quad (15)$$

Note that the quantities on the right side of this equation are all expressible analytically in terms of derivatives of the F_i, and so as functions of x, y and ϵ. Hence by solving the equation set (15), we can obtain a formula for $\frac{\partial^2 G_1}{\partial \theta_1^2}$ which only contains of the derivatives of F_1, F_2 and F_3. The formulas for $\frac{\partial^2 G_1}{\partial \theta_2^2}$ and $\frac{\partial^2 G_1}{\partial \theta_3^2}$can be obtained in the same way. Substituting these formulas for the derivatives of G_1 in equation (11) we can finally obtain the analytical expressions for the bias in x ($E(\delta x)$) including only the derivatives of F_1, F_2 and F_3. This results in much easier calculation than computing an analytic expression for G and obtaining derivatives. The derivation of the analytic expression for $E(\delta y)$ is similar.

2. More than Three Anchors

When there are more than three anchors, the situation is similar to the three anchors case except that the extra variable ε is no longer a scalar. Instead it is a vector which can be defined as follows.

$$\varepsilon = [e_1, e_2, ..., e_i]^T \qquad i = m - 2 \quad (16)$$

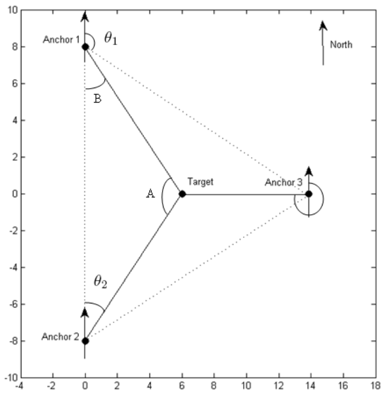

Fig. 3. Geometry in simulation

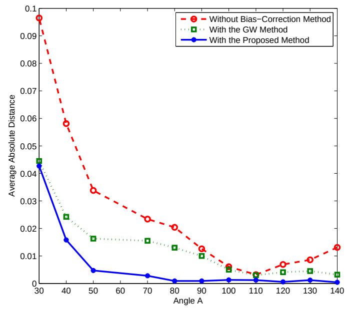

Fig. 4. Comparison of the GW method and the proposed method

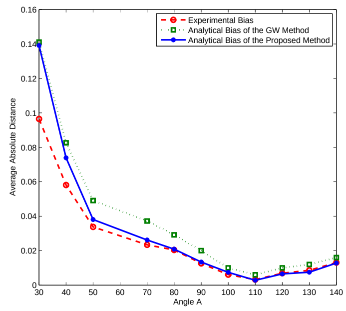

Fig. 5. Comparison of Experimental bias and analytical bias

where e_i denotes a coefficient to set correctly the moved distance in each dimension of the normal, and m denotes the number of anchors.

The calculations are a straightforward variation on those for the three anchors situation. We omit the details here.

III. Simulation Results

In this section the results of Monte Carlo simulations will be provided. Some assumptions on the simulation will be first noted. Next the comparison of the proposed method and the GW method with two types of simulations will be presented. The simulation results verify the proposed method can correct the bias very well while performing better than the GW method. Given space limitations we only illustrate the simulation with three anchors. However the simulation can be easily extended to more anchors and the simulation results will remain similar.

- The three anchors are fixed at $(0, 8)$, $(0, -8)$ and $(8\sqrt{3}, 0)$ respectively (See Fig. 3).
- The measurement errors for θ_1, θ_2 and θ_3 are produced by independent Gaussian distributions ($\mu = 0$ and $\sigma^2 = 1$). Though the simulations have been done in different level of noise, we do not show the details here because of the space limitation.
- All the simulation results are obtained from 5000 Monte Carlo experiments.

Two types of simulation have been done. In the first type we fix the value of y of the target at zero while changing the value of x, i.e. we adjust the angle A (shown in Fig. 3) The variation of angle A is from 15° to 300°. Following are the simulation results.

Fig. 4 illustrates the comparison of the average absolute distance between the estimated position of target and the true position in three situations: without a bias-correction method, with the GW method and with the proposed bias-correction method. Evidently, both the GW method and the proposed method can reduce the localization bias for angle A ranging from 30° to 140°. Furthermore the curve denoting the results with the proposed method is below the GW curve all the time, which demonstrates the performance of the proposed method is better than the GW method. However, when the angle A is too large or too small neither the GW method nor our method can work (see TABLE I). At that time the target is far away from the three anchors. Quite apart from issues of bias correction, localization algorithms cannot work satisfactorily in these cases because the target and the three anchors can be considered as nearly collinear [10, 13]. From TABLE I (which shows average absolute error between true position and

Angle A (Degree)	15°	20°	245°	300°
Without Bias-Correction Method	3.6257	0.3953	0.0125	0.1732
With the Proposed Method	6.1543	0.2629	0.0056	0.3042
With the GW Method	16.3051	0.53	0.0352	0.4763

TABLE I
Comparison of Average Absolute Error With No Bias correction, correction via the GW Method and correction via the Proposed Method

estimated position) we can also obtain that when the angle A is 20°, the proposed method is still effective while the GW method is not. The proposed method continues to be effective until the angle A is reduced to 15°. Similar observation can be made for when the angle A is 245° and 300°. This shows that the proposed method has a wider region of applicability than the GW method. This also demonstrates that, from another standpoint, the performance of the proposed method is better than the GW method.

Fig. 5 depicts a comparison of experimental bias and analytical bias. From 30° to 140°, the analytical bias of the proposed method is closer to the experimental bias than the bias of the GW approach. This also verifies, from another point of view, that the proposed method is more effective than the GW method.

In the second type of simulation we set the value of x of the target as 8 while adjusting the value of y, which means the angle B (shown in Fig. 3) is changing : the variation of the angle B is between 45° and 170°.

Fig. 6 shows the comparison when angle B is changing. From the figure we can see (conclude would also be very appropriate) that the proposed approach can correct the bias very well from 45° and 140° while the applicability region of the GW method is only from 45° to 100°. In all cases,

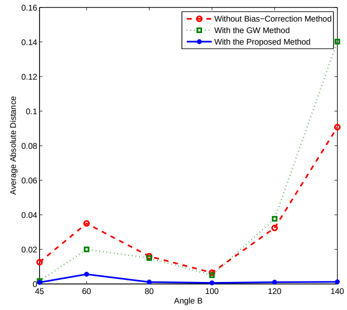

Fig. 6. Comparison of the GW method and the proposed method

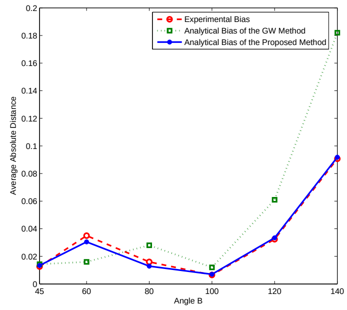

Fig. 7. Comparison of Experimental bias and analytical bias

Angle B (Degree)	140°	160°	170°
Without Bias-Correction Method	0.0907	0.7718	4.4474
With the Proposed Method	0.0012	0.1531	6.4083
With the GW Method	0.1402	1.7101	12.6213

TABLE II

COMPARISON OF AVERAGE ABSOLUTE ERROR WITH NO BIAS CORRECTION, CORRECTION VIA THE GW METHOD AND CORRECTION VIA THE PROPOSED METHOD

the performance of the proposed method is better than the GW approach. Nevertheless, when the angle B is too large our method is no longer effective (see TABLE II). At that time, the target is in the far field and we can consider the three anchors and the target as nearly collinear, which means that localization may not be practically possible. Again from TABLE II (which shows average absolute error between true position and estimated position) we can obtain that the region of applicability of the proposed method is larger than for the GW. Further Fig. 7 illustrates the comparison of experimental bias and analytical bias. Again, the comparison results also demonstrate the better performance of the proposed method than the GW method.

From the above simulation results we can observe that the proposed method can correct the bias very well at least for the noise level we have considered in most situations except a near-collinear one. At that time, localization algorithms are often less effective or noneffective. In other words the bias-correction approach presented in this paper is consistent with the applicability of localization algorithms. Furthermore, by comparing with the GW method, we conclude that the proposed method not only performs better than the GW method in the same situation but also has larger applicability area. More simulation has been done with different level of noise by changing the variance of measurement errors, the simulation results remain similar demonstrating the better performance of our method than the GW method. Our conjecture for the reason for better performance is that in the GW method truncation of Taylor series occurs three times while we only truncate the Taylor series once in our method. Using more terms before truncation may lead to improved precision. More analysis will be done in the future.

IV. CONCLUSION

An approach to reduce the bias in localization algorithms is presented in this paper. The proposed method analytically formulates the bias in an easy way mixing Taylor series and Jacobian matrices. We analyze the proposed approach with three and more anchors based on bearing-only localization algorithms. However it is easy to extend the method to other kinds of localization algorithms. For example, use of the proposed method based on distance-measurements has been studied in [11]. In addition we compare the proposed method with the GW method based on simulation. Monte Carlo experiments illustrate our method can correct the bias very well except in the nearly collinear situation. At that time the localization algorithms are often less effective or even noneffective [10, 13]. Our future work is to further improve the performance of the proposed method (such as via using high order terms of a Taylor series) and try to extend it to three dimensional space.

REFERENCES

[1] I.F. Akyildiz, W. Su, Y. Sankarasubramaniam, E. Cayirci. Wireless sensor networks: a survey. Computer Networks, 38:393-422, 2002.

[2] G. Mao, B. Fidan and B. D.O. Anderson. Wireless sensor network localization techniques. Computer Networks, 51:2529-2553, 2007.

[3] A. N. Bishop, B. D. O. Anderson, B. Fidan. Bearing-only localization using geometrically constrained optimization. IEEE Transactions on Aerospace and Electronic Systems, 45(1): 308-320, 2009.

[4] K. Doğançay. Online Optimization of Receiver Trajectories for Scan-Based Emitter Localization. IEEE Transactions on Aerospace and Electronic Systems, 43(3): 1117-1125, 2007.

[5] K. Doğançay and D. A. Gray. Bias compensation for least-squares multi-pulse TDOA localization algorithms. ISSNIP Conference, 51-56, 2005.

[6] S. P. Drake and K. Doğançay. Some Applications of Tensor Algebra to Estimation Theory. 3rd International Symposium on Wireless Pervasive Computing, ISWPC 2008, 106-110, 2008.

[7] M. Gavish and A.J. Weiss. Performance analysis of bearing-only target location algorithms. IEEE Transactions on Aerospace and Electronic Systems, 28(3): 817-827, 1992.

[8] W. H. Foy. Position-Location Solution by Taylor-Series Estimation. IEEE Transactions on Aerospace and Electronic Systems, AES-12(2): 187-194, 1976.

[9] J. L. Melsa and D. L. Cohn. Decision and Estimation Theory. McGraw-Hill Inc., 1978.

[10] B. Fidan, S. P. Drake, G. Mao, B. D. O. Anderson and A. A Kannan. Collinearity problems in passive target localization using direction finding sensors. Processing of the 5th International Conference on Intelligent Sensors, Sensor Networks and Information Processing, 2009.

[11] Y. Ji, C. Yu and B. D. O. Anderson. Bias-correction in localization algorithms. IEEE Global Communication Conference, 2009.

[12] D. J. Torrieri. Statistical Theory of Passive Location Systems. IEEE Transactions on Aerospace and Electronic Systems, 20(2): 183-198, 1984.

[13] Y. Ji, C. Yu and B. D. O. Anderson. Geometric Dilution of Localization and Bias-Correction Methods. ICCA 2010. Accepted.

[14] S. Guolin, C. Jie, G. Wei, K.J.R. Liu. Signal processing techniques in network-aided positioning: a survey of state-of-the-art positioning designs. IEEE Signal Processing Magazine, 22(4):12-23, 2005.

[15] A. Amar and A. J. Weiss. Localizaion of Narrowband Radio Emitters Based on Doppler Frequency Shifts. IEEE Transactions on Signal Processing, 56(11): 5500-5508, 2008.