ORTHONORMAL SUBBAND CODER DESIGN USING POLYNOMIAL EIGENVALUE DECOMPOSITION

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ABSTRACT

We consider the design of M-channel orthonormal filter banks using polynomial matrix EVD techniques. Modifications are proposed to a time-domain, polynomial EVD technique, known as SBR2, which enables it to be applied effectively to the task of FIR paraunitary filter bank design for use in subband coding. This algorithm is compared to a well-known benchmark FIR compaction filter design method. We show that higher coding gains are obtainable with our technique for a small number of algorithm iterations.

1. INTRODUCTION

Orthonormal filter banks have been extensively studied for subband coding [1]-[11]. Subband coding has been exploited in an increasing number of applications, including digital communications, image and audio coding [6], noise reduction and channel coding [11]. For the case where the order of the filters is unconstrained, it is known that a *principal component filter bank* (PCFB) exists and is an optimal orthonormal (paraunitary) filter bank for this problem [2], [3]. This is also true when the filter orders are constrained to be not greater than the number of subband channels. In this case, the Karhunen-Loeve transform (KLT) or the singular-value decomposition (SVD) provide the optimal solution. The PCFB also exists for the special case of the two-channel filter bank. However, in general, it is believed that the PCFB does not exist for the intermediate case where order-constrained filters, i.e. finite impulse response (FIR) filters, are used [7].

A number of authors have proposed methods for the design of suboptimal (near-optimal) constrained-order orthonormal filter banks [4]-[10]. A common approach has been to calculate an optimal FIR compaction filter [4]-[6], [8],[10], for a given input power spectral density (PSD), then use an appropriate completion strategy to construct the filter bank, such as that in [5]. Typically, the filter is chosen to optimise a specific objective function, such as coding gain or energy compaction. As a consequence, all such methods require the numerical optimisation of non-linear and non-convex functions. Moulin *et al.* [5] formulate the FIR compaction filter design problem as a semi-infinite linear programming problem. Kirac and Vaidyanathan propose a more efficient way of obtaining FIR compaction filters in [6], called the window method. In [8], Tuqan and Vaidyanathan proposed a semi-definite programming method based on a state-space description of the compaction filter, and was shown to be globally optimal. However, this algorithm is also computational more costly and complex than the window method. A drawback with all these techniques is that they suffer from the ambiguity caused by the non-uniqueness of the FIR compaction filter. In essence, different compaction filter spectral factors lead to different filter banks, which in turn yield different performances. As such, all such spectral factors need to be tested for their performance [10].

Other authors have presented paraunitary filter bank design methods in the context of signal subspace analysis of broadband signals [12], [13]. In [12], the fixed degree parameterisation proposed by Vaidyanathan [1] is exploited. An alternative design can be obtained by generalisation of the eigenvalue decomposition (EVD) to polynomial matrices, as proposed in [14], [15]. This algorithm is called the second order sequential best rotation (SBR2) algorithm. It has been successfully used in applications where the EVD has traditionally been employed, including subspace decomposition. In this paper, we present a novel method of designing orthonormal filter banks for subband coding that uses an adaptation of the SBR2 algorithm. A more thorough treatment of the new algorithm is left for a journal publication.

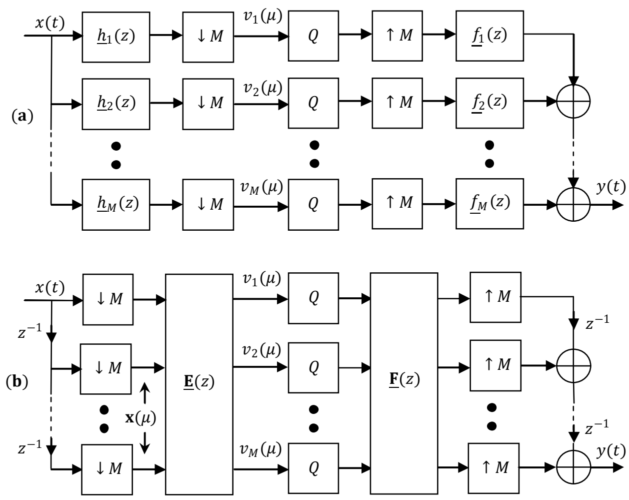

Fig. 1. (a) M–channel uniform, maximally decimated filter bank and (b) its polyphase representation.

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2. OPTIMAL FILTER BANKS

An *M*–channel subband coder is shown in Fig. 1(a) and its polyphase form is shown in Fig. 1(b). This is a maximally decimated uniform filter bank; our discussions are limited to this type of subband coder. We further assume that the filter bank is orthonormal, i.e., the matrix $\mathbf{E}(e^{j\omega})$ in Fig.1(b) is unitary for all ω. In other words, $\underline{\mathbf{E}}(z)$ is paraunitary [1], i.e. $\underline{\mathbf{E}}(z)\underline{\tilde{\mathbf{E}}}(z) = \underline{\tilde{\mathbf{E}}}(z)\underline{\mathbf{E}}(z) = \mathbf{I}$, where $\underline{\mathbf{E}}(z)$ is an $M \times M$ polynomial matrix and $\underline{\tilde{\mathbf{E}}}(z) = \underline{\mathbf{E}}^H(z^{-1})$. If $\underline{\mathbf{F}}(z)$ is chosen such that $\underline{\mathbf{E}}(z)\underline{\mathbf{F}}(z) = cz^{-\tau}\mathbf{I}$, for some constant c and integer τ, then the subband coder is a perfect reconstruction filter bank. That is, with no subband-processing, $y(t) = x(t-\tau)$ for all $t, \tau \in \mathbb{Z}$.

A PCFB offers an optimal solution to two subband coding problems. Firstly, assuming optimal bit allocation, it is an optimal orthonormal subband coder in the sense of maximising the well-known *coding gain* [2]:

$$G = \frac{(1/M)\sum_{l=1}^{M}\sigma_l^2}{(\prod_{l=1}^{M}\sigma_l^2)^{1/M}}, \tag{1}$$

where $\sigma_l^2 = E\{|v_l(\mu)|^2\}$, $1 \le l \le M$, is the variance of $v_l(t)$: the output of $\underline{h}_l(z)$, $E\{\cdot\}$ denotes the expectation operator and $\mu = Mt$ denotes the low-rate time index. Secondly, it minimises the reconstruction error for a proper subset of the set of subband channels. Vaidyanathan has shown that the outputs of a PCFB simultaneously satisfy:

(i) *Strong decorrelation.* The subband signals, $v_l(t)$, are decorrelated at all relative time lags, i.e.,

$$E\{\textstyle\sum_\mu v_l(\mu)v_m(\mu+\tau)\} = 0, \tag{2}$$

for $l \neq m$ and all τ.

(ii) *Spectral majorisation.* Let the PSD of $v_l(t)$ be denoted as $S_{ll}(e^{j\omega})$. For all ω, the set $\{S_{ll}(e^{j\omega})\}$ has the property,

$$S_{11}(e^{j\omega}) \ge S_{22}(e^{j\omega}) \ge \cdots \ge S_{MM}(e^{j\omega}), \tag{3}$$

where the subbands are numbered such that $\sigma_l^2 \ge \sigma_{l+1}^2$.

3. POLYNOMIAL MATRIX EVD

Correlation between signals is a type of redundancy which can be exploited to achieve compression. If the signals are only correlated at zero relative time lag, then the KLT (or SVD) can be used to decorrelate the signals. The decorrelation process converts the form of the redundancy from correlation between the signals to disparity between the signal powers. At this stage, it is possible to achieve compression by discarding low power channels.

In the case of an *M*-channel filter bank (depicted in Fig. 1(b)), provided the input samples are uncorrelated over any lag $|\tau| < M$ so that the subband channels are uncorrelated for any relative delay, the matrix $\underline{\mathbf{E}}(z)$ required for the KLT/SVD is a unitary matrix applied to the vector $\mathbf{x}(t)$. The orthogonality condition implies that the transformation is energy preserving.

However, if the subband signals are correlated for lags $\tau \neq 0$ in (2), decorrelation by a unitary matrix is not sufficient for accurate signal subspace estimation; and strong decorrelation is necessary. To achieve this, the transformation applied must be a matrix of polynomials (a bank of FIR filters), as represented by $\underline{\mathbf{E}}(z)$. It is desirable to have a suitable polynomial matrix EVD (PEVD) algorithm to generate a transformation of the form: $\underline{\mathbf{S}}(z) = \underline{\mathbf{H}}(z)\underline{\mathbf{R}}(z)\underline{\tilde{\mathbf{H}}}(z)$, where $\underline{\mathbf{R}}(z)$ is an estimate of the cross-spectral density matrix for the input signals and $\underline{\mathbf{S}}(z)$ is approximately diagonal and provides an estimate of the cross-spectral density matrix for the transformed signals. Such a $\underline{\mathbf{H}}(z)$ can be found by the SBR2 algorithm [15].

3.1 Sequential Best Rotation Algorithm

The SBR2 algorithm constitutes a simple scheme for generating polynomial (FIR) paraunitary matrices to achieve the strong decorrelation of multiple channels. The structure of the filter bank produced by the technique is an immediate generalisation of the paraunitary matrix decomposition found by Vaidyanathan in [1]. For the 2×2 case, the paraunitary matrix may be expressed as,

$$\begin{aligned}\underline{\mathbf{H}}(z) &= \underline{\mathbf{P}}_L(z)\underline{\mathbf{P}}_{L-1}(z)\ldots\underline{\mathbf{P}}_0(z)\\ &= \mathbf{Q}_L\underline{\mathbf{\Lambda}}^{\tau_L}(z)\mathbf{Q}_{L-1}\underline{\mathbf{\Lambda}}^{\tau_{L-1}}(z)\ldots\mathbf{Q}_0\underline{\mathbf{\Lambda}}^{\tau_0}(z)\end{aligned} \tag{4}$$

where $\underline{\mathbf{P}}_\ell(z) = \mathbf{Q}_\ell\underline{\mathbf{\Lambda}}^{\tau_\ell}(z)$ is an elementary paraunitary matrix composed of a 2×2 unitary matrix $\mathbf{Q}_\ell$ and a polynomial matrix $\underline{\mathbf{\Lambda}}^{\tau_\ell}(z) = \begin{bmatrix}1 & 0\\ 0 & z^{\tau_\ell}\end{bmatrix}$ for which the integer parameter τ_ℓ can be negative or positive. The SBR2 algorithm operates in an iterative manner. At each step, the algorithm applies a generalised similarity transformation given by: $\underline{\mathbf{R}}''(z) = \underline{\mathbf{P}}_\ell(z)\underline{\mathbf{R}}(z)\underline{\tilde{\mathbf{P}}}_\ell(z)$.

This elementary paraunitary transformation constitutes one stage of the SBR2 algorithm designed to zero the dominant off-diagonal coefficient of $\underline{\mathbf{R}}(z)$. The algorithm continues by making the substitution $\underline{\mathbf{R}}''(z) \leftarrow \underline{\mathbf{R}}(z)$. In practice, this iterative process is repeated until the magnitude of the dominant off-diagonal coefficient, $|r_{np}(\tau_\ell)|$, of $\underline{\mathbf{R}}(z)$ is sufficiently small, at which point the polynomial matrix is declared to be diagonal. The polynomial matrix generated by (4) is paraunitary since each stage is paraunitary. The algorithm intrinsically aims to perform PEVD on the (sample) cross-spectral density matrix $\underline{\mathbf{R}}(z)$.

3.2 Alternative Cost Function

The SBR2 tends to strongly decorrelate signals with large power at the expense of signals that have relatively lower power. This limits the extent to which strong decorrelation and spectral majorisation is performed. This problem can be alleviated by the use a cost function which is proportionately, equally sensitive to changes in any of the signals, such as the coding gain measure in (1). Hence, we use (1) as our cost-function but base it on sample statistics. A thorough treatment of this cannot be provided here due to limited space. We call this is the modified SBR algorithm.

4. CROSS-SPECTRAL COVARIANCE ESTIMATION FOR DEMULTIPLEXED SIGNALS

The SBR2 algorithm can be classed as a 'blind' technique since it does not use knowledge about the signals or the mixing matrix. Furthermore, its formulation is not based on knowledge of the input signal statistics save for the minor requirement that the mean value of the signals is zero. Therefore, the performance of the filter bank it designs depends on the accuracy of its estimate of the true space-time covariance matrix for input signals.

The modified SBR2 algorithm, introduced in Sect. 3, can be used directly to construct an $M \times M$ paraunitary polynomial matrix $\underline{\mathbf{E}}(z) = \underline{\mathbf{H}}(z)$ for the demultiplexed signals $\underline{\mathbf{x}}(z)$ in Fig. 1(b). The output subband signals from $\underline{\mathbf{E}}(z)$ may be expressed as $\underline{\mathbf{v}}(z) = \underline{\mathbf{E}}(z)\underline{\mathbf{x}}(z)$. However, if the input signal $x(t)$ is stationary, this scheme can be improved upon. In this case, the statistics of the demultiplexed input signal are such that the cross-spectral density matrix $\underline{\mathbf{A}}(z)$ is pseudocirculant. Knowledge of this structure is implicitly exploited by conventional filter bank design algorithms, such as the window method. In the following we investigate the structure of $\underline{\mathbf{A}}(z)$ with a view to improving the covariance matrix estimate.

4.1 Cross-Spectral Density Matrix

Consider the subband coder in Fig. 1(b). The blocked samples from the demultiplexer are

$$\mathbf{x}(\mu) = [x_1(\mu), x_2(\mu), \dots, x_M(\mu)]^T \tag{5}$$

where $x_k(\mu) = x(Mt + k - 1)$, $1 \le k \le M$, are the demultiplexed signals. We assume that the signal $x(t)$ is wide-sense stationary (WSS): A stochastic process, $\chi(t)$, is said to be WSS if and only if [1] $E\{\chi(t)\} = E\{\chi(t+\tau)\}$, $\forall\, t, \tau \in \mathbb{Z}$; and $E\{\chi(t)\chi^*(t+\tau)\} = \alpha(\tau)$, where $\alpha(\tau)$ is the autocovariance function of $\chi(t)$. Note that we assume $E\{\chi(t)\} = 0, \forall\, t$.

The $M \times M$ cross-spectral density matrix for the demultiplexed signals is:

$$\underline{\mathbf{A}}(z) = \sum_{\tau=\tau_1}^{\tau_2} \mathrm{A}(\tau) z^{\tau} = \begin{bmatrix} \underline{a}_{11}(z) & \cdots & \underline{a}_{1M}(z) \\ \vdots & \ddots & \vdots \\ \underline{a}_{M1}(z) & \cdots & \underline{a}_{MM}(z) \end{bmatrix}, \tag{6}$$

where $\underline{a}_{lm}(z) = \sum_{\tau=-\infty}^{\infty} a_{lm}(\tau) z^{-\tau}$ and $a_{lm}(\tau) = E\left\{x_l(\tau) x_m^*\,(\mu+\tau)\right\}$ are the cross-covariances between subband signals.

4.2 Parahermitian Nature of $\underline{A}(z)$

It is easy to show that $\underline{\mathbf{A}}(z)$ is parahermitian. We have that

$$\begin{aligned} a_{l,m}(\tau) &= E\{x_l(\tau) x_m^*\,(\mu+\tau)\} \\ &= E\{x_m(\tau) x_l^*\,\,(\mu-\tau)\}^* \\ &= a_{m,l}^*(-\tau) \end{aligned} \tag{7}$$

and $\underline{a}_{l,m}(z) = \sum_{\tau=-\infty}^{\infty} a_{l,m}(\tau) z^{-\tau} \;= \underline{\tilde{a}}_{m,l}(z)$, therefore $\underline{\mathbf{A}}(z) = \underline{\tilde{\mathbf{A}}}(z)$.

4.3 Pseudocirculant Matrices

An $M \times M$ cross-spectral density matrix $\underline{\mathbf{A}}(z)$ with entries $\underline{a}_{l,m}(z)$ is said to be pseudocirculant if there exists polynomials $\underline{\varphi}_0(z), \underline{\varphi}_1(z), \dots, \underline{\varphi}_{M-1}(z)$ such that [1]

$$\underline{a}_{l,m}(z) = \begin{cases} \underline{\varphi}_{m-l}(z), & 1 \le l \le m \le M \\ z^{-1}\underline{\varphi}_{m-l+N}(z), & 1 \le m < l \le M. \end{cases} \tag{8}$$

In words, $\underline{\mathbf{A}}(z)$ is a circulant matrix except that the entries below the main diagonal are multiplied by z^{-1}.

In the following, we show that the true cross-spectral density matrix, $\mathbf{A}(z)$, of the demultiplexer outputs, $\underline{\mathbf{x}}(z)$, (see Fig. 1(b)) is a pseudocirculant matrix for a WSS input signal. A typical term from the true cross-spectral density matrix of $x(t)$, $\underline{\mathbf{A}}(z)$, is:

$$\begin{aligned} \underline{a}_{l,m}(z) &\propto E\{(\textstyle\sum_\mu x_l(\mu) z^{-\mu})(\sum_\mu x_m(\mu) z^{-\mu})\} \\ &= E\left\{(\textstyle\sum_t x(Mt+l-1) z^{-t}) \times (\sum_t x(Mt+m-1) z^{-t})\right\} \\ &= \textstyle\sum_\tau E\left\{\sum_t x(M(t+\tau)+l-1) \times x(Mt+m-1)\right\} z^{-\tau}. \end{aligned} \tag{9}$$

Hence

$$\begin{aligned} \underline{a}_{l,m}(z) &\propto \textstyle\sum_\tau a([M(t+\tau)+l-1] - [Mt+m-1]) z^{-\tau} \\ &= \textstyle\sum_\tau a(M\tau+m-l) z^{-\tau}. \end{aligned} \tag{10}$$

So, setting $\underline{\varphi}_k(z) = \sum_\tau a(M\tau + k - 1) z^{-\tau}$, we see that $\underline{\mathbf{A}}(z)$ is pseudocirculant.

4.4 Estimation of $\underline{\mathbf{A}}(z)$

Due to the pseudocirculant structure of $\underline{\mathbf{A}}(z)$, there is extra (useful) information about $\underline{a}(z)$ in the combination of related entries, which can be used to determine $\underline{a}(z)$. Given $\tau, T \in \mathbb{Z}$, $0 \le t_1 \le T$, the sample autocovariance function for the input signal $x(t)$ may be expressed as:

$$\underline{r}(z) = \textstyle\sum_{\tau=-t_1}^{t_1} \left[\frac{1}{T}\sum_{t=0}^{T-1}[x(t)x^*(t+\tau)]\right] z^{-\tau}, \tag{11}$$

The sample cross-spectral density matrix for the demultiplexed signals $\mathbf{x}(t)$ is given by

$$\underline{\mathbf{R}}(z) = \textstyle\sum_{\tau=-t_1}^{t_1} \mathbf{R}(\tau) z^{-\tau} \tag{12}$$

where

$$\mathbf{R}(\tau) = \tfrac{M}{T}\textstyle\sum_{\mu=0}^{T-1} \mathbf{x}(\tau)\mathbf{x}^{\mathrm{H}}(\mu+\tau) \;\in \mathbb{C}^{M\times M}. \tag{13}$$

The SBR2 algorithm can be modified to exploit the pseudocirculant structure of $\underline{\mathbf{A}}(z)$. The set of diagonally related elements of $\underline{\mathbf{R}}(z)$ are different estimates of the same true cross-covariance. Therefore, to improve the estimate of $\underline{\mathbf{A}}(z)$, averaging may be performed across the associated coefficients in $\underline{\mathbf{R}}(z)$ – taking account of the delay between terms above the diagonal and those below. We define

$$\varphi_k(\tau) = \left(\frac{1}{M}\right)\left(\sum_{l=1}^{M-k} r_{l,l+k}(\tau) + \sum_{l=M-k+1}^{M} r_{l,l+k-M}(\tau+1)\right) \quad (14)$$

$0 \le k \le M-1$, and a typical entry of the new (averaged) sample covariance matrix $\underline{\mathbf{R}}'(z)$ as

$$\underline{r}'_{l,m}(z) = \begin{cases} \sum_\tau \varphi_{m-l}(\tau) z^{-\tau} & , \ 1 \le l \le m \\ \sum_\tau \varphi_{m-l+M}(\tau) z^{-\tau-1} & , \ 1 \le m < l. \end{cases} \quad (15)$$

The modified SBR2 algorithm can be applied to our improved estimate of the cross-spectral density matrix. This combined system yields the *SBR2 coder*. A diagram of the process blocks constituting the SBR2 coder is shown in Fig. 2.

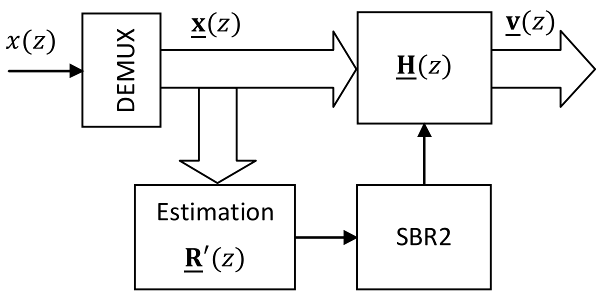

Fig. 2. The SBR2 coder.

5. SIMULATION RESULTS

In this section, we present simulation results that quantify the data encoding performance of the SBR2 coder. The coding gain in (1) is used to assess its performance. We compare our algorithm to the window method [6].

The data simulated for the following experiments are based on examples given in [6], [5]. The algorithms were tested on an ARMA(5) input process with a PSD as represented by the dashed curve in Fig. 3. This type of process is regarded as a good model for many practical signals such as image and speech signals. The input signal $x(t)$ was generated by filtering a binary phase-shift keying (BPSK) sequence with unit variance and zero mean of length 2000 samples: each sample takes the value ± 1 with a probability of 1/2. The BPSK sequence was filtered with an order 5 Yule-Walker IIR filter. The ARMA process had the following poles and zeros:

$$p = [0.1195, 0.8990e^{\pm j2.1472}, 0.8824e^{\pm j0.5594}]$$
$$z = [\pm 0.9992, -0.45416, 1.0020e^{\pm j1.3305}]$$

Unless stated otherwise, experiments quantifying coding gain performances were repeated over 50 realisations and the mean over the trials were taken. The SBR2 coder was applied to the improved estimate of the cross-spectral density matrix, $\mathbf{R}'(z)$, with entries as in (15), and with a window of length of $2t_1 + 1 = 41$, as in (12), which produced the best results.

Example 1: Frequency response. A comparison of the coding gain achieved by the SBR2 coder for $L = 11$ and the window method for $N = 11$ was made. For this dataset, the SBR2 coder achieves a coding gain of 1.46dB higher than that obtained using the window method. Fig. 3 shows the frequency response of the filters $\underline{h}_1(z)$ and $\underline{h}_2(z)$ produced by the SBR2 coder for $L = 11$ as the solid curve and the dotted curve, respectively. It can be seen that the algorithm has designed a multiband compaction filter with passbands that coincide with the dominant signal energies, which is commensurate with high compaction gains.

Example 2: Dependence on L. Fig. 4 gives a comparison of the coding gain performance between the two-channel filter bank designed using the window method and that produced by the SBR2 coder. The abscissa on this figure represents both the number of SBR2 iterations L and filter order N (window method). The dotted (horizontal) line represents the ideal coding gain. As expected, the maximum coding gain attained by the algorithms are below the ideal values. An important result is that, for the given input process, the filter banks constructed by the SBR2 coder generally attain a higher coding gain than those of the window method.

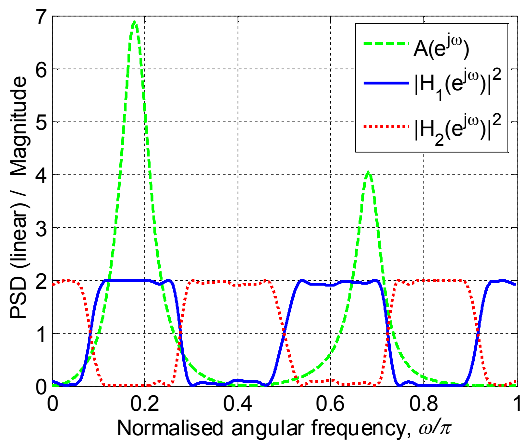

Fig. 3. The frequency responses of a two-channel filter bank designed by the SBR2 coder with $L = 11$ for an ARMA(5) process.

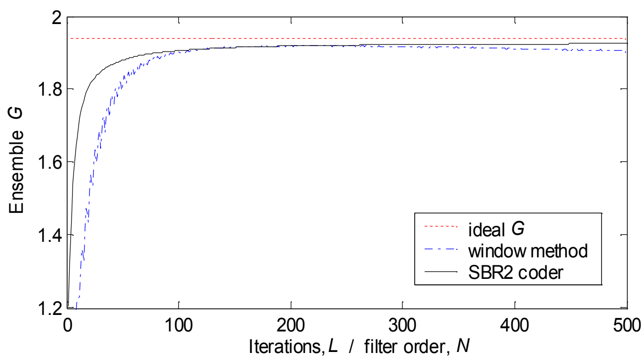

Fig. 4. Comparison of the coding gain performance of the window method and the SBR2 coder for the 2-channel case and the ARMA(5) process.

6. CONCLUSIONS

In this paper, an adaptation of a polynomial matrix EVD algorithm, namely the SBR2 algorithm, has been proposed that takes advantage of the special structure of the subband covariance matrix to design multi-band orthonormal subband coders. We also propose a new cost function for use with the SBR2 algorithm that improves the diagonalisation and data compression performances of the algorithm.

The resultant algorithm, called the SBR2 coder, can converge to a solution that yields a perfect reconstruction filter bank which is approximately optimal for subband coding in a small number of iterations; the suboptimality of the algorithm diminishes as the number of steps increases. The SBR2 coder has been shown to outperform a well-known algorithm, called the window method, for the two-channel case and for a set of benchmark problems.

It is envisaged that the SBR2 algorithm can be extended naturally for application to the problem of multichannel subband coding for applications such as MIMO digital communications using sensor arrays.

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