INDICATORS OF COLOUR IMAGE QUALITY FOR $\eta\sigma\lambda$-SPACES

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ABSTRACT

We take the standing that a *characterization* of an image, together with a knowledge of the class the image belongs to, is required information to assess the quality of the image. We consider characterizations of images in colour spaces of the hue-saturation-luminance ($\eta\sigma\lambda$ for short) type. We use *word descriptors*, the entropy of hue circular histograms and other indexes that summarize the characterization process. We introduce two spaces of the type $\eta\sigma\lambda$; one is *spherical*; in the other, the saturation is not normalized by the luminance (unlike the HSV, HSL and HSI spaces).

1. INTRODUCTION

We consider the no-reference assessment of colour image quality. The hue, saturation (or colourfulness) and luminance variables are denoted here as η, σ and λ, respectively; $\eta\sigma\lambda$ spaces are more intuitive than RGB colour space and this is an advantage when estimating colour image quality. Even though the luminance component is the main determinant of image quality, the chromatic aspects are important as well. We consider the main aspects of image quality to be *readability* and *aesthetics*.

To assess image quality, we initially *characterize* the image along several dimensions. Then, depending on the class (e.g. natural scenes) the image is considered to be in, the quality assessment process can proceed.

Location-Dispersion plots [5] along each of the dimensions η, σ and λ, provide useful characterizations of colour images. For the three components of hue, saturation and luminance and for a window that is placed at a large and representative set of positions across the image, we compute corresponding values of *location* (e.g. the midrange μ, or the circular average) and of *dispersion* (e.g. the linear range ρ, or the circular range); indeed, using scatter plots, we plot the dispersion as a function of the location. The hue component, being of a circular nature, requires of the use of special tools, such as circular averages. The information in such scatter plots is then summarized with the help of *word descriptors.*

We present two new colour spaces; one has a double-cone image space and is labeled $\rho\mu$; in it, the range (i.e. the max minus the min) of the triple (R, G, B) is used as an unnormalized measure of saturation, while the midrange (i.e. the average of the max and the min) is used as a measure of luminance. Pixelwise plots of saturation versus luminance are then used to chromatically characterize colour images. Also, a spherical colour space, where the colour attributes being made explicit are those of *hue, colourfulness and brightness* is presented; it is called *Runge* space and is labeled $\eta\kappa\lambda$. A simple application of colour modification in virtual restoration, using this space, is shown.

The readability of an image depends on its of *contrast* contents while its aesthetics is dependant both on hue location and on *variety* of luminance and saturation. We measure contrast along the hue, saturation and luminance components; likewise, we measure variety as the nonuniformity of circular histograms of the hue variable, and use codes for the ways the $\rho\mu$*-triangle* is covered.

2. $\eta\sigma\lambda$-TYPE SPACES

The $\eta\sigma\lambda$ variables can be derived from RGB values in several ways, as in the spaces HSV, HSL and HSI. Usually, $\eta\sigma\lambda$ spaces normalize the saturation component by the luminance component; this results in saturation artifacts at luminance values near 0, near 1, or both.

Geometrically, the luminance of a colour point in RGB cube is a measure of its distance to the origin while the saturation is a measure of the distance to the achromatic line (in Runge space, defined below, the saturation is given by a distance to *intermediate gray* which has RGB coordinates [1/2, 1/2, 1/2]); the hue is a measure of the angle that is measured on the projection on the plane that contains the basis elements R, G and B, with axis the achromatic line, and measured with respect to R.

2.1 Important subsets of the RGB cube

Call the origin [0, 0, 0] of the RGB cube *pure black*, the point [1, 1, 1] *pure white*, the line segment between them the *achromatic segment* Φ and the plane through the origin orthogonal to Φ, the *chromatic plane* Π. Call the faces of the cube with points with $min(R,G,B) = 0$, the *dark corner* of the cube, and those with $max(R,G,B) = 1$, the *light corner*. The *star* (i.e. a set of edges) of those edges of the cube with zero median is the *black corner* of the cube while the star of the edges with unitary median is the *white corner* of the cube. Call the polygon formed by the edges of the cube of points (R,G,B) with $min(R,G,B) = 0$ and $max(R,G,B) = 1$, the *chromatic hexagon*. Finally, call each triangle that has Φ as one its sides and a point on the chromatic hexagon as corresponding opposite vertex, a *(constant-) hue triangle* or a *chromatic triangle*. Colour points on Φ (called *achromatic colours*) have an undefined hue and the colours on each chromatic triangle have the same hue.

Using barycentric coordinates (with reference to the tetrahedron with vertices pure black, red, green and blue) for the points (R,G,B) on each chromatic triangle having a vertex (r,g,b) on the chromatic hexagon, one has $(R,G,B) = \lambda_1(1,1,1) + \lambda_2(r,g,b) + \lambda_3(0,0,0) = \lambda_1(1,1,1) + \lambda_2(r,g,b)$. Therefore, the points in the cube with a given ordering of the components R, G and B (there are six such orderings) form a tetrahedron with *slices* given by chromatic triangles.

2.2 The spaces HSL, HSI, HSV and $\eta\rho\mu$: caveats

In the spaces HSV, HSI and HSL, the points on the dark corner, different from pure black, have saturation $\sigma = 1$ and, since the saturation of pure black (and of each point on the achromatic line) is defined to be 0, one has a discontinuity of the corresponding transformations from RGB space.

The range $\rho(R,G,B) = max(R,G,B) - min(R,G,B)$ and the midrange $\mu(R,G,B) = \frac{max(R,G,B)+min(R,G,B)}{2}$ of the triple (R,G,B) are respectively measures of saturation [1] and of luminance that we use below. The midrange is the luminance component of HSL colour space. We denote as ρ_1 the quasirange $median(R,G,B) - min(R,G,B)$ and we denote as μ_2 the "upper midrange" $\frac{max(R,G,B)+median(R,G,B)}{2}$.

The saturation component of the space HSL is defined as $\sigma = \frac{\rho}{2\mu}$, if $0 < \mu <= 0.5$ and $\sigma = \frac{\rho}{2(1-\mu)}$, if $1 > \mu >= 0.5$. The *circular* hue component η is coded in the interval $[-\frac{1}{6}, \frac{5}{6}]$, and is defined as $\eta = \frac{1}{6}\frac{G-B}{\rho}$ for the orderings RBG and RGB; $\eta = \frac{1}{6}\frac{B-R}{\rho} + \frac{1}{3}$, for the orderings GRB and GBR, and $\eta = \frac{1}{6}\frac{R-G}{\rho} + \frac{2}{3}$, for the orderings BGR and BRG.

For constant values of the luminance component μ, near 0 and near 1, a very small change of ρ results in a large change of HSL's saturation.

For the space HSI, the luminance component is given by the projection $[I,I,I]$ of the colour point $\mathbf{C} := [R,G,B]$ on the line Φ and is given by $I = \frac{R+G+B}{3}$. The projection of the colour point on the plane Π is given by $\frac{1}{3}[2R-G-B, 2G-R-B, 2B-R-G]$. The hue component is defined as the angle that the projection $P_{\Pi}(\mathbf{C})$ makes with the projection [2/3, -1/3, -1/3] of pure red. The cosine of H is then given by $\alpha := cos(H) = \frac{2R-G-B}{2\sqrt{R^2+G^2+B^2-RG-RB-GB}}$, which can be derived as the cosine in a dot product (our edition of [2], on page 94, has a typo for this formula). Alternatively, writing $P_{\Pi}(\mathbf{C}) = [a,b,-(a+b)]$, we have $\mathbf{C} = [I,I,I] + [a,b,-(a+b)]$ and the alternate expression $\alpha = \frac{\sqrt{3}a}{2\sqrt{a^2+b^2+ab}}$ results. One gets $\eta = arcos(\alpha)$, if $G \geq B$, and $H = -arcos(\alpha)$ if $G \leq B$.

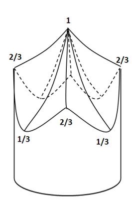

Figure 1: Caped cylinder of image HSI space.

The saturation component of the HSI space is given by $S := 1 - \frac{min(R,G,B)}{I} = \frac{2}{3}\frac{\mu_2 - min}{I}$. The set of values that the saturation-luminance pair (S,I) can take, depends on the value of the hue component H. This posses problems since, for the modification of the hue at large values of I, it may be necessary to modify the saturation as well. For a given hue H, the possible values of the pair (S,I) are bounded below by the line segment $I = 0$, with $S \in [0,1]$; on the left, by the axis $S = 0$ with $I \in [0,1]$, on the left and above, by the vertical segment $\{(S,I) : S = 1, I \in [0, I_0]\}$ and a segment of a hyperbola $\{(S,I) : I = \frac{I_0}{I_0+(1-I_0)S}, S \in [0,1]\}$; I_0 is indirectly a function of the hue H; more directly, it is a function of the median med' of the point on the chromatic hexagon that is vertex of the chromatic triangle that contains the point, and we have $I_0 = \frac{1+med'}{3}$ see Figure 1.

For the space HSV, the saturation is given by $1 - \frac{min}{max} = \frac{\rho}{max}$ and the luminance component is given by the *max*. The hue is the same as that for the HSL system; in the three cases HSL, HSI and HSV, the hue is constant for the points on each chromatic triangle. Even tough a geometrically uniform space (HSV range space is a cylinder) and the possible values of the pair $(max, \frac{\rho}{max})$ are those in $[0,1]^2$, in this square, the "lines of constant ρ" are segments of a hyperbola connecting the points $(\rho, 1)$ and $(1, \rho)$. Each of these hyperbolas intersect the line $s = max$ at the value $max = \sqrt{\rho}$; thus, for values of ρ close to 0, the distance from the origin of the square to the point of intersection grows rather abruptly with ρ. For small ρ, we are near the achromatic line, and we have a sharp decrease of saturation from 1 towards 0, as the max moves away from zero.

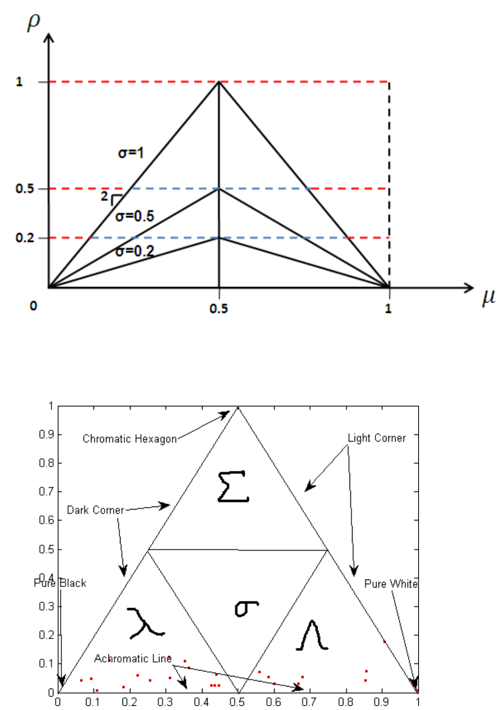

Figure 2: Above, Luminance-Saturation triangular slice Δ (same for each hue angle) of $\rho\mu$ colour space (by spinning the triangle across the base of the triangle, a double cone results); values of corresponding HSV's saturation σ are indicated. Below, images of important subsets of the RGB cube.

To avoid the discontinuity issues that arise from the normalization of the saturation by the luminance, we simply use the space with components η, ρ and μ; see Figure 2. Each chromatic triangle of the RGB cube maps in a bijective fashion to the $\rho\mu$-triangle. Points on the chromatic hexagon get mapped to $(\mu, \rho) = (0.5, 1)$. The achromatic line is mapped to the base of the triangle. Points on the faces of the dark (resp. light) corner get mapped to the left (resp. right) edge of the triangle. The $\rho\mu$-triangle is subdivided as shown in Fig 2, into four subtriangles, called *Highly Chromatic* (labeled Σ), *Moderately Chromatic* (labeled σ), *Light* (labeled Λ) and *Dark* (Labeled λ). In Section 3.2, the labels of the regions are ordered according to their resulting relative weights.

2.3 Runge $\eta\kappa\lambda$ space

We propose as well a space that separately codes hue, colourfulness and luminance; it is, geometrically, a solid 3-ball and is named after Otto Runge (1777 - 1810). Initially, shift the cube so that the central point (intermediate gray) ends up at the origin, then, with center the origin, radially *contract* the cube to a ball of radius 1/2, finally, rotate the ball so that the achromatic diameter points upwards. The colour attributes that are made explicit by using spherical coordinates r, θ, ϕ on the resulting *Runge ball* are those of hue ($\eta = \theta$), colourfulness ($\kappa = 1 - \gamma$, where $\gamma = 1 - 2r$ is the *grayness*) and lightness ($\lambda = \frac{\pi - \phi}{\pi}$). One has $\theta \in [0, 2\pi)$ and $\lambda, \gamma, \kappa \in [0, 1]$. Thus we get a tridimensional ball, centered at the origin and with the achromatic axis points vertically upwards. Power laws can be exploited for lightness and grayness correction, while hue correction requires the use of *circular tools;* see [4].

2.3.1 Matlab Program Code

Matlab code for the routines that convert rectangular RGB coordinates to Runge's spherical $r\theta\varphi$ coordinates, and back, are given below.

```
function[A] = RGB2RUNGE(v)
M= [.5+.5/sqrt(3), -.5+.5/sqrt(3), -1/sqrt(3);
-.5+.5/sqrt(3), 0.5+.5/sqrt(3), -1/sqrt(3);
1/sqrt(3),1/sqrt(3),1/sqrt(3)];
       x=[v(1)-0.5, v(2)-0.5, v(3)-0.5 ];
       if x(1)==0 & x(2)==0 & x(3)==0
            y=x;
       else
            xx=[abs(x(1)), abs(x(2)), abs(x(3))];
            k=max(xx)/sqrt(x(1)^2+x(2)^2+x(3)^2);
            y=k*x;
            z= (M*y')';
            w(1)= sqrt(z(1)^2+z(2)^2+z(3)^2); %r
            w(2)= angle(z(1)+ i*z(2) );    %teta
            w(3)=
            angle(z(3) + i*sqrt(z(1)^2+z(2)^2));
       end
       A=w;

function[A] = RUNGE2RGB(w)
M= [.5+.5/sqrt(3), -.5+.5/sqrt(3), -1/sqrt(3);
-.5+.5/sqrt(3), 0.5+.5/sqrt(3), -1/sqrt(3);
1/sqrt(3),1/sqrt(3),1/sqrt(3)];
       z(3)= w(1)*cos(w(3)); %r cos fi
       z(1)= (w(1)*sin(w(3)))*cos(w(2));
       z(2)= (w(1)*sin(w(3)))*sin(w(2)); %
       y= (M'*z')';
       if y(1)==0 & y(2)==0 & y(3)==0
            x=y;
       else
            yy=[abs(y(1)), abs(y(2)), abs(y(3))];
            k=
            sqrt(y(1)^2 + y(2)^2 + y(3)^2)/max(yy);
            x=k*y;
            color= [x(1)+.5, x(2)+.5, x(3)+.5];
       end
       A= color;
```

To illustrate the use of Runge Colour Space we implement a hue circular shift and power-law corrections of grayness and lightness. We modify the grayness component γ to γ^p, the lightness component component λ to λ^q while the hue contrast of the image is locally enhanced using the formula $h_0 \leftarrow h_0 + \alpha(h_0 - \bar{h})$ where h_0 is the hue of the central pixel of the window, $\bar{h}$ is a circular location measure (e.g. the circular mean or the circular median [4]) of the hues in the window and α is a control parameter; the ordering of the corrections is immaterial. For example, consider the application of this tool, using a window of 5×5 (image is of size 614×436) and control parameters $p = 1.2$, $q = 1.1$ and $\alpha = 1.5$, as shown in Figure 3.

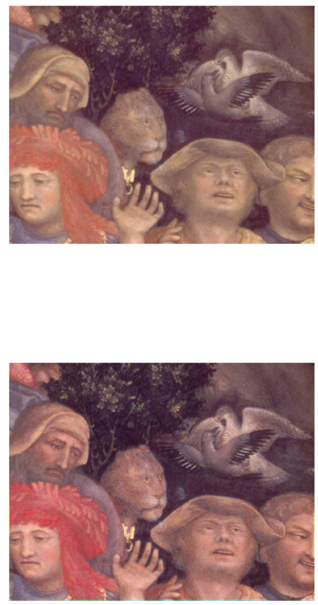

Figure 3: Above, original image Grotta. Below, colour modification with $\theta - 0.2$, $\gamma^{1.2}$, $\lambda^{1.1}$, and hue contrast enhancement with $\alpha = 1.5$ (below).

3. IMAGE CHARACTERIZATION

We consider three main types of characterization. Location versus dispersion scatter plots along each of the dimensions of hue, saturation and luminance with corresponding word descriptors; pixelwise distributions of saturation versus luminance in $\eta\rho\mu$ colour space with corresponding word descriptors, and circular histograms where the corresponding entropy measures the uniformity of the histogram. We base our discussion on the set of images shown in Figure 4; the first four images are of a better quality than the last four.

3.1 Location-Dispersion in $\eta\rho\mu$ space

We consider here the amount of local (i.e. on the basis of a moving window) contrast (or dispersion) that is present at each level (location), for each of the three components of luminance, saturation and hue; we work in $\eta\rho\mu$ space. As shown elsewhere [5], when the location is measured with the midrange and the dispersion with the range, the location-dispersion pair lives in a triangle, which we further subdivide into four subtriangles: A, up or high dispersion; B, left or

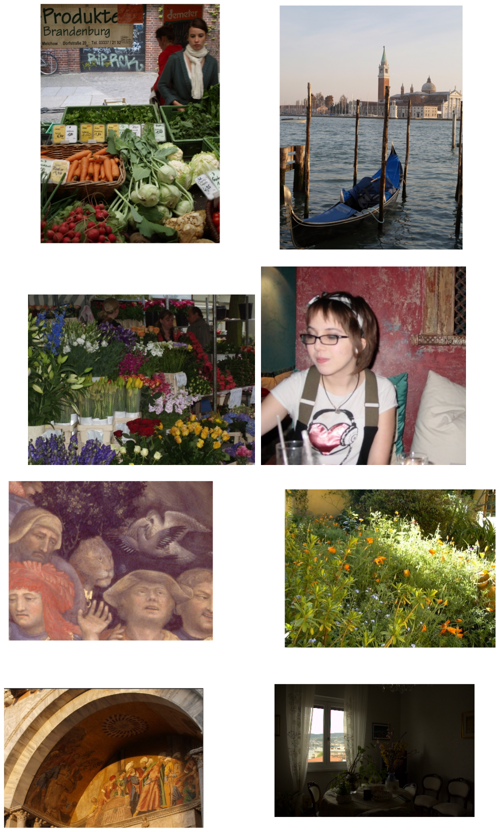

Figure 4: Set of images considered: Mercado1, Venezia1, Mercado2, Gabriela (photo by Nary Kim), Grotta, Ababoles, Venezia2 and Window.

low location; C, right or high location and, D, center or intermediate. (This applies for the dimensions of luminance and saturation but not for the hue dimension, see below.)

Along the luminance dimension, we have the distributions shown in Figure 5, to which the data in Table 1 correspond. For the location-dispersion characterization in the

Table 1: Location-Dispersion Characterization for Luminance, in $\eta\rho\mu$ space

Image	A	B	C	D	word
mercado1	3.58	59.11	22.11	15.17	BCDA
venezia1	1.68	30.24	27.57	40.48	DBCA
mercado2	3.08	77.52	14.14	5.24	BCDA
gabriela	3.10	48.03	18.11	30.75	BDCA
grotta	0.06	45.87	29.75	24.31	BCDA
ababoles	17.78	65.11	14.67	2.42	BACD
venezia2	0.83	66.92	18.60	13.64	BCDA
window	7.28	86.12	2.89	3.68	BADC

dimension of saturation, we have the data shown in table 2, where the percentages of occupancy of the subtriangles A, B, C, and D is given. Images with the saturation entirely in the A (e.g. images Gabriela and Venezia1) subtriangle have a visually interestingly smooth characteristic. In this sense, images Venezia2 and Ababoles are rough.

Now, consider the local variation (circular dispersion) of hue as a function of local hue. The local hue is measured with the circular mean while the local dispersion is measured with a circular range by measuring first the *circle gap* and then the circular range is given by $2\pi - gap$; for more details, see [4]. In Figure 6, a 5×5 window is used; as the window size is increased, the width of the clusters decreases. The occurrence of clusters at certain hues may suggest a hue shift, as a colour correction technique, but this depends on the specific image under consideration.

Table 2: Location-Dispersion Characterization for Saturation, in $\eta\rho\mu$ space

Image	A	B	C	D	word
mercado1	0.03	96.25	3.39	0.30	BCDA
venezia1	0	99.98	0.01	0	BCD'A'
mercado2	0.61	97.00	2.16	0.20	BCAD
gabriela	0	100	0	0	BC'D'A'
grotta	0	99.68	0.31	0	BCD'A'
ababoles	1.59	87.53	9.99	0.87	BCAD
venezia2	0.03	82.01	9.96	7.98	BCDA
window	0	99.29	0.70	0	BCD'A'

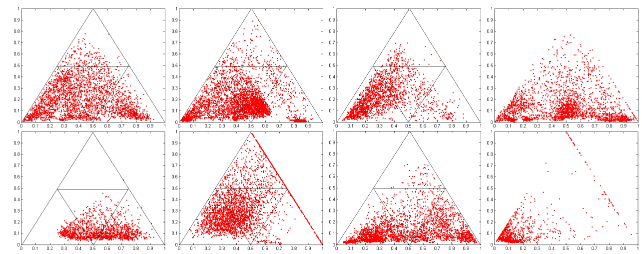

Figure 5: Location-Dispersion plots for luminance: Mercado1, Venezia1, Mercado2 and Gabriela (above), and, Grotta, Ababoles, Venezia2 and Window (below).

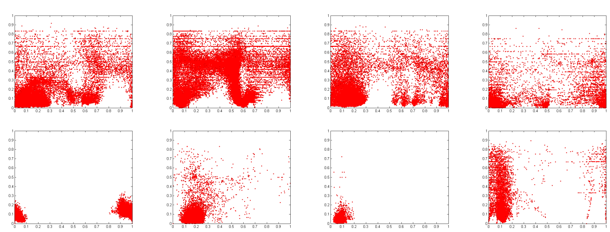

Figure 6: Location-Dispersion plots corresponding to the hue component; window = 5×5. Above: Mercado1, Venezia1, Mercado2, Gabriela; below: Grotta, Ababoles, Venezia2 and Window. The horizontal hue axis should be interpreted as circular. (The plot exists on a cylinder.)

3.2 Pixelwise saturation vs. luminance characterization

The triangular shape of the luminance-saturation subspace of $\rho\mu$ double-cone space indicates that the saturation must have small values for luminance values near 0 and near 1 and that full saturation is only possible at medium luminance; this agrees with the fact that, in natural scenes and under photopic conditions, very bright and very dark colours tend to look desaturated. We have found that natural images tend to have a unique word descriptor, namely $\lambda\sigma\Lambda\Sigma$ (a majority of dark pixels followed by a second majority of lowly chromatic pixels, followed by a minority percentage of light pixels, followed by a minority of highly chromatic pixels.) In fact, corresponding to the images in Figure 4 we have, respectively, the luminance-saturation word descriptors $\lambda\sigma\Lambda\Sigma$, $\lambda\sigma\Lambda\Sigma$, $\lambda\sigma\Lambda\Sigma$, $\lambda\Lambda\sigma\Sigma'$, $\sigma\lambda\Lambda\Sigma$, $\lambda\sigma\Lambda\Sigma$, $\lambda\sigma\Sigma\Lambda$ and $\lambda\sigma\Lambda\Sigma$; this points out to a preponderance of low values of saturation followed by one of intermediate values of both saturation and luminance. To make the descriptor more informative the horizontal line should perhaps be moved down. Observe the distributions shown in Figure 7.

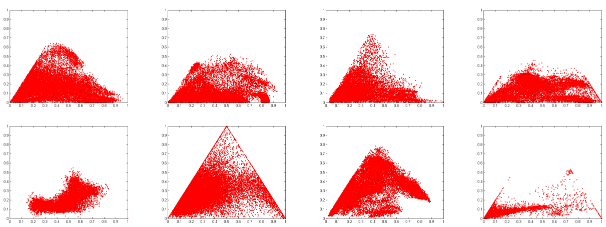

Figure 7: Pixelwise distribution of saturation as a function of luminance, in $\rho\mu$ space, for the images Mercado1, Venezia1, Mercado2 and Gabriela (above), Grotta, Ababoles, Venezia2 and Window (below). A typical code is $\lambda\sigma\Lambda\Sigma$.

3.3 Circular histograms of hue

After grouping the pixel hues into 8 groups labeled *ry* (reddish oranges), *yr* (yellowish oranges), *yg* (yellowish cetrines), *gy*(greenish cetrines), *gb* (greenish cyans), *bg* (bluish cyans), *br* (bluish purples) and *rb* (reddish purples), we get an 8-bin circular hue histogram for each image, as shown in Figure 8. In addition, we call pixels with chromaticities *yr* and *ry*, *warm* while those in with chromaticities *gb* and *bg*, *cool*; also, we call pixels with chromaticities *yg* and *gy* *sour* and those with chromaticities *br* and *rb* *sweet*; see Table 3. To measure the degree of uniformity of the hue histograms, which in turn tells us how variegated the images are, we normalize by the number of pixels and compute the 0-1 entropy given by the formula $-\frac{1}{2.0794}\sum p_i ln(p_i)$; see Table 3. An entropy below 0.5 is evidence of a monochromatic (prevalence of one of the groups over the others) image.

Table 3: Hue Characterization

Image	Degree of balance	Dominant chromaticities
mercado 1	0.8633	balanced
venezia1	0.7113	cool and warm
mercado2	0.8451	warm and sour
gabriela	0.6987	warm and sweet
grotta	0.3564	warm and sweet
ababoles	0.3869	warm and sour
venezia2	0.3257	warm
window	0.4611	warm

4. CONCLUSION

We have introduced a *double-cone* colour space $\rho\mu$ that allows for a more accurate chromatic characterization of images by leaving the saturation unnormalized, and a *spherical* Runge space of hue, colourfulness and lightness $\eta\kappa\lambda$, that is perceptually homogeneous and is suitable for image colour modification. We have presented set of characterizations of

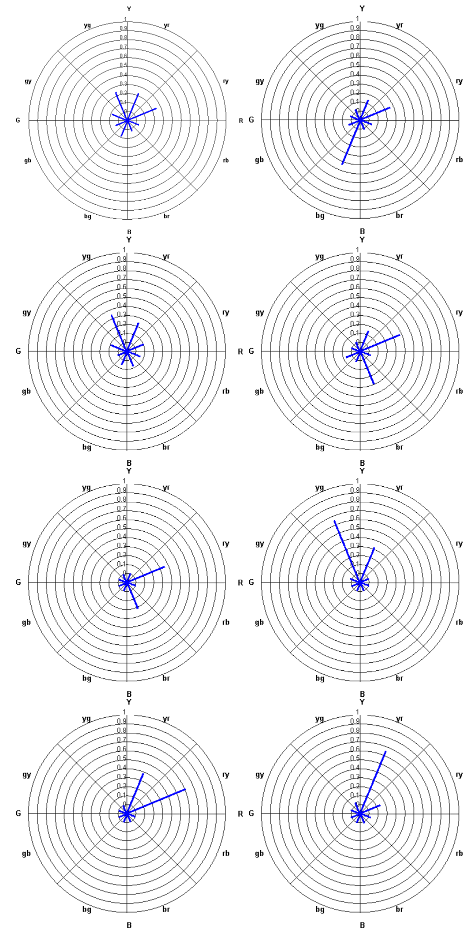

Figure 8: Hue histograms corresponding to images in Figure 4

colour images in terms of the distributions of dispersion versus location and also measures of colour variegation such as the entropy of the circular hue histogram. The corrector of an image should take into consideration both the image and the characterizations in order to apply appropriate correction tools; e.g. neither a low saturation nor a low hue entropy are necessarily indicators of low image quality. Thus, automatic correction requires homogeneous databases of images.

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