Sound Zoning in an Ad-hoc Wireless Acoustic Sensor and Actuator Network

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***Abstract*—In this paper, the pressure matching (PM) method to sound zoning is considered in an ad-hoc wireless acoustic sensor and actuator network (WASAN) consisting of multiple audio-devices with loudspeakers and microphones. The goal of sound zoning is to simultaneously create different zones with different dominant sounds. To obtain this, a network-wide objective involving the acoustic coupling between all the loudspeakers and microphones is presented where the optimal solution is obtained by solving a quadratically constraint quadratic Program (QCQP). To allow for distributed processing, a Gauss-Seidel type algorithm is proposed. It requires only that all the nodes have access to the different microphone signals, but other than this there is no need for communication between different nodes or with a fusion center (FC). The algorithm is referred to as the distributed adaptive PM algorithm (DA-PM). The algorithm is proven to converge to the optimal solution, as also illustrated by Monte Carlo simulations and evaluated in a simulated acoustic environment.**

***Index Terms*—Pressure Matching, Sound Zoning, Wireless Sensor and Actuator Network (WASAN), Quadratically Constraint Quadratic Program (QCQP)**

I. INTRODUCTION

An ad-hoc wireless acoustic sensor and actuator network (WASAN) consists of multiple audio-devices, equipped with microphones and loudspeakers, where wireless links make it possible for the devices to communicate and cooperate to perform a certain audio processing task. Ad-hoc mean that the relative positions of the different audio-devices, also referred to as nodes, are undefined, unknown and can change during the operation. When the nodes are able to manipulate their loudspeaker signals, new applications like active noise cancellation [1], [2] and sound zone control [3]–[5] emerge. This paper focuses on sound zoning in an ad-hoc WASAN.

The goal of sound zoning is to simultaneously create different zones with different dominant sounds. This is controlled by measuring the pressure in these zones by means of so-called error microphones. In a WASAN, each zone can decide independently which sound signals belong to its required sound signals and which sound signals are interfering sound signals that should be suppressed. To create the desired sound zones, FIR filters are designed to pre-filter all the loudspeaker signals in the WASAN. Design criteria for these filters involve the signal distortion for the required sound signals and the resulting interfering sound power. In the acoustic contrast control (ACC) method [6], the ratio between the power of the residual interfering sound signals and the power of the required sound signals is minimized. In the pressure matching (PM) method [7], the signal distortion of the required sound signals is explicitly taken into account. Hybrid methods [4], [8] combining the ACC and PM criteria are also available.

This paper focuses on the PM method where the FIR filters are designed by minimizing an objective involving the acoustic coupling between all the loudspeakers and error microphones in the WASAN, without exceeding the maximum power of each loudspeaker. The optimal solution can be found by solving a quadratically constraint quadratic program (QCQP). Solving the QCQP requires gathering all acoustic information in a fusion center (FC), which however might become intractable as the number of nodes grows large. To allow for distributed processing, a Gauss-Seidel type algorithm is proposed. It requires only that all the nodes have access to the different microphone signals, but other than this there is no need for communication between different nodes or with a FC. The algorithm is referred to as the distributed adaptive PM algorithm (DA-PM). The algorithm is proven to converge to the optimal solution, as also illustrated by Monte Carlo simulations and evaluated in a simulated acoustic environment.

The paper is organized as follows. The problem formulation and the centralized PM method are presented in Section II and III respectively. In Section IV, the distributed algorithm is presented. Computer simulations to illustrate the convergence of the presented algorithm, are provided in Section V. Conclusions are given in Section VI.

II. SIGNAL MODEL AND PROBLEM FORMULATION

A WASAN is considered where P localized error microphones are recording the sound pressure in P different sound

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zones. K nodes, equipped with L loudspeakers each, are also scattered over the considered region in an ad-hoc fashion. The nodes are assumed to have access to the different error microphone signals (e.g., the error microphones can broadcast their signals using wireless links), but other communications between the nodes will not be needed. Each node has also access to S different sound signals $x_s(t)$, with t the time index. Each sound signal belongs either to the required sound signals $\mathcal{R}_p$ or the interfering sound signals $\mathcal{I}_p$ of sound zone p.

The goal of the PM method is to produce a desired sound pressure $d_p(t)$ in sound zone p, defined as the sound pressure produces by a virtual source playing all required sound signals $x_s(t)$ in $\mathcal{R}_p$, i.e.

$$d^p(t) = \sum_{n=0}^{N-1} h^p_{\mathrm{v}}(n) \sum_{s\in\mathcal{R}_p} x_s(t-n) = \sum_{s\in\mathcal{R}_p} \underbrace{\sum_{n=0}^{N-1} h^p_{\mathrm{v}}(n) x_s(t-n)}_{d^p_s(t)} . \tag{1}$$

Here $\mathbf{h}^p_{\mathrm{v}} = [h^p_{\mathrm{v}}(0) \ ... \ h^p_{\mathrm{v}}(N-1)]^T$ models the impulse response between the virtual source and error microphone p as an N-th order FIR filter and $(.)^T$ is the transpose operator.

To produce the desired pressures, loudspeaker l of node k in the WASAN will play a signal y_{kl} obtained as a convolution of the known S sound signals and S unknown M-th order PM filters $\{\mathbf{w}_{kl,s}\}_{s=1...S}$, i.e.

$$y_{kl}(t) = \sum_{s=1}^{S}\sum_{m=0}^{M-1} w_{kl,s}(m) x_s(t-m) = \sum_{s=1}^{S} \mathbf{w}^T_{kl,s}\mathbf{x}_s(t) \tag{2}$$

with $\mathbf{w}_{kl,s} = [w_{kl,s}(0) \ ... \ w_{kl,s}(M-1)]^T$ and $\mathbf{x}_s = [x_s(t) \ ... \ x_s(t-M+1)]^T$. The sound pressure $e^p(t)$ received at error microphone p is then given by the sum of all loudspeaker signals convolved with the impulse responses between the loudspeakers and the error microphones, i.e.

$$\begin{aligned} e^p(t) &= \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{n=0}^{N-1} h^p_{kl}(n) y_{kl}(t-n) + n^p(t) \\ &= \sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{l=1}^{L} \mathbf{w}^T_{kl,s}\mathbf{H}^p_{kl}\tilde{\mathbf{x}}_s(t) + n^p(t) \\ &= \sum_{s=1}^{S}\sum_{k=1}^{K} \mathbf{w}^T_{k,s}\mathbf{H}^p_{k}\tilde{\mathbf{x}}_s(t) + n^p(t) \\ &= \sum_{s=1}^{S} \underbrace{\mathbf{w}^T_{s}\mathbf{H}^p\tilde{\mathbf{x}}_s(t)}_{e^p_s(t)} + n^p(t). \end{aligned} \tag{3}$$

where $\mathbf{h}^p_{kl}$ is the N-th order impulse response between loudspeaker l of node k and error microphone p and where $n^p(t)$ denotes additive noise[1] uncorrelated with the sounds and $\mathbf{H}^p_{kl}$ is a Toeplitz matrix defined as

$$\mathbf{H}^p_{kl} = \begin{bmatrix} \boxed{\mathbf{h}^{p,T}_{kl}} & 0 & \dots & 0 \\ 0 & \boxed{\mathbf{h}^{p,T}_{kl}} & \dots & 0 \\ \vdots & \vdots & \ddots \quad \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \boxed{\mathbf{h}^{p,T}_{kl}} \end{bmatrix} \in \mathbb{R}^{M\times(M+N-1)}. \tag{4}$$

The other quantities used in (3) are defined as

- $\mathbf{H}^p_k = [\mathbf{H}^{p,T}_{k1} \ ... \ \mathbf{H}^{p,T}_{kL}]^T$, $\mathbf{H}^p = [\mathbf{H}^{p,T}_1 \ ... \ \mathbf{H}^{p,T}_K]^T$
- $\mathbf{w}_{k,s} = [\mathbf{w}^T_{k1,s} \ ... \ \mathbf{w}^T_{kL,s}]^T$, $\mathbf{w}_s = [\mathbf{w}^T_{1,s} \ ... \ \mathbf{w}^T_{K,s}]^T$
- $\tilde{\mathbf{x}}_s(t) = [x_s(t) \ ... \ x_s(t-M-N+2)]^T$.

In equation (2) and (3), the PM filters are considered to be time-invariant [9]. Note that this assumption is approximately satisfied if the coefficients of the PM filters change slowly compared to the time scale of the system to be controlled, i.e. the impulse responses $\mathbf{h}^p_{kl}$.

It is also assumed that the sound signals are uncorrelated. Consequently $d^p_s(t)$ can only be obtained from $e^p_s(t)$. The optimal PM filters $\hat{\mathbf{W}} = [\hat{\mathbf{w}}_1 \ ... \ \hat{\mathbf{w}}_S]$ can be found from the following optimization problem:

$$\begin{aligned} &\min_{\mathbf{W}} \sum_{p=1}^{P}\left(\sum_{s\in\mathcal{R}_p} E\{||d^p_s(t)-e^p_s(t)||^2\} + \mu\sum_{s\in\mathcal{I}_p} E\{||e^p_s(t)||^2\}\right) \\ &s.t. \ \sum_{s=1}^{S} \mathbf{w}^T_{kl,s} \underbrace{E\{\mathbf{x}_s(t)\mathbf{x}_s(t)^T\}}_{\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}} \mathbf{w}_{kl,s} \leq P_{kl} \quad \forall k,l. \end{aligned} \tag{5}$$

Here $E\{.\}$ denotes the expected value operator and is implemented by (recursive) time-averaging over a time window. The power of each loudspeaker signal $y_{kl}(t)$ is also constrained by a maximal power P_{kl}, i.e. $E\{y^2_{kl}(t)\} \leq P_{kl}$. The parameter μ is included as a trade-off between the reproduction error for the required sound signals and the residual interfering sound power [4].

III. Centralized algorithm

The objective function of (5) , denoted by $J(\mathbf{W})$, can, after some matrix manipulations, be written as

$$J(\mathbf{W}) = \sum_{s=1}^{S}\left(\mathbf{w}^T_s\mathbf{R}_s\mathbf{w}_s - 2\mathbf{w}^T_s\mathbf{r}_s + E\{|d^p_s(t)|^2\}\right) \tag{6}$$

where

$$\mathbf{R}_s = \sum_{p\in\mathcal{B}_s} \mathbf{H}^p\mathbf{R}_{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s}\mathbf{H}^{p,T} + \mu\sum_{p\in\mathcal{D}_s} \mathbf{H}^p\mathbf{R}_{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s}\mathbf{H}^{p,T} \tag{7}$$

$$\mathbf{r}_s = \sum_{p\in\mathcal{B}_s} \mathbf{H}^p E\{\tilde{\mathbf{x}}_s(t) d^p_s(t)\} \tag{8}$$

$$\mathbf{R}_{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s} = E\{\tilde{\mathbf{x}}_s(t)\tilde{\mathbf{x}}_s(t)^T\} \tag{9}$$

with $\mathcal{B}_s$ and $\mathcal{D}_s$ denoting the collection of error microphones with $s \in \mathcal{R}_p$ and $s \in \mathcal{I}_p$ respectively. These sets are often referred to as the bright and dark zone for sound s.

[1] The noise is the result of background noise, quantization and thermal noise of the error microphone and can also consist of training sequences transmitted by loudspeakers to estimate the impulse response.

Note that (5) is a convex QCQP and has a unique optimal solution $\hat{\mathbf{W}}$ since $\mathbf{R}_s$ and $\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}$ are positive definite[2] for all sources. This solution can be found using well know methods, e.g. interior-point methods [10]. The optimal solution will fulfill the KKT conditions

$$\begin{aligned}
\forall s: &\left(\mathbf{R}_s + \sum_{k=1}^{K}\sum_{l=1}^{L} \hat{\lambda}_{kl}\mathbf{E}_{kl}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{kl}^T\right)\hat{\mathbf{w}}_s = \mathbf{r}_s \\
\forall k,l: &\sum_{s=1}^{S} \hat{\mathbf{w}}_{kl,s}^T\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\hat{\mathbf{w}}_{kl,s} \leq P_{kl} \qquad (10)\\
\forall k,l: &\hat{\lambda}_{kl}\left(\sum_{s=1}^{S} \hat{\mathbf{w}}_{kl,s}^T\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\hat{\mathbf{w}}_{kl,s} - P_{kl}\right) = 0.
\end{aligned}$$

The matrix $\mathbf{E}_{kl} = [\mathbf{0}\ \mathbf{I}_M\ \mathbf{0}]^T \in \mathbb{R}^{MKL\times M}$ is an all zero matrix where the identity matrix is placed on the positions corresponding to indexes k and l.

However, to compute these optimal PM filters $\hat{\mathbf{W}}$, all impulse responses $\mathbf{h}_{kl}^p$ have to be available at a central FC. This FC then has to solve the QCQP (5), which is an optimization problem with $KMLS$ variables. This introduces a single point of failure, and furthermore, requires a high computational capacity in the FC. To alleviate these problems, an efficient distributed algorithm is presented in the next section, which is then shown to converge to the same optimal PM filters.

IV. Distributed Algorithm

Before presenting the distributed algorithm, the following Gauss-Seidel procedure for solving the QCQP is discussed. Here i denotes the iteration index and q is the updating node:

1) Initialize $\mathbf{W}^0$ randomly without violating any constraint in (5), set $i \leftarrow 0$ and $q \leftarrow 1$.
2) Solve the reduced dimensional QCQP to obtain $\mathbf{w}_{k,s}^{i+1}$ for all the nodes:

$$\begin{aligned}
\min_{\mathbf{W}}\ &\sum_{s=1}^{S} \mathbf{w}_s^T\mathbf{R}_s\mathbf{w}_s - 2\mathbf{w}_s^T\mathbf{r}_s + E\{|d_s^p(t)|^2\} \\
s.t.\ &\sum_{s=1}^{S} \mathbf{w}_{ql,s}^T\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{w}_{ql,s} \leq P_{ql} \quad \forall l \qquad (11)\\
&\mathbf{w}_{k,s} = \mathbf{w}_{k,s}^i \quad \forall s, k\neq q.
\end{aligned}$$

3) $i \leftarrow i+1$, $q \leftarrow$mod$(q,K)+1$ and return to step 2.

The following result holds for this Guass-Seidel procedure.

Theorem IV.1. *If $\mathbf{R}_s$ has full rank $\forall s$ and the sequence $\{\mathbf{W}^i\}_{i\in\mathbb{N}}$ is generated by the Gauss-Seidel procedure defined above, then*

$$\lim_{i\to\infty}\mathbf{W}^i = \hat{\mathbf{W}}. \qquad (12)$$

Proof. The proof starts by defining $\mathbf{p}_s^i = \mathbf{w}_s^{i+1} - \mathbf{w}_s^i = \mathbf{E}_q(\mathbf{w}_{q,s}^{i+1} - \mathbf{w}_{q,s}^i)$ $\forall s$. From the first KKT condition of (11) (Lagrangian stationarity), it is clear that there exist $\{\lambda_{ql}^{i+1} \geq 0\}_{l\in\mathcal{A}_q^{i+1}}$ such that $\mathbf{W}^{i+1}$ fulfills $\forall s$

$$\mathbf{E}_q^T\left(\mathbf{R}_s\mathbf{w}_s^{i+1} + \mathbf{r}_s + \sum_{l\in\mathcal{A}_q^{i+1}}\lambda_{ql}^{i+1}\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{w}^{i+1}\right) = \mathbf{0} \qquad (13)$$

where $\mathcal{A}_q^{i+1}$ is the set of active constraints. This leads to the following identities $\forall s$:

$$\mathbf{p}_s^{i,T}\left(\mathbf{R}_s\mathbf{w}_s^{i+1} + \mathbf{r}_s\right) = -\sum_{l\in\mathcal{A}_q^{i+1}}\lambda_{ql}^{i+1}\mathbf{p}_s^{i,T}\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{w}_s^{i+1}. \qquad (14)$$

Since $\mathbf{W}^i$ is a feasible solution of (11), it can be written as

$$\begin{aligned}
P_{ql} \geq &\sum_{s=1}^{S}\mathbf{w}_s^{i,T}\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{w}_s^i \\
= &\sum_{s=1}^{S}\left(\mathbf{w}_s^{i+1,T}\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{w}_s^{i+1} + \mathbf{p}_s^{i,T}\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{p}_s^i\right.\\
&\left.-2\mathbf{p}_s^{i,T}\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{w}_s^{i+1}\right). \qquad (15)
\end{aligned}$$

Here $l\in\mathcal{A}_q^{i+1}$ implies that $\sum_{s=1}^{S}\mathbf{w}_s^{i+1,T}\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{w}_s^{i+1} = P_{ql}$, resulting in the inequalities $\forall l\in\mathcal{A}_q$:

$$2\sum_{s=1}^{S}\mathbf{p}_s^i\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{w}_s^{i+1} \geq \sum_{s=1}^{S}\mathbf{p}_s^i\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{p}_s^i. \qquad (16)$$

It can be verified using the definition of the cost function $J(\mathbf{W})$, (14) and (16) that

$$\begin{aligned}
J(\mathbf{W}^i) - J(\mathbf{W}^{i+1}) &= \sum_{s=1}^{S}\left(\mathbf{p}_s^{i,T}\mathbf{R}_s\mathbf{p}_s^i - 2\mathbf{p}_s^{i,T}(\mathbf{R}_s\mathbf{w}_s^{i+1} + \mathbf{r}_s)\right)\\
&\geq \sum_{s=1}^{S}\left(\mathbf{p}_s^{i,T}\mathbf{R}_s\mathbf{p}_s^i + \sum_{l\in\mathcal{A}_q^{i+1}}\lambda_{ql}^{i+1}\mathbf{p}_s^{i,T}\mathbf{E}_{ql}\mathbf{R}_{\mathbf{x}_s\mathbf{x}_s}\mathbf{E}_{ql}^T\mathbf{p}_s^i\right)\\
&\geq \sum_{s=1}^{S}\lambda_{\text{min}}(\mathbf{R}_s)||\mathbf{p}_s^i||^2 \geq \lambda_{\text{min}}\sum_{s=1}^{S}||\mathbf{p}_s^i||^2 \geq 0 \qquad (17)
\end{aligned}$$

where $\lambda_{\text{min}}(\mathbf{R}_s)$ denotes the smallest eigenvalue of $\mathbf{R}_s$ and $\lambda_{\text{min}} = \min_{s=1\ldots S}\lambda_{\text{min}}(\mathbf{R}_s)$. Since $\mathbf{W}^{i+1}$ minimizes the cost function under the same constraints as $\mathbf{W}^i$, it must hold that $J(\mathbf{W}^{i+1}) \leq J(\mathbf{W}^i), \forall i > 0$. Therefore, and since the cost function $J(\mathbf{W})$ is bounded below by zero, it holds that

$$\sum_{i=0}^{\infty}\left(J(\mathbf{W}^i) - J(\mathbf{W}^{i+1})\right) = J(\mathbf{W}^0) - \lim_{i\to\infty}J(\mathbf{W}^i) < \infty. \qquad (18)$$

Using $\lambda_{\text{min}} > 0$, this leads to the result

$$\sum_{i=0}^{\infty}\sum_{s=1}^{S}||\mathbf{p}_s^i||^2 \leq \infty \Rightarrow \lim_{i\to\infty}||\mathbf{W}^{i+1} - \mathbf{W}^i||^2 = 0. \qquad (19)$$

This shows that the sequence converges to a fixed point $\mathbf{W}^{\infty}$, i.e. solving (11) for every updating node will result in the same solution $\mathbf{W}^{\infty}$. It remains to show that this fixed point is indeed $\hat{\mathbf{W}}$. This can be shown by writing out the KKT condition for

[2]The optimal solution will no longer be unique when $\mathbf{R}_s$ is positive semi-definite. The distributed algorithm presented in Section IV will in this case converge to a particular minimizer of the problem, but it is not possible to define which one, therefore positive definiteness is assumed.

this fixed point $\forall q$ as is partly done in (13). Grouping these equations will result in a set of KKT conditions as in (10), for which only $\hat{\mathbf{W}}$ can be the solution since this solution is unique. $\square$

The optimization problem in (11) can, after some manipulations and removing constants in the objective function, be written as

$$\min_{\mathbf{W}_q} \sum_{s=1}^{S} \mathbf{w}_{q,s}^T \mathbf{R}_{q,s} \mathbf{w}_{q,s} - 2\mathbf{w}_{q,s}^T \left(\mathbf{r}_{q,s} - \mathbf{H}_q^p \mathbf{r}_s^i + \mathbf{R}_{q,s} \mathbf{w}_{q,s}^i\right)$$
$$s.t. \sum_{s=1}^{S} \mathbf{w}_{ql,s}^T \mathbf{R}_{\mathbf{x}_s\mathbf{x}_s} \mathbf{w}_{ql,s} \leq P_{ql} \quad \forall l \tag{20}$$

with $\mathbf{W}_q = [\mathbf{w}_{q,1} \ldots \mathbf{w}_{q,S}]$ and

$$\mathbf{R}_{q,s} = \sum_{p\in\mathcal{B}_s} \mathbf{H}_q^p \mathbf{R}_{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s} \mathbf{H}_q^{p,T} + \mu \sum_{p\in\mathcal{D}_s} \mathbf{H}_q^p \mathbf{R}_{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s} \mathbf{H}_q^{p,T} \tag{21}$$
$$\mathbf{r}_{q,s} = \sum_{p\in\mathcal{B}_s} \mathbf{H}_q^p E\{\tilde{\mathbf{x}}_s(t) d_s^p(t)\} \tag{22}$$
$$\mathbf{r}_s^i = \sum_{p\in\mathcal{B}_s} \mathbf{R}_{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s} \mathbf{H}^{p,T} \mathbf{w}_s^i + \mu \sum_{p\in\mathcal{D}_s} \mathbf{R}_{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s} \mathbf{H}^{p,T} \mathbf{w}_s^i. \tag{23}$$

Although (20) is much easier to solve than (5) since it has only LSM optimization variables, (23) still requires the knowledge of all impulse responses $\mathbf{h}_{kl}^p$ and current PM filters in the WASAN. Luckily (23) can also be computed from the error microphone signals as

$$\mathbf{r}_s^i = \sum_{p\in\mathcal{B}_s} E\{\tilde{\mathbf{x}}(t) e^p(t)\} + \mu \sum_{p\in\mathcal{D}_s} E\{\tilde{\mathbf{x}}(t) e^p(t)\}. \tag{24}$$

Based on this observation, a distributed adaptive PM algorithm (DA-PM) can be proposed as defined in Algorithm 1. The steps that are presented in Algorithm 1 are more general than the steps performed in the Gauss-Seidel procedure. To avoid confusion, the iteration index is now put between brackets (i). The differences are that in each iteration (i) of Algorithm 1 a set of nodes $\mathcal{S}^i$ is chosen to perform an update of there local PM filters. Also, instead of updating each local filter with the newly computed filter, a smooth combination between the newly computed filter and the filter from the previous iterations is used, using a predefined smoothing parameter α^i.

Proving convergence of Algorithm 1 can be done similarly as in [11], which is omitted here for conciseness. The conditions for α_i to guarantee convergence are that

$$\alpha^i \in (0,1]; \quad \lim_{i\to\infty} \alpha^i; \quad \sum_{i=0}^{\infty} \alpha^i = \infty \tag{25}$$

In practice this can be relaxed to a condition $\alpha^i \leq \alpha^\star$, where $\alpha^\star$ depends on the eigenvalues of $\mathbf{R}_s$ $\forall s$, and so $\alpha^\star$ is scenario-dependent. The condition for determining the sets $\mathcal{S}^i$ for $i = 0\ldots\infty$ is that none of the nodes permanently stops the updating process of its parameters. This means that when the number of times a node k belongs to $\mathcal{S}^i$ is counted for $i = 0\ldots\infty$, this number will be unbounded. It is noted that this requires no centralized controller, each node can on the fly decide when and how often it performs an update of its parameters.

The above Gauss-Seidel procedure is obtained by choosing $\mathcal{S}^i = \mathrm{mod}(i, K) + 1$ and $\alpha_i = 1$ in Algorithm 1.

Algorithm 1: Distributed Adaptive PM algorithm (DA-PM)

1 - Initialize $\mathbf{w}_{k,s}^{(0)}$ $\forall k, s$ without violating any constraint in (5).
 - $i \leftarrow 0$.
2 - Each node k produces the loudspeaker signals for $t = 1\ldots T$:

$$y_{kl}(iT + t) = \sum_{s=1}^{S} \mathbf{w}_{kl,s}^{(i),T} \mathbf{x}(iT + t) \quad \forall l. \tag{26}$$

3 - The error microphone signals $e^p(iT + t)$ $\forall p$ are transmitted to all nodes $k \in \mathcal{S}^i$.
4 - Each node $q \in \mathcal{S}^i$ performs the following operations:
  1) Collect T observations of $e^p(iT + t)$ $\forall p$ and estimate $\mathbf{r}_s^{(i)}$ using (24).
  2) Compute a new estimates of $\mathbf{h}_{ql}^p$ $\forall l, p$ or reuse a previous estimate.
  3) Estimate $\mathbf{R}_{q,s}$ and $\mathbf{r}_{q,s}$ using (21) and (22).
  4) Solve QPCP (20) to obtain $\mathbf{w}_{q,s}^{\text{new}}$ $\forall s$.
  5) Update the local PM filters as
$$\mathbf{w}_{q,s}^{(i+1)} = (1 - \alpha^i)\mathbf{w}_{q,s}^{(i)} + \alpha^i \mathbf{w}_{q,s}^{\text{new}} \quad \forall s. \tag{27}$$
5 - All other nodes $k \notin \mathcal{S}^i$ do not update their PM filters:
$$\mathbf{w}_{k,s}^{(i+1)} = \mathbf{w}_{k,s}^{(i)} \quad \forall s. \tag{28}$$
6 - $i \leftarrow i + 1$ and return to step 2.

The estimation of the impulse responses $\mathbf{h}_{kl}^p$ in step 4.2 is necessary to track possible changes in the impulse responses and can be done using training sequences. The updating node q then adds a training signal $\delta_{ql}(t)$[3] that is uncorrelated with the sounds, to its loudspeaker signals $\mathbf{y}_{ql}(iT+t)$ and estimates the impulse responses as:

$$\mathbf{h}_{ql}^p = E\{\boldsymbol{\delta}_{ql}(t)\boldsymbol{\delta}_{ql}(t)^T\}^{-1} \sum_{t=1}^{T} \boldsymbol{\delta}_{ql}(t) e^p(iT + t) \quad \forall l, p \tag{29}$$

with $\boldsymbol{\delta}_{ql}(t) = [\delta_{ql}(t) \ldots \delta_{ql}(t - N + 1)]^T$.

The advantages of the DA-PM are that the local computational complexity is much more relaxed and that there is no need for commutation between the nodes or communication with a FC. The disadvantage is that it requires multiple iterations to converge to the optimal solution, and will hence experience a slower tracking speed.

V. SIMULATIONS

To demonstrate the convergence of the DA-PM, 50 Monte Carlo simulations are performed on a WASAN with $K = 6$ nodes, $L = 4$ loudspeakers per node and $P = 10$ error microphones. Each error microphone randomly picks 1 out of 3 sound signals as its required sound signal and defines

[3] Since this will distort the signal received at the error microphones, it can also be useful to use colored noise (e.g. a noise signal with the spectrum below the hearing threshold for all frequencies) to estimate the impulse responses

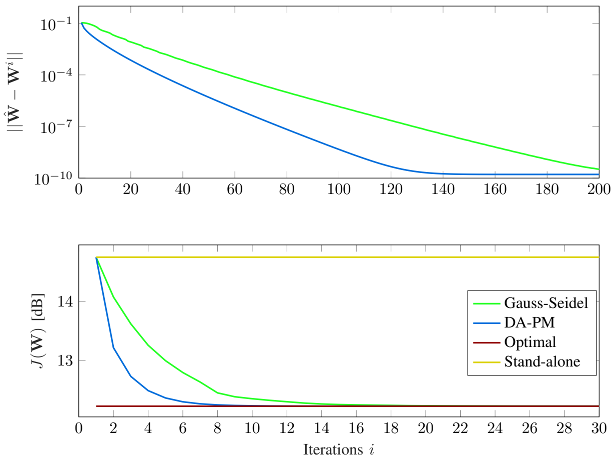

Fig. 1: Evolution of $||\hat{\mathbf{W}}-\mathbf{W}^i||$ and $J(\mathbf{W})$ for the DA-PM and Gauss-Seidel procedure, compared to the optimal and stand-alone solution.

its desired sound pressure as the clean required sound. The sounds are generated as Gaussian noise with zero mean and unit variance of $T = 1000$ samples. P_{kl} is set to 0.1 to activate the constraints and $\mu = 2$. The impulse responses are modeled as random impulse responses with zero mean and unit variance of order $N = 10$ and the PM filters have order $M = 10$ as well. In the simulations, the proposed DA-PM with $S^i = \{1, ..., K\}$ and $\alpha_i = 0.5$ for all iterations, is compared to the Gauss-Seidel procedure, the optimal PM filters and the stand-alone PM filters, obtained when each node locally solves (11) with the PM filters of the other nodes put to zero. Figure 1 shows $||\hat{\mathbf{W}} - \mathbf{W}^i||$ and $J(\mathbf{W})$ for the different filters in each iterations, averaged over 50 Monte Carlo runs. Convergence can be observed and it is clear that the proposed DA-PM method converges faster to the optimal solution than the Gauss-Seidel procedure.

To evaluate the performance of the DA-PM, an acoustic scenario is simulated using the image method [12] in a room of dimension $5m \times 5m \times 3m$ and with a reverberation time $T_{60} = 0.222$. There are 4 nodes, each having an array 4 loudspeakers, spaced $0.5m$ from each other, and each located at $0.5m$ from a wall. The virtual source is located in the center of the room and 3 error microphones are located in a radius of $0.5m$ from this center, each requiring a different sound signal. The 3 different sound signals are 5 seconds of speech from the HINT database. P_{kl} is also set to 0.1 and at iterations 40 and 80, all loudspeakers and error microphones are moved $0.2m$ in a random direction, realizing a change in the impulse responses. M is set to 50. The used performance measure is the distortion ratio, defined as

$$\mathrm{DR} = \sum_{p=1}^{P} \frac{E\{||d_p(t) - \sum_{s=1}^{S} \mathbf{w}_s^T \mathbf{H}^p \tilde{\mathbf{x}}_s(t)||^2\}}{E\{||d_p(t)||^2\}}. \tag{30}$$

The results for the different algorithms are shown in Figure 2. It is clear that the Gauss-Seidel procedure and the DA-PM converge to the optimal solution and that they can track changes

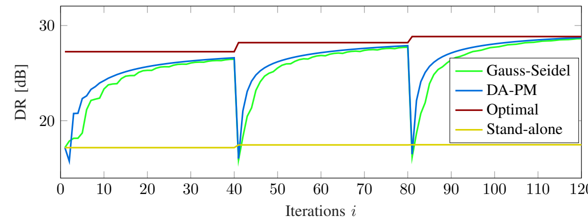

Fig. 2: Evolution of DR for the DA-PM and Gauss-Seidel procedure, compared to the optimal and stand-alone solution.

in the acoustic environment. The DA-PM also outperforms the stand-alone solution after 2 iterations.

VI. Conclusion

In this paper, PM based sound zoning has been considered in a WASAN. A distributed adaptive PM algorithm has been proposed, avoiding the high communication and computational requirements of a centralized algorithm. The algorithm has been shown to converge to the solution of the centralized algorithm, by means of Monte Carlo simulations and evaluated in a simulated acoustic scenario.

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