KALMAN FILTER DESIGN FOR APPLICATION TO AN INS ANALYSING SWIMMER PERFORMANCE

Tanya Le Sage, Axel Bindel, Paul Conway, Laura Justham, Sian Slawson and Andrew West

Sports Technology Institute, Loughborough University
Loughborough Park, Loughborough, Leicestershire, LE11 3TU
phone: + (44) 1509 564812, email: T.Le-Sage@lboro.ac.uk

ABSTRACT

Performance analysis is essential for the ongoing development of elite swimmers. Current methods of analysis do not provide the coach with an accurate system for determining a swimmer's position. This research was conducted to allow transmission, processing and presentation of the swimmer's attitude, velocity and position. A Kalman filter was integrated into a wireless system to track these parameters.

1. INTRODUCTION

Research has been conducted previously to enable analysis of the swimming stroke. Maglischo produced typical velocity profiles of a swimmer's hand for each individual stroke [1]. Seifert used such profiles to identify that an abrupt change in the coordination pattern of front crawl occurred at the critical velocity of 1.8 m/s [2]. Many studies have focused on post processing the data rather than monitoring performance in real-time and as such do not allow accurate determination of the position of the swimmer [include references].

At present the majority of methods used to analyse swimming technique are vision-based systems. Quintic [3] is an example of vision-based software where the analyst uses a pre-recorded video file and then manually digitizes key occurrences within the recording. The disadvantages of this and other such systems are the parallax errors induced by the use of video cameras, inaccurate measurements due to light reflection on the water surface and the large amount of time (and hence cost) it takes to process the data. Manual digitization is a time consuming process and does not allow real-time feedback to the coaches or swimmers. Wireless sensor devices have also been developed for use in a swimming environment. An example of this was presented by Davey [4], where a system was developed using a tri-axis accelerometer to monitor stroke technique. Ohgi used a similar system to measure wrist acceleration of swimmers [5]. Although both these systems used sensor devices for monitoring the swimmer, neither used a wireless sensor network (WSN) nor embedded processing to analyse the stroke technique of multiple swimmers in real-time. Both systems used a data logging accelerometer to capture the data, which meant that information could not be viewed in real time. These systems rely on post processing that increases the analysis time significantly and subsequently coaches are unable to offer immediate feedback to the swimmers. In addition, neither system offers a measurement of the swimmer's velocity or position in relation to the length of the pool.

The research presented within this paper has been carried out at Loughborough University, UK, and focuses on the development of a Kalman filter for use on an inertial navigation system (INS), which contains a tri-axis accelerometer and a tri-axis gyroscope, in order to characterise swimmer performance. The system under development has been produced to provide real-time data feedback to coaches on poolside and allows coaches to extract useful data with regards to each individual swimmer's performance. The filter provides the coach with the velocity, attitude and position of the swimmer with respect to the length of the swimming pool. A WSN has previously been developed that allows real-time data transmission to swimming coaches and subsequently their swimmers in a training environment. It was developed to operate as a network of nodes to allow analysis of multiple swimmers performance during a training session.

2. INERTIAL NAVIGATION SYSTEMS

Inertial navigation uses sensors to sense rotational and translational motion with respect to an inertial frame [6]. An INS usually contains three axes of acceleration and three axes of angular rotation. The purpose of an INS is to determine the angular motion of an object using gyroscopic sensors, from which its attitude relative to a reference frame may be derived and to measure specific force using accelerometers. The specific force measurements are resolved into the reference frame using the knowledge of attitude derived from the information provided by the gyroscopes. The force resulting from the gravitational field is evaluated and the specific force measurements are integrated to obtain estimates of the velocity and position of the object being tracked. The inertial measurements are recorded in the body frame (b-frame). The b-frame is the orthogonal axis set, which is aligned with the roll, pitch and yaw axes of the object in which the navigation system is installed, i.e. the orientation of the swimmer (see Figure 1). The b-frame is converted into the inertial frame (i-frame) in this application. The i-frame has its origin at the centre of the Earth and has axes that are non-rotating with respect to the fixed stars, defined by the axes Ox_i, Oy_i, Oz_i. The axis Oz_i is coincident with the Earth's polar axis (which is assumed to be invariant in direction, see Figure 2) [6].

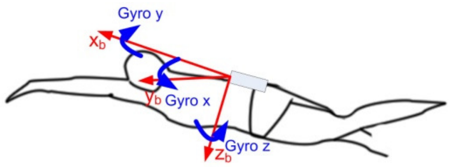


Figure 1: Body reference frame for the wireless sensor

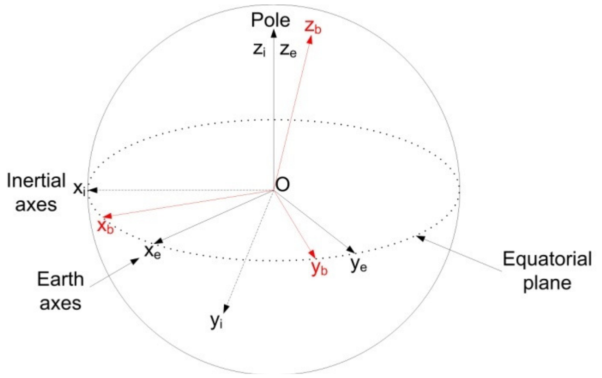


Figure 2: Frames of reference

In the application presented within this paper the INS was applied to a *strapdown* system. In strapdown systems the inertial sensors are mounted rigidly onto the device, which in this application is the swimmer's body. Accelerometers measure the sum of the acceleration with respect to inertial space and the acceleration due to gravitational acceleration. Hence, the measurement provided by the accelerometers must be combined with knowledge of the gravitational field to determine the acceleration of the object with respect to inertial space [6]. The main errors inherent within a micro-electrical-mechanical-system (MEMS) accelerometer are fixed bias, scale-factor and cross-coupling errors [7]. Fixed bias refers to the offset of the accelerometer signal from the true value when the applied acceleration is zero. It is possible to estimate the fixed bias by measuring the long term average of the accelerometer's output when it is not undergoing any acceleration. It is necessary to know the precise orientation of the device with respect to the gravitational field in order to measure the bias. In practice this can be achieved by calibration routines in which the device is mounted on a turntable, whose orientation can be controlled extremely accurately [7]. The scale-factor errors are inaccuracies in the ratio of a change in the output signal to a change in the input acceleration that is to be measured, and the cross-coupling errors are erroneous accelerometer outputs resulting from accelerometer sensitivity to accelerations applied normal to the input axis. They are commonly expressed as a ratio of output error to input rate, in parts per million (ppm) [6].

Rate gyroscopes are used to sense the rate of turn by a vehicle or structure about some predefined axis (see figure 1). MEMS gyroscopes experience fixed bias, acceleration-dependent bias and scale-factor errors. The fixed bias is the average output from the gyroscope when it is not undergoing any rotation, i.e. the offset of the output from the true value. Acceleration-dependent bias is proportional to the magnitude of applied acceleration.

Edwards [8] demonstrated that seemingly small aliased content could cause appreciable errors in the integrated waveforms. He stated that all experimentally collected waveforms of finite duration, and that have been discretised by an analog to digital convertor, will contain aliased content to some degree. Thong [9] supported this work and suggested that the errors in integration depended not only on the noise level but also on the system sampling frequency and effects such as drift due to temperature fluctuations. A common method to minimise the errors associated with the accelerometer and gyroscope signal is the use of filtering (see for example, Koukoulas, [10], Hernandez, [11] and Jo, [12]). Filtering reduces the errors associated with integration of a signal, in this case integration of the acceleration in order to obtain velocity and double integration to obtain position.

A recursive least squares adaptive noise cancelling method was applied by Hernandez [8] to estimate electrical signals coming from an accelerometer embedded in a bus. The results allowed estimation of acceleration and velocity by using a computer controller system, without the necessity of buying expensive precision electronic instruments for direct hardware implementation. The recursive least squares algorithm also has the advantage of computational simplicity. However the least squares algorithm assumes that the criteria is one of fitting data, and not minimizing the estimation error [10]. The Kalman filter brings into consideration the problems associated with the least squares method. It minimises the estimation error and uses information regarding a priori knowledge of parameters.

The instantaneous "state" of a linear dynamic system perturbed by white noise is estimated using a Kalman filter using a state space representation [13]. White noise is a stationary random process having a constant spectral density function [14]. An advantage of the Kalman filter is that it is recursive and hence estimates are updated upon receipt of each measurement. This means that there is no need to save past data. It can also be easily configured to handle extraneous data points and model changes. The Kalman filter can be used for predictive design of sensors systems and to optimize the use of sensor outputs [15]. An overview of the Kalman filtering process can be seen in Figure 3.

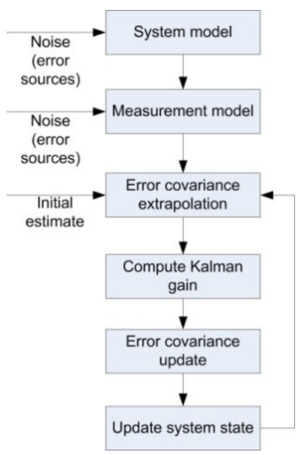


Figure 3: Overview of Kalman filtering process

For the sake of the reader's convenience, the discrete linear Kalman filter equations are summarised below:

System model:
$\boldsymbol{x}_k = \boldsymbol{\Phi}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1},\ w_k \sim N(0,\boldsymbol{Q}_k)$
Measurement model:
$\boldsymbol{z}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k,\ v_k \sim N(0,R_k)$
State estimate extrapolation:
$\hat{\boldsymbol{x}}_k(-) = \boldsymbol{\Phi}_{k-1}\hat{\boldsymbol{x}}_{k-1}(+)$
Error covariance extrapolation:
$\boldsymbol{P}_k(-) = \boldsymbol{\Phi}_{k-1}\boldsymbol{P}_{k-1}(+)\boldsymbol{\Phi}_{k-1}^T + \boldsymbol{Q}_{k-1}$
State estimate update:
$\hat{\boldsymbol{x}}_k(+) = \hat{\boldsymbol{x}}_k(-) + \overline{\boldsymbol{K}}_k[z_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_k(-)]$
Error covariance update:
$\boldsymbol{P}_k(+) = [\boldsymbol{I} - \overline{\boldsymbol{K}}_k\boldsymbol{H}_k]\boldsymbol{P}_k(-)$
Kalman gain matrix:
$\overline{\boldsymbol{K}}_k = \boldsymbol{P}_k(-)\boldsymbol{H}_x^T[\boldsymbol{H}_k\boldsymbol{P}_k(-)\boldsymbol{H}_k^T + \boldsymbol{R}_k]^{-1}$

Where:
$\boldsymbol{x}_k$ = (n x 1) process state vector at time t_k
$\boldsymbol{\Phi}_k$ = (n x n) state transition matrix relating $\boldsymbol{x}_k$ to $\boldsymbol{x}_{k+1}$
$\boldsymbol{w}_k$ = (n x 1) vector – assumed to be a white sequence with known covariance structure
$\boldsymbol{z}_k$ = (m x 1) vector measurement at time t_k
$\boldsymbol{H}_k$ = (m x n) matrix giving the ideal (noiseless) connection between the measurement and the state vector at time t_k
$\boldsymbol{v}_k$ = (m x 1) measurement error – assumed to be a white sequence with known covariance structure and having zero crosscorrelation with the $\boldsymbol{w}_k$ sequence
$(-)$ = a priori estimate
$(+)$ = a posterior estimate

Kim [16] and Vaganay [17] both used an extended Kalman filter to combine measurements from accelerometers and gyroscopes in order to produce an estimate of position. Kim developed a real-time orientation estimation algorithm based on measurements from three MEMS accelerometers and three MEMS rate gyroscopes. The approach was based on relationships between the quaternion representing the platform orientation, the measurement of gravity from the accelerometers, and the angular rate measurement from the gyroscopes. The performance of the Kalman filter was evaluated in terms of the roll, pitch and yaw angles. An optical position tracking device measured the position of three LEDs attached to the IMU, providing verification for the orientation tracker. Vaganay [17] developed an attitude estimation system based on inertial measurements for a mobile robot. He used two accelerometers and three gyroscopes and implemented an extended Kalman filter to combine the measurements from both.

3. KALMAN FILTER APPLICATION

3.1 Tracking orientation

The overview of the navigation equations that are to be used to track the orientation and position of the swimmer can be seen in Figure 4.

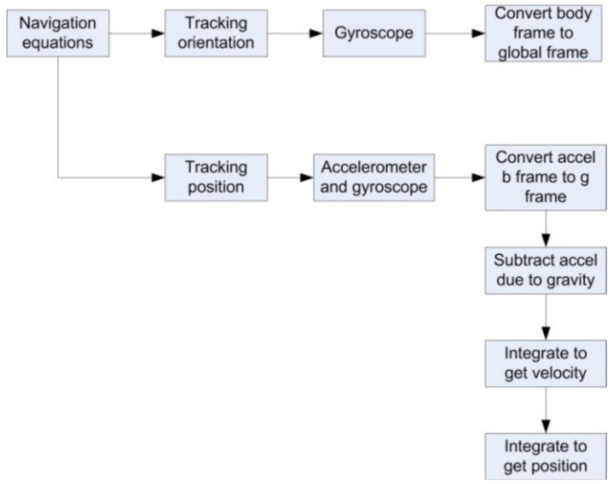


Figure 4: Overview of navigation equations used

The orientation, or attitude, of an INS relative to the global frame of reference is tracked by 'integrating' the angular velocity signal $\omega_b(t) = (\omega_{bx}(t), \omega_{by}(t), \omega_{bz}(t))^T$ obtained from the system's rate-gyroscopes. In order to specify the orientation of an INS a direction cosine attitude representation is used. In the direction cosine representation the attitude of the body frame relative to the global frame is specified by a 3 x 3 rotation matrix **C**, in which each column in a unit vector along one of the body axes specified in terms of the global axes [6].

Initially the difference between the previous and present raw gyroscope output in the x, y and z axes are determined:

$$\delta\omega_{bx}(t) = \omega_{bx}(t) - \omega_{bx}(t\text{-}1)$$
$$\delta\omega_{by}(t) = \omega_{by}(t) - \omega_{by}(t\text{-}1)$$
$$\delta\omega_{bz}(t) = \omega_{bz}(t) - \omega_{bz}(t\text{-}1)$$

where $\boldsymbol{\psi} = \omega_{bx}$, $\boldsymbol{\Phi} = \omega_{by}$ and $\theta = \omega_{bz}$ and w_{bx}, w_{by}, and w_{bz} are the raw gyroscope outputs in the x, y and z axes at time t respectively. To track the attitude of the INS C is tracked through time. The attitude at time t is given by $C(t)$ and the rate of change of C at t by

$$\dot{\boldsymbol{C}} = \lim_{\delta \to 0}(\boldsymbol{C}(t + \delta t) - \boldsymbol{C}(t))/\delta t$$

$C(t+\delta t)$ can be written as the product of two matrices:

$$\boldsymbol{A}(t) = \boldsymbol{I} + \boldsymbol{\delta\psi}$$

where $\boldsymbol{A}(t)$ is a direction cosine matrix which relates the b-frame at time t to the b-frame at time $t+\delta t$ [6] and:

$$\boldsymbol{\delta\psi} = \begin{bmatrix} 0 & -\delta\psi & \delta\theta \\ \delta\psi & 0 & -\delta\phi \\ -\delta\theta & \delta\phi & 0 \end{bmatrix}$$

The angular velocity signals obtained from the gyroscopes were integrated by the INS attitude algorithm, propagating errors in the gyroscope signal through to the calculated orientation. White noise and uncorrelated bias errors are the main causes of an error in the orientation. White noise causes an angle random walk whose standard deviation grows proportionally to the square root of time. An uncorrelated bias causes an error in orientation which grows linearly with time [6]. These errors are accounted for using the Kalman filter.

3.2 Tracking position

To track the position of the INS the acceleration signal $a_b(t) = (a_{bx}(t), a_{by}(t), a_{bz}(t))^T$ obtained from the accelerometers was projected into the global frame of reference:

$$\boldsymbol{a}_g(t)=\boldsymbol{C}(t)a_b(t)$$

Acceleration due to gravity was subtracted and the remaining acceleration integrated once to obtain velocity, and again to obtain displacement:

$$vg(t) = vg(0) + \int_0^t ag(t) - g dt$$

$$sg(t) = sg(0) + \int_0^t vg(t) dt$$

where vg is the velocity of the swimmer, sg is the displacement of the swimmer and g is the acceleration due to gravity in the global frame. In this system $1g$ must be subtracted from the (globally) vertical acceleration signal to remove acceleration due to gravity before the signal is integrated. A tilt error ε causes a component of the acceleration due to gravity with magnitude $g.sin(\varepsilon)$ to be projected onto the horizontal axes.

3.3 Implementation of a Kalman filter

The overview of the system which is to be used to calculate the required variables (velocity, position, pitch and roll of the swimmer) can be seen in Figure 5.

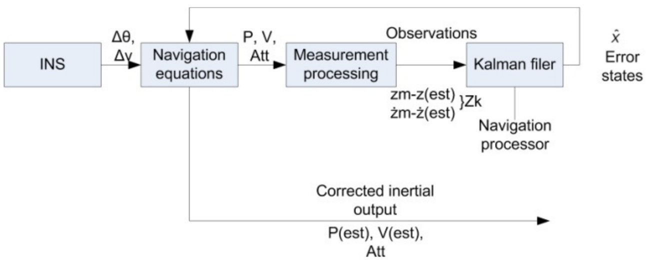


Figure 5: Overview of system for navigation of a swimmer

The swimmer application required 14 states within the Kalman filter: x_1 = east position, x_2 = east velocity, x_3 = north position, x_4 = north velocity, x_5 = pitch, x_6 = roll, x_7 = gyro bias east, x_8 = gyro bias north, x_9 = gyro scale factor east, x_{10} = gyro scale factor north, x_{11} = accelerometer bias east, $x12$ = accelerometer bias north, x_{13} = accelerometer scale factor east, x_{14} = accelerometer scale factor north. These states have been used to create the state transition matrix (**Φ**) for the filter. Secondly an initial estimate of the process noise covariance has been calculated, where **Q**(k) is the covariance of the white noise w(k). The sensor noise covariance, **R**(k) was also calculated. This covariance has been determined using the power spectral density (PSD) values obtained from the vendors with regards to each sensor. The measurement sensitivity matrix (**H**) was determined, defining the linear relationship between the state of the dynamic system and measurements to be made. It was constructed as a 6 x 6 matrix, in order to allow determination of the parameters x_1, x_2, x_3, x_4, x_5 and x_6. An initial estimate for $\boldsymbol{P}(k)(+)$ was created. This initial estimate was large so that $\hat{x}(k)(-)$ could be approximated to zero.

4. USE CASE FOR A SWIMMER ANALYSIS TOOL

An overview of the Kalman filter implementation for the swimmer application can be seen in Figure 6. Initially a trigger was implemented onto the WSN which synchronized the high speed video and WSN and initiated the Kalman filter when the buzzer was pressed to instigate the dive. The function was implemented in the embedded programming on the wireless node which sent an interrupt via a TTL signal to the access point (AP) when the trigger was enabled. The embedded code initialised the trigger, starting the recording on the rising edge of the signal. The INS and Kalman filter were used to track the position and attitude of the swimmer as they travelled along one length of the pool. The turn of the swimmer was identified as the largest peak on real-time filtered data (see Figure 7), using an equation to determine the maxima and minima points. The swimmers turn was used to identify a distance of 50m had been reached, in the x-axis, and to update the Kalman filter parameters for the next length.

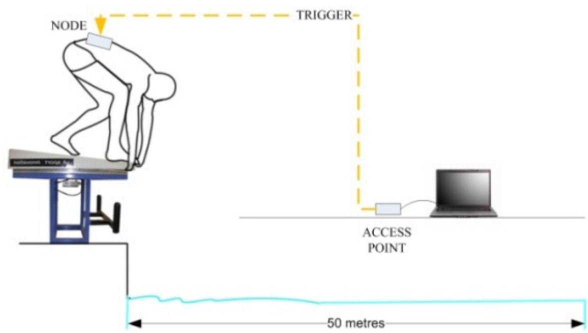


Figure 6: Implementation of the Kalman filter

A low pass Butterworth filter embedded onto the node was used to ascertain the time at which the swimmer's feet touched the wall. Setting a filter frequency of 0.6Hz was sufficient to achieve this. Real-time embedded filtered data on 100m of front crawl stroke with 3 turns can be seen in Figure 7. The largest peaks in the data have been identified as the swimmer's turn at the wall.

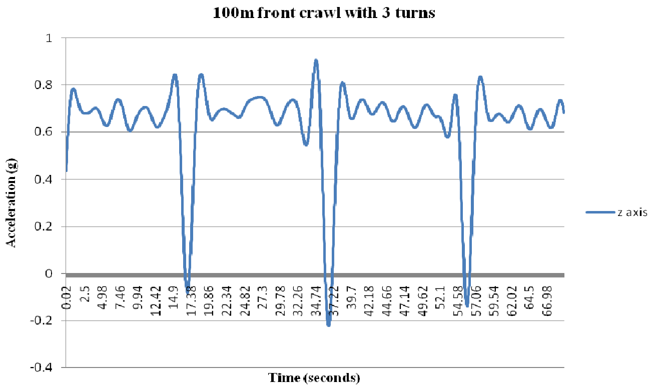
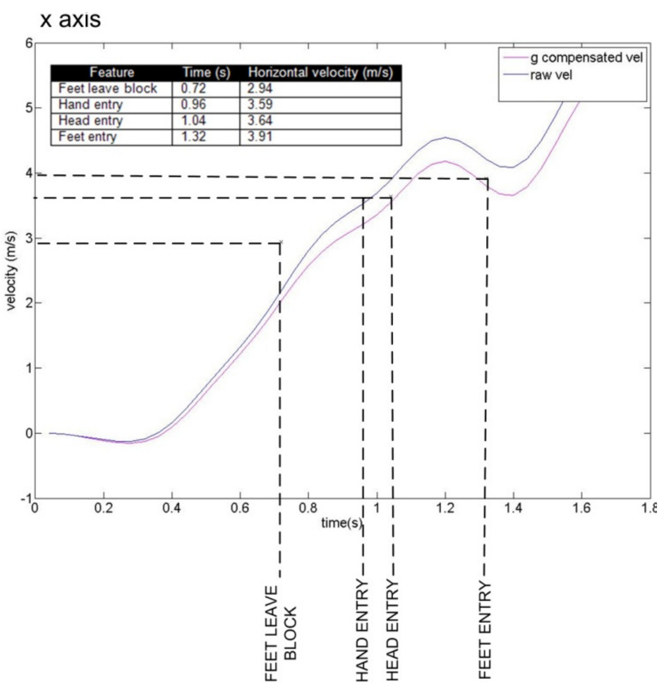


Figure 7: Butterworth filter on 4 lengths of front crawl data

Using the Butterworth filter to determine the exact time when the swimmer finished each length it was possible to update the Kalman filter. The time at which the swimmer's feet hit the wall was identified and correlated with a position of 50m in the x axis in the inertial frame.

At present video data has only been collected to supplement sensor information gathered during a swimmer's dive. The raw and corrected velocities obtained from the sensor readings were compared with video data. The corrected velocity refers to the data in the inertial frame, with the implementation of a Kalman filter. From Figure 8 it can be seen that as the length of time increased the corrected velocities tended closer to the velocities obtained using the high speed video camera.



5. CONCLUSION

The system developed within this paper provides a methodology that provides coaches with information in regard to a swimmer's position with respect to the length of the pool. It provides a means for determining the attitude and velocity of the swimmer at each time interval. Ongoing and future work involves validation tests to determine the accuracy of the algorithms.

REFERENCES

[1] E. Maglischo. *Swimming even faster.* Mountain View, CA. Mayfield Publishing Company. 1993.
[2] L. Seifert. "Effect of swimming velocity on arm coordination in the front crawl: a dynamic analysis," *Journal of Sports Sciences,* vol.22, no. 7. 2004.
[3] Quintic Consultancy Ltd. *Putting sports science into practice.* Available: http://www.quintic.com. Accessed: 03.10.2010
[4] N. Davey. "An accelerometer-based system for elite swimming performance analysis," *Proceedings of SPIE the International Society for Optical Engineering,* vol. 5649, no. 1. 2005.
[5] Y. Ohgi. "Microcomputer-based data logging device for accelerometry in swimming," *Engineering of Sport,* vol. 4. 2002.
[6] D. H. Titterton, J. L. Weston. "Strapdown inertial navigation technology," *Progress in Astronautics and Aeronautics.* MIT Lincoln Laboratory, Massachusetts. 2004.
[7] O. J. Woodman, *An introduction to inertial navigation.* University of Cambridge Computer Laboratory. 2007.
[8] T. Edwards. "Effects of aliasing on numerical integration," *Mechanical systems and signal processing,* vol. 21. Elsevier. 2005.
[9] Y. K. Thong, et al. "Dependence of inertial measurements of distance on accelerometer noise," *Measurement science and technology,* vol. 13. IOP Publishing Ltd, UK. 2002
[10] T. Koukoulas. "Binary low, high and band pass amplitude filters with full and quantized phase in the presence of disjoint noise," *Lasers in engineering,* vol. 15. Old City Publishing Inc. 2005
[11] W. Hernandez. "Improving the response of an accelerometer by using optimal filtering," *Sensors and actuators A,* vol. 88. Elsevier. 2000
[12] G. Jo. "Underwater navigation system with velocity measurement by a receding horizon Kalman filter," *Seiken symposium conference 38*, vol. 3. 2004.
[13] R. G. Brown, P. Y. Hwang. *Introduction to random signals and applied Kalman filtering, second edition.* John Wiley & Sons, Inc. USA. 1992
[14] M. S. Grewal, A. P. Andrews. *Kalman filtering. Theory and Practice using MATLAB, Third Edition.* John Wiley & Soncs, Inc. New jersey. 2008.
[15] M. S. Grewal. "Application of Kalman filtering to GPS, INS and navigation," *Short course notes.* Kalman Filtering Consultant Associates. 2010.
[16] A. Kim, M. F. Golnaraghi. "A quaternion-based orientation estimation using an inertial measurement unit," *IEEE Xplore.* 2004.
[17] J. Vaganay, et al. "Mobile robot attitude estimation by fusion of inertial data," *IEEE Xplore.* 1993.