A Robust RLS Implementation of the ANC Block in GSC Structures

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***Abstract*—Adaptive beamforming, and the minimum variance distortionless response (MVDR) beamformer in particular, is widely used in speech enhancement applications. We consider the enhancement of a single desired speaker in a vehicle (e.g. the driver), in road noise environment. For our problem of a fixed look direction and continuous noise tracking, the generalized sidelobe canceler (GSC) decomposition was shown to be computationally efficient for the implementation of the MVDR criterion. To address robustness issues that arise due to array imperfection, the projected least mean squares (LMS) algorithm, commonly adopted for the adaptive noise canceler (ANC) block realization, was shown to be effective. Here, due to the high dynamics of vehicle road and cabin noises, we propose utilizing a recursive least squares (RLS) flavor algorithm for the realization of the ANC block. To address the robustness issue, we introduce the Modified RLS algorithm. The Modified RLS converges to the required amount of diagonal loading which is associated with the array immunity to imperfection. The proposed diagonal loading algorithm can be easily employed in any diagonal loading problems utilizing RLS based adaptive filtering (AF). We present its operation considering audio signals recorded in a vehicle, demonstrating the way artifacts are mitigated when applied.**

***Index Terms*—beamforming, diagonal loading, acoustics, robustness**

I. Introduction

The minimum variance distortionless response (MVDR) beamformer is a commonly used technique for speech enhancement in audio communication and speech recognition applications. It is targeted at noise reduction and interference cancellation, while constrained to maintain the desired speech signal undistorted. Due to variations in the microphone manufacturing process [1], numerical errors resulting from finite precision [2] or design implications [3], robustness concerns arises and are often addressed as an additional constraint on the norm of the beamformer.

In time-varying noise environments, the generalized sidelobe canceler (GSC) structure is commonly utilized for the realization of the MVDR beamformer for real-time implementation to enable efficient noise adaptation [4]. A common method for the realization of noise canceling adaptation filter is the least mean squares (LMS) algorithm [5]. It has a complexity of $\mathcal{O}(M)$, for M being the number of microphones, and with a simple scalar normalization can achieve the required amount of robustness, that is, the desired maximal beamformer norm. This normalization can be shown to be equivalent to the commonly used diagonal loading technique [6]. In high noise dynamics, and especially for directional interferences, the LMS method may achieve low performance due to its limited convergence rate and tracking capabilities. To overcome this, a recursive least squares (RLS) flavor tracking algorithm can be utilized instead [7]. Though its implementation complexity is of $\mathcal{O}(M^2)$, it assures fast convergence rate with no dependence of noise directionality [5].

Introducing robustness constraints when the RLS is utilized is not as straightforward as it is for the LMS algorithm. The simple normalization does not apply in the RLS case and the alternative of an iterative solution may not be applicable in many real-time applications. Here, we propose a Modified RLS procedure which converges to the optimal level of diagonal loading λ. In the proposed algorithm, the value of λ is recursively estimated. In each new frame, the noise spatial correlation matrix is updated with some underlying level of loading which recursively converges to λ, resulting in the required norm constraint. A key feature of our proposed algorithm is that the exact diagonal loading is neither known nor required, and its level is achieved implicitly, relying on the required beamformer norm. In previous study [8], the level of diagonal loading in each time frame is restricted to a small value. This restriction can delay the loading value convergence to the required value. In the proposed algorithm any diagonal loading value can be applied, making it suitable for all possible scenarios. The proposed diagonal loading algorithm use is not restricted only to the discussed use case, and can be easily employed in any diagonal loading problems utilizing RLS-based adaptive filtering (AF). We describe the algorithm in details and show that only slight modification to the traditional RLS algorithm is required.

II. Problem Formulation

Consider a desired speech signal, contaminated by additive noise comprising of any combination of coherent, diffuse and spatially white noise signals, impinging on an array consisting of M microphones. The microphone signals are sampled at a sampling rate of f_s and transformed into the short time Fourier transform (STFT) using a window of length K with overlap η between frames. The transformed microphone signals are stacked into an M dimensional vector per time-frequency bin

$$\mathbf{x}(\ell,k)=\mathbf{h}(k)s(\ell,k)+\mathbf{v}(\ell,k),$$

where ℓ and k are time-frame and frequency bin indices, respectively, $s(\ell,k)$ is the desired signal, $\mathbf{v}(\ell,k)$ is the noise component vector and $\mathbf{h}(k)$ is a vector of acoustic transfer functions (ATFs) and is assumed to be time-independent (static source).

Assuming that the speech and noise signals are statistically independent, the spatial covariance matrix of the received signals is

$$\mathbf{\Phi}_{\mathbf{x}}(\ell,k)=\sigma_s^2(\ell,k)\mathbf{h}(k)\mathbf{h}^H(k)+\mathbf{\Phi}_{\mathbf{v}}(\ell,k), \quad (1)$$

where $\sigma_s^2(\ell,k)$ is the power spectral density (PSD) of the desired signal, $\mathbf{\Phi}_{\mathbf{v}}(\ell,k)$ is the spatial covariance matrix of the noise component and $(\cdot)^H$ denotes the conjugate-transpose operator.

Given an a priori knowledge of $\mathbf{h}(k)$[1], the problem at hand is to design a beamformer $\mathbf{w}(\ell,k)$ which is robust to array imperfection, such that the output signal

$$y(\ell,k)=\mathbf{w}^H(\ell,k)\mathbf{x}(\ell,k) \quad (2)$$

is enhanced, according to the MVDR criterion, defined in the following section. For brevity, the frequency bin index is omitted hereafter.

III. Background

The MVDR beamformer is defined in the STFT domain as the solution to the following optimization problem:

$$\mathbf{w}(\ell)=\text{argmin}_{\mathbf{w}}\mathbf{w}^H\mathbf{\Phi}_{\mathbf{v}}(\ell)\mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H\mathbf{h}=1. \quad (3)$$

Its closed-form solution is given by:

$$\mathbf{w}(\ell)=\frac{\mathbf{\Phi}_{\mathbf{v}}^{-1}(\ell)\mathbf{h}}{\mathbf{h}^H\mathbf{\Phi}_{\mathbf{v}}^{-1}(\ell)\mathbf{h}}. \quad (4)$$

To increase the robustness of the beamformer to array imperfection, additional constraint on the norm of the beamformer is introduced to the optimization criterion in (3) [6]:

$$\mathbf{w}(\ell)=\text{argmin}_{\mathbf{w}}\mathbf{w}^H\mathbf{\Phi}_{\mathbf{v}}(\ell)\mathbf{w} \text{ s.t. } \mathbf{w}^H\mathbf{h}=1 \text{ and } \|\mathbf{w}\|^2\leq\delta^2, \quad (5)$$

where δ^2 is a design requirement which limits the maximal sensitivity to uncorrelated errors and $\|\cdot\|$ is the l_2 norm.

The obtained closed-form solution of (5) is given by

$$\mathbf{w}(\ell)=\frac{\left(\mathbf{\Phi}_{\mathbf{v}}(\ell)+\lambda_\ell\mathbf{I}\right)^{-1}\mathbf{h}}{\mathbf{h}^H\left(\mathbf{\Phi}_{\mathbf{v}}(\ell)+\lambda_\ell\mathbf{I}\right)^{-1}\mathbf{h}}, \quad (6)$$

where λ_ℓ is the Lagrange multiplier related to δ^2 for time index ℓ and $\mathbf{I}$ is an $M\times M$ identity matrix.

The GSC structure splits the constrained optimization in (5) into two tasks, namely maintaining the desired signal undistorted and reducing noise [11]. The beamformer is accordingly split into

$$\mathbf{w}(\ell)=\mathbf{q}-\mathbf{B}\hat{\mathbf{f}}(\ell), \quad (7)$$

where $\mathbf{q}$ is the $M\times 1$ fixed beamformer (FBF), $\mathbf{B}$ is the $M\times(M-1)$ blocking matrix (BM) and $\hat{\mathbf{f}}(\ell)$ is the $(M-1)\times 1$ adaptive noise canceler (ANC).

[1] The estimation of $\mathbf{h}(k)$ or alternatively its RTF representation, is widely researched [9], [10] and is not addressed in the remainder of the paper.

The FBF is responsible for satisfying the distortionless response constraint. Matrix $\mathbf{B}$ is designed to block the desired signal, i.e. $\mathbf{B}^H\mathbf{h}=\mathbf{0}$. Therefore, theoretically, the $(M-1)$ dimensional signal at its output contains only noise components, and is hence denoted the *noise reference signal*. The ANC is designed to minimize the variance of the noise component at the output $y(\ell)$, by estimating and subtracting the noise component at the output of the FBF using the noise reference signals. It is defined as the solution to the unconstrained minimum mean square error (MMSE) problem:

$$\hat{\mathbf{f}}(\ell)=\text{argmin}_{\hat{\mathbf{f}}}(\mathbf{q}-\mathbf{B}\hat{\mathbf{f}})^H(\mathbf{\Phi}_{\mathbf{v}}(\ell)+\lambda_\ell\mathbf{I})(\mathbf{q}-\mathbf{B}\hat{\mathbf{f}}). \quad (8)$$

The well-known closed-form solution of (8) is the Wiener filter

$$\hat{\mathbf{f}}(\ell)=(\mathbf{B}^H(\mathbf{\Phi}_{\mathbf{v}}(\ell)+\lambda_\ell\mathbf{I})\mathbf{B})^{-1}\mathbf{B}^H(\mathbf{\Phi}_{\mathbf{v}}(\ell)+\lambda_\ell\mathbf{I})\mathbf{q}. \quad (9)$$

By further assuming orthogonal GSC decomposition, such that $\mathbf{B}^H\mathbf{q}=\mathbf{0}$, and unitary matrix $\mathbf{B}$, (9) becomes:

$$\begin{aligned}\hat{\mathbf{f}}(\ell)&=(\mathbf{B}^H\mathbf{\Phi}_{\mathbf{v}}(\ell)\mathbf{B}+\lambda_\ell\mathbf{I})^{-1}\mathbf{B}^H\mathbf{\Phi}_{\mathbf{v}}(\ell)\mathbf{q}\\&=(\mathbf{\Phi}_{\mathbf{u}}(\ell)+\lambda_\ell\mathbf{I})^{-1}\boldsymbol{\phi}_{\mathbf{u}d}(\ell),\end{aligned} \quad (10)$$

where we denoted the noise reference signal by $\mathbf{u}\triangleq\mathbf{B}^H\mathbf{v}$, $d\triangleq\mathbf{v}^H\mathbf{q}$ and $\boldsymbol{\phi}_{\mathbf{u}d}$ is the cross correlation vector between $\mathbf{u}$ and d.

Subject to the same assumptions, a projected LMS type algorithm proposes a recursive solution to (10) and can be adopted for reducing the computational complexity when the noise field is time-varying and the optimal ANC filter is continuously changing. It address the equivalent optimization problem of $\hat{\mathbf{f}}$, as the constrained optimization problem of the form [2]:

$$\hat{\mathbf{f}}(\ell)=\text{argmin}_{\hat{\mathbf{f}}}(\mathbf{q}-\mathbf{B}\hat{\mathbf{f}})^H\mathbf{\Phi}_{\mathbf{v}}(\ell)(\mathbf{q}-\mathbf{B}\hat{\mathbf{f}}) \quad \text{s.t.} \quad \|\hat{\mathbf{f}}\|^2\leq\hat{\delta}^2, \quad (11)$$

where we defined $\hat{\delta}^2\triangleq\delta^2-\|\mathbf{q}\|^2$. The norm constraint defines a multidimensional sphere that is centered at the origin, and therefore its projection can be implemented by a simple scalar normalization [6].

IV. Proposed Solution

Let us first recall the traditional RLS procedure that proposes a computationally efficient solution to:

$$\tilde{\mathbf{f}}(\ell)=\mathbf{\Phi}_{\mathbf{u}}^{-1}(\ell)\phi_{\mathbf{u}d}(\ell). \quad (12)$$

The RLS procedure avoids a direct matrix inversion and utilizes the Woodbury matrix lemma assuming rank-1 update of $\mathbf{\Phi}_{\mathbf{u}}(\ell-1)$, that is

$$\mathbf{\Phi}_{\mathbf{u}}(\ell)=\mu\mathbf{\Phi}_{\mathbf{u}}(\ell-1)+(1-\mu)\mathbf{u}(\ell)\mathbf{u}^H(\ell), \quad (13)$$

for obtaining $\mathbf{\Phi}_{\mathbf{u}}^{-1}(\ell)$ with some forgetting factor μ. It can be implemented with complexity of $\mathcal{O}(M^2)$ by considering the values of $\mathbf{\Phi}_{\mathbf{u}}^{-1}(\ell-1)$ and $\mathbf{u}(\ell)$.

By further assuming rank-1 update of $\boldsymbol{\phi}_{\mathbf{u}d}(\ell-1)$ with μ, a time update equation for $\tilde{\mathbf{f}}(\ell)$ is obtained. The algorithm is described in [5].

The problem we are addressing here is a modification of (12) as presented in (10). Instead of efficiently inverting $(\mu\mathbf{\Phi}_{\mathbf{u}}(\ell-1)+(1-\mu)\mathbf{u}(\ell)\mathbf{u}^H(\ell))$ to obtain $\tilde{\mathbf{f}}(\ell)$, we are required to invert a matrix of the form

$(\mu\boldsymbol{\Phi}_{\mathbf{u}}(\ell-1)+(1-\mu)\mathbf{u}(\ell)\mathbf{u}^H(\ell)+\lambda_\ell\mathbf{I})$ to obtain $\hat{\mathbf{f}}(\ell)$, which does not comply with the rank-1 assumption.

The proposed Modified RLS algorithm is described in the next sections. We will show that with a slight modification of the traditional RLS procedure, we can recursively add some amount of diagonal loading on each time-frame, until the desired amount of loading is reached. This loading control is incorporated into the traditional RLS procedure.

A. Modified Matrix Inversion

Define:

$$\hat{\boldsymbol{\Phi}}_{\mathbf{u}}(\ell) \triangleq \boldsymbol{\Phi}_{\mathbf{u}}(\ell)+\lambda_\ell\mathbf{I}. \tag{14}$$

In a previous study [8], a Taylor series approximation was used to solve for $\hat{\boldsymbol{\Phi}}_{\mathbf{u}}^{-1}(\ell)$. This approach imposes a strict constraint of $\lambda_\ell \ll 1$ for the Taylor approximation to hold valid. This restriction can delay the loading value convergence to the required level. Furthermore, the loaded value was realized in the solution for $\hat{\mathbf{f}}(\ell)$ without changing the properties of $\hat{\boldsymbol{\Phi}}_{\mathbf{u}}(\ell)$ which does not ensure its stability in case it is ill conditioned.

As will be presented next, in our proposed solution, any required value for diagonal loading can be applied. Furthermore, it is primarily applied on $\hat{\boldsymbol{\Phi}}_{\mathbf{u}}(\ell)$, ensuring the stability of $\hat{\boldsymbol{\Phi}}_{\mathbf{u}}^{-1}(\ell)$.

Assuming rank-1 update for obtaining $\boldsymbol{\Phi}_{\mathbf{u}}(\ell)$, (14) becomes:

$$\begin{aligned}\hat{\boldsymbol{\Phi}}_{\mathbf{u}}(\ell) &= \mu\boldsymbol{\Phi}_{\mathbf{u}}(\ell-1)+(1-\mu)\mathbf{u}(\ell)\mathbf{u}^H(\ell)+\lambda_\ell\mathbf{I}\\ &= \mu(\boldsymbol{\Phi}_{\mathbf{u}}(\ell-1)+\lambda_{\ell-1}\mathbf{I})+(1-\mu)\mathbf{u}(\ell)\mathbf{u}^H(\ell)+\Delta\lambda_\ell\mathbf{I}\\ &= \mu\hat{\boldsymbol{\Phi}}_{\mathbf{u}}(\ell-1)+(1-\mu)\mathbf{u}(\ell)\mathbf{u}^H(\ell)+\Delta\lambda_\ell\mathbf{I}\\ &= \mathbf{R}(\ell)+\Delta\lambda_\ell\mathbf{I}\end{aligned} \tag{15}$$

where

$$\Delta\lambda_\ell \triangleq \lambda_\ell-\mu\lambda_{\ell-1} \tag{16}$$

and

$$\mathbf{R}(\ell) \triangleq \mu\hat{\boldsymbol{\Phi}}_{\mathbf{u}}(\ell-1)+(1-\mu)\mathbf{u}(\ell)\mathbf{u}^H(\ell). \tag{17}$$

Equation (15) proposes that the additional required loading in time-frame ℓ, that is $\Delta\lambda_\ell$, can be applied to the traditional RLS output $\mathbf{R}(\ell)$, rather than the addition of the total required loading λ_ℓ to $\boldsymbol{\Phi}_{\mathbf{u}}(\ell)$ as suggested in (14).

Initially, we wish to modify the traditional RLS procedure to efficiently obtain

$$\hat{\boldsymbol{\Phi}}_{\mathbf{u}}^{-1}(\ell) = (\mathbf{R}(\ell)+\Delta\lambda_\ell\mathbf{I})^{-1}. \tag{18}$$

For that, we propose the following relation:

$$\begin{aligned}\hat{\boldsymbol{\Phi}}_{\mathbf{u}}^{-1}(\ell) &= (\mathbf{R}(\ell)+\Delta\lambda_\ell\mathbf{I})^{-1}\\ &= \left(\mathbf{R}(\ell)+\Delta\lambda_\ell\sum_{i=1}^{M}\mathbf{e}_i\mathbf{e}_i^T\right)^{-1},\end{aligned} \tag{19}$$

where $\mathbf{e}_i$ is the standard basis vector with i^{th} coordinate equals 1 and $(\cdot)^T$ denotes the transpose operator.

Defining the partially diagonal loaded matrix $\mathbf{R}_p$, $p \le M$:

$$\mathbf{R}_p(\ell) \triangleq \mathbf{R}(\ell)+\Delta\lambda_\ell\sum_{i=1}^{p}\mathbf{e}_i\mathbf{e}_i^T, \tag{20}$$

it can be expressed as:

$$\begin{aligned}\mathbf{R}_p(\ell) &= \mathbf{R}(\ell)+\Delta\lambda_\ell\sum_{i=1}^{p-1}\mathbf{e}_i\mathbf{e}_i^T+\Delta\lambda_\ell\mathbf{e}_p\mathbf{e}_p^T\\ &= \mathbf{R}_{p-1}(\ell)+\Delta\lambda_\ell\mathbf{e}_p\mathbf{e}_p^T.\end{aligned} \tag{21}$$

Equation (21) states that $\mathbf{R}_p$ can be viewed as a rank one update of the $\mathbf{R}_{p-1}$ matrix, thus its inverse can be calculated efficiently using the Woodbury inversion lemma:

$$\begin{aligned}\mathbf{R}_p^{-1}(\ell) &= \left(\mathbf{R}_{p-1}(\ell)+\Delta\lambda_\ell\mathbf{e}_p\mathbf{e}_p^T\right)^{-1}\\ &= \mathbf{R}_{p-1}^{-1}(\ell)-\frac{\mathbf{R}_{p-1}^{-1}(\ell)\mathbf{e}_p\mathbf{e}_p^T\mathbf{R}_{p-1}^{-1}(\ell)}{\Delta\lambda_\ell^{-1}+\mathbf{e}_p^T\mathbf{R}_{p-1}^{-1}(\ell)\mathbf{e}_p}\\ &= \mathbf{R}_{p-1}^{-1}(\ell)-\frac{\mathbf{r}_{p-1}(p)\mathbf{r}_{p-1}^H(p)}{\Delta\lambda_\ell^{-1}+r_{p-1}(p,p)},\end{aligned} \tag{22}$$

where $r_{p-1}(i,j)$ is the element of $\mathbf{R}_{p-1}^{-1}(\ell)$ in the (i,j) position, and $\mathbf{r}_{p-1}(i)$ is the i^{th} column of $\mathbf{R}_{p-1}^{-1}(\ell)$. Equation (22) defines a recursion step which solves (19) in M iterations to obtain $\hat{\boldsymbol{\Phi}}_{\mathbf{u}}^{-1}(\ell)$.

Finally, the filter $\hat{\mathbf{f}}(\ell)$ is obtained. For that, we define the indeterminate filter $\mathbf{f}(\ell)$ as:

$$\mathbf{f}(\ell) = \mathbf{R}^{-1}(\ell)\phi_{\mathbf{u}d}(\ell). \tag{23}$$

By considering Eq. (12), it can be noted that it is the result of utilizing the traditional RLS steps.

This relation can be used with (15) to solve for the final weights value

$$\begin{aligned}\hat{\mathbf{f}}(\ell) &= \hat{\boldsymbol{\Phi}}_{\mathbf{u}}^{-1}(\ell)\phi_{\mathbf{u}d}(\ell)\\ &= \hat{\boldsymbol{\Phi}}_{\mathbf{u}}^{-1}(\ell)\left(\mathbf{R}(\ell)+\Delta\lambda_\ell\mathbf{I}-\Delta\lambda_\ell\mathbf{I}\right)\mathbf{f}(\ell)\\ &= \left(\mathbf{I}-\Delta\lambda_\ell\hat{\boldsymbol{\Phi}}_{\mathbf{u}}^{-1}(\ell)\right)\mathbf{f}(\ell).\end{aligned} \tag{24}$$

This completes the proposed inversion algorithm.

The modified matrix inversion process is summarized in Algorithm 1. The recursion step complexity is the same as the standard RLS algorithm i.e. $\mathcal{O}(M^2)$. Since M recursion steps are required to complete the diagonal loading process, the entire algorithm complexity is of $\mathcal{O}(M^3)$. In order to reduce the complexity burden of the algorithm only one recursion step can be taken in each time frame, thereby completing the entire recursion after M frames. This method will maintain the proposed algorithm complexity at $\mathcal{O}(M^2)$ while delaying the loading effect by M frames. Furthermore, this computation is only required when the square norm of the ANC exceeds $\hat{\delta}$ and results with an Hermitian matrix, which can further reduce the computational load. It should be noted that the proposed diagonal loading algorithm is a general algorithm and its use is not restricted to the GSC robustness problem. The proposed algorithm can be used in all problems utilizing an RLS based AF which requires diagonal loading e.g. multi reference acoustic echo cancellation (AEC) with correlated reference channels.

B. Obtaining $\Delta\lambda$

Generally, the desired amount of loading λ is not known and instead the requirement is given in terms of the beamformer norm, or equivalently in terms of the norm of the ANC filter.

In this section, we propose to derive the required $\Delta\lambda_\ell$ for recursively converging to the desired ANC norm.

We first consider an iterative procedure which at the n^{th} iteration introduces additional loading of $\Delta\lambda_\ell^n$. The stopping criterion is such that ensures $\|\hat{\mathbf{f}}(\ell)\|^2 \leq \hat{\delta}^2$ (see (11)). We parameterize the noise canceller at time ℓ at the n^{th} iteration by λ_ℓ^n, denoting it by $\hat{\mathbf{f}}(\lambda_\ell^n)$ and define the objective function $g(\lambda_\ell^n)$:

$$g(\lambda_\ell^n) = \|\hat{\mathbf{f}}(\lambda_\ell^n)\|^2 - \hat{\delta}^2. \tag{25}$$

Under the assumption of positive definite $\mathbf{\Phi}_{\mathbf{u}}$, the norm of the ANC can be rewritten as:

$$\begin{aligned} ||\hat{\mathbf{f}}(\lambda_\ell^{n-1})||^2 &= \left\| \sum_{i=0}^{M-1} \frac{1}{\lambda_\ell^{n-1} + \gamma_i} \mathbf{u}_i \mathbf{u}_i^H \phi_{\mathbf{u}d} \right\|^2 \\ &= \sum_{i=0}^{M-1} \left(\frac{1}{\lambda_\ell^{n-1} + \gamma_i} \right)^2 ||\mathbf{u}_i \mathbf{u}_i^H \phi_{\mathbf{u}d}||^2 \\ &= \sum_{i=0}^{M-1} \left(\frac{1}{\lambda_\ell^{n-1} + \gamma_i} \right)^2 |\mathbf{u}_i^H \phi_{\mathbf{u}d}|^2 \\ &= \sum_{i=0}^{M-1} \left(\frac{1}{\lambda_\ell^{n-1} + \gamma_i} \right)^2 \alpha_i^2 \end{aligned} \tag{26}$$

where $\gamma_i > 0$ are the eigenvalues of $\mathbf{\Phi}_{\mathbf{u}}$, $\mathbf{u}_i$ its eigenvectors for $i = 0, \ldots, M-1$ and we defined $\alpha_i \triangleq |\mathbf{u}_i^H \phi_{\mathbf{u}d}|$.

For the iterative update we use Newton-Raphson method

$$\lambda_\ell^n = \lambda_\ell^{n-1} - \frac{g(\lambda_\ell^{n-1})}{g'(\lambda_\ell^{n-1})}. \tag{27}$$

Based on (26), the derivative $g'(\lambda_\ell^{n-1})$ can be lower bounded by:

$$\begin{aligned} g'(\lambda_\ell^{n-1}) &= -2 \sum_{i=0}^{M-1} \alpha_i^2 \frac{1}{(\lambda_\ell^{n-1} + \gamma_i)^3} \\ &\geq -2 \frac{1}{\lambda_\ell^{n-1}} \sum_{i=0}^{M-1} \frac{\alpha_i^2}{(\lambda_\ell^{n-1} + \gamma_i)^2} \\ &= \frac{-2}{\lambda_\ell^{n-1}} ||\hat{\mathbf{f}}(\lambda_\ell^{n-1})||^2. \end{aligned} \tag{28}$$

Substituting (28) in (27) proposes the following update step:

$$\lambda_\ell^n = \lambda_\ell^{n-1} + \frac{\lambda_\ell^{n-1}}{2} \left(1 - \frac{\hat{\delta}^2}{||\hat{\mathbf{f}}(\lambda_\ell^{n-1})||^2} \right). \tag{29}$$

The inequality in (28) implies that the step-size actually used is smaller or equal to that suggested by Newton-Raphson method (i.e smaller than $-g(\lambda_\ell^{n-1})/g'(\lambda_\ell^{n-1})$). The direction of the step size is maintained equal to that suggested by the Newton-Raphson method due to the assumption of a positive definite $\mathbf{\Phi}_{\mathbf{u}}$.

For the recursive implementation, we substitute the iteration index n with the time index ℓ and propose the recursive form of (29) to be:

$$\lambda_\ell = \lambda_{\ell-1} + \frac{\lambda_{\ell-1}}{2} \left(1 - \frac{\hat{\delta}^2}{||\mathbf{f}(\ell)||^2} \right). \tag{30}$$

Substituting (30) in (16) yields:

$$\begin{aligned} \Delta\lambda_\ell &= \lambda_{\ell-1}(1-\mu) + \frac{\lambda_{\ell-1}}{2} \left(1 - \frac{\hat{\delta}^2}{\|\mathbf{f}(\ell)\|^2} \right) \\ &= \lambda_{\ell-1} \left(\left(\frac{3}{2} - \mu \right) - \frac{\hat{\delta}^2}{2\|\mathbf{f}(\ell)\|^2} \right). \end{aligned} \tag{31}$$

The Modified RLS is summarized in Algorithm 2. Negative values of $\Delta\lambda_\ell$ suggest that sufficient amount of loading has been achieved and no additional loading should be applied for the ℓ^{th} time frame.

Algorithm 1 Modified Matrix Inversion

Input: $\mathbf{R}^{-1}(\ell)$, $\Delta\lambda_\ell, \mathbf{f}(\ell)$
Output: $\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell)$, $\hat{\mathbf{f}}(\ell)$
$\mathbf{R}_0^{-1} = \mathbf{R}^{-1}(\ell)$
for $1 \leq p \leq M$ **do**
  $\mathbf{R}_p^{-1} = \mathbf{R}_{p-1}^{-1}(\ell) - \frac{\mathbf{r}_{p-1}(p)\mathbf{r}_{p-1}^H(p)}{\Delta\lambda_\ell^{-1} + r_{p-1}(p,p)}$
end for
$\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell) = \mathbf{R}_M^{-1}$
$\hat{\mathbf{f}}(\ell) = \left(\mathbf{I} - \Delta\lambda_\ell \hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell) \right) \mathbf{f}(\ell)$
return $\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell)$, $\hat{\mathbf{f}}(\ell)$

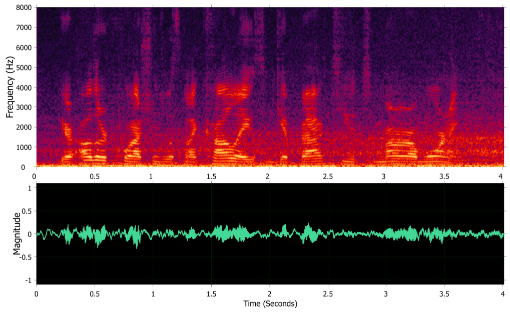
Algorithm 2 Modified RLS

Input: $\hat{\mathbf{f}}(\ell-1)$, $\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell-1)$, $\lambda_{\ell-1}$, $\mathbf{u}(\ell)$, $d(\ell)$, $\tilde{\delta}$, μ
Output: $\hat{\mathbf{f}}(\ell)$, $\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell)$, λ_ℓ
$\mathbf{k}_\ell = \frac{\mu^{-1}\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell-1)\mathbf{u}(\ell)}{(1-\mu)^{-1} + \mu^{-1}\mathbf{u}^H(\ell)\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell-1)\mathbf{u}(\ell)}$
$e_\ell = d^*(\ell) - \mathbf{u}^H(\ell)\hat{\mathbf{f}}(\ell-1)$
$\mathbf{f}(\ell) = \hat{\mathbf{f}}(\ell-1) + \mathbf{k}e_\ell$
$\mathbf{R}^{-1}(\ell) = \mu^{-1}\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell-1) - \mu^{-1}\mathbf{k}\mathbf{u}^H(\ell)\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell-1)$
$\Delta\lambda = \lambda_{\ell-1}\left(\left(\frac{3}{2} - \mu \right) - \frac{\hat{\delta}^2}{2\|\mathbf{f}(\ell)\|^2} \right)$
if $\Delta\lambda > 0$ **then**
  Apply Algorithm 1 to obtain $\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell)$ and $\hat{\mathbf{f}}(\ell)$
  $\lambda_\ell = \mu\lambda_{\ell-1} + \Delta\lambda$
else
  $\lambda_\ell = \mu\lambda_{\ell-1}$
  $\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell) = \mathbf{R}^{-1}(\ell)$
  $\hat{\mathbf{f}}(\ell) = \mathbf{f}(\ell)$
end if
return $\hat{\mathbf{f}}(\ell)$, $\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell)$, λ_ℓ

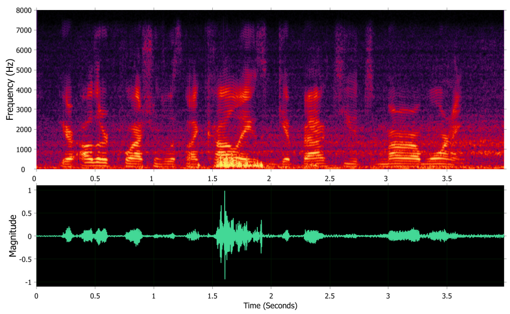
V. Analysis with Speech Signals

The proposed algorithm is evaluated in a vehicle, in driving noise scenario, which consists of road noise as well as transient noises transmitted by passing vehicles or horns. We also challenge the algorithm by opening the windows, turning on the air conditioning or activating the wipers. The driver is considered to be the single desired speaker and is instructed to speak and move naturally. The desired speech signal is contaminated by the additive noise and captured by a microphone array.

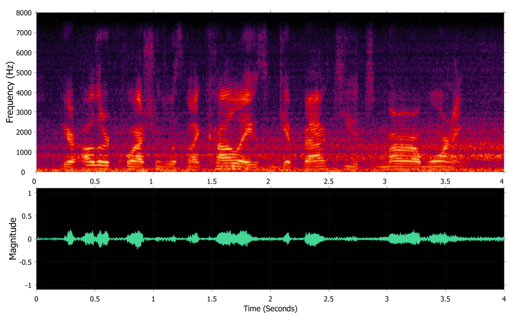
We apply the GSC algorithm with the RLS procedure for ANC realization, comparing the performance of the traditional RLS form with our proposed algorithm. The microphone signals are sampled at $f_s = 16$KHz. We use an STFT with



(a) Input signal.



(b) Traditional RLS algorithm output.



(c) Proposed Modified RLS algorithm output.

Fig. 1: Traditional vs Modified RLS algorithm.

hamming window of length $K = 512$ with an overlap of $\eta = 50\%$ between successive frames. The RLS forgetting factor is set to $\mu = 0.98$ and for the proposed algorithm, the ANC norm square is bound by $\hat{\delta} = 10$dB.

While for the most part of the utterance the traditional RLS based GSC does not exhibit any artifacts, we identify sudden bursts associated with transient noises. Figure 1b depicts an example of such artifacts. It can be clearly observed at around 1.5sec for the traditional RLS at the lower frequencies. The input signal, shown in Figure 1a, shows no anomalies around that time-frames, implying that the identified artifacts are attributed to the beamforming operation.

The proposed Modified RLS algorithm controls these artifacts through $\hat{\delta}$. In Figure 1c it can be observed that the sudden burst is mitigated. Since the algorithm is only active in relevant time-frequency bins (that is when $\Delta\lambda_\ell > 0$), the overall noise reduction is maintained.

VI. Summary

In this contribution we proposed the Modified RLS procedure for the realization of the ANC block of the GSC structure. We showed that with minor modification to the traditional RLS procedure, the RLS algorithm can be adjusted to recursively maintain sufficiently small norm ensuring mitigation of artifacts attributed to array imperfection.

The proposed algorithms has two drawbacks. Unlike the projected LMS, we are not assured that the norm constraint is satisfied for each time-frame. The second drawback is around the computational complexity of $\hat{\mathbf{\Phi}}_{\mathbf{u}}^{-1}(\ell)$ which is of $\mathcal{O}(M^3)$. However, we must note that this computation is only required when the square norm of the ANC exceeds $\hat{\delta}$ and results with an Hermitian matrix, which can further reduce the computational load. This property can significantly reduce multiplication complexity in comparison to complex matrices multiplication. Furthermore, the modified matrix inversion algorithm can be split between several time frames, thereby reducing its calculation complexity to $\mathcal{O}(M^2)$.

While the described drawbacks must be admitted, our proposed algorithm is an enabler for an RLS-based ANC implementation. Without any robustness considerations, artifacts should be expected. These artifacts, even if transients and are rare, can impact the listening quality. On top of the algorithm being essential, it is easily implemented and it holds the important ability to achieve the desired performance without requiring the explicit value of λ_ℓ, which has intricate relation to the known $\hat{\delta}$. Furthermore, the algorithm use is not restricted to the discussed scenario and can be easily implemented in any RLS based system.

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