Cell-Free Massive MIMO with Multiple-Antenna Users under I/Q Imbalance

James A. C. Sutton, Hien Quoc Ngo, and Michail Matthaiou
Institute of Electronics, Communications and Information Technology (ECIT), Queen's University Belfast, U.K.
Email:{jsutton03, hien.ngo, m.matthaiou}@qub.ac.uk

***Abstract*—Cell-free massive multiple-input multiple-output (MIMO) is a core technology for future wireless communication systems, since it harnesses benefits from massive MIMO and distributed MIMO systems. In order to be practically viable, cell-free massive MIMO systems are expected to make use of many low cost, low quality components. As a consequence, this makes cell-free massive MIMO susceptible to hardware imperfections, such as in-phase and quadrature-phase imbalance (IQI). Motivated by this, we provide a derivation for the achievable spectral efficiency of each user in a cell-free massive MIMO system with IQI and multiple-antenna users. Our analysis demonstrates that the performance loss from IQI is noticeable under high IQ mismatch. Furthermore, our analytical and numerical results showcase that the system performance improves without bound for a perfect IQ matching scenario, however, if IQI is present the performance saturates even for an increasing number of access points.**

I. Introduction

Massive multiple-input multiple-output (MIMO) is a core technology for the fifth generation (5G) wireless communication systems. Specifically, large gains in spectral efficiency (SE), energy efficiency, and reliability are achieved by massive MIMO, from the utilization of a huge number of base station (BS) antennas serving a comparatively small number of users [1]. Cell-free massive MIMO was recently introduced to increase the connectivity in massive MIMO [2]. It consists of a large number of geographically distributed access points (APs), serving a smaller number of users across an area. The APs cooperate using a back/front-haul network and serve the users without cells. The nature of this operational setup allows cell-free massive MIMO to simultaneously harness benefits from both massive MIMO and distributed MIMO, such as lower pathloss, macrodiversity gains and improved coverage [2]. Therefore, it is of particular research interest for the development of future wireless communication systems and has naturally attracted much research attention [3]–[5].

In a practical system, there will always be imperfections due to the hardware that cause undesired disruptions in the system. Such hardware imperfections will be frequently present in low quality components. Cell-free massive MIMO systems are expected to make use of low cost, low precision components. Hence, it is important to model and analyze the effects of such imperfections on cell-free massive MIMO. The in-phase and quadrature-phase imbalance (IQI) is one such hardware imperfection, that refers to the mismatch between the real and imaginary parts of the complex signal. In the massive MIMO space, there exists research that accounts for the effects of IQI, see, for example, [6]–[9]; comparatively, there is little research on the impact of IQI on cell-free massive MIMO systems. The work in [10], [11] competes a range of investigations into cell-free massive MIMO systems with hardware impairments which include the effects of IQI. However, these works assume single-antenna users and a Gaussian-type model for the aggregate impact of hardware impairments.

Inspired by the above discussion, in our work, we analyze the performance of cell-free massive MIMO systems under IQI at the APs, where both the APs and the users are equipped with multiple antennas. We consider simple processing at the APs and users where there are no IQI compensation techniques used. To complete this analysis, new expressions for the minimum mean-square error (MMSE) estimation and the achievable SE are derived. An asymptotic analysis for an infinite number of APs is also presented. We show that when the number of APs grows with bound, the IQI effect persists. This suggests more advanced IQI compensation techniques should be taken into consideration to fully exploit the benefits of cell-free massive MIMO.

Notation: Superscripts $()^*$, $()^H$, $()^T$ and $()^{-1}$ express the conjugate, conjugate-transpose, transpose and inverse, while $\mathrm{tr}\,()$ and $|.|$ represent the trace and determinant of a matrix, respectively. Finally, $\otimes$ expresses the Kronecker product.

II. System Model

Consider a cell-free massive MIMO system comprised of M APs and K users located in a large area. Each AP deploys L antennas and each user has N antennas. The system operates in time-division duplex (TDD) mode. Let $\mathbf{G}_{mk} \in \mathbb{C}^{L\times N}$ be the channel matrix between the kth user and the mth AP, i.e.,

$$\mathbf{G}_{mk} = \sqrt{\beta_{mk}}\mathbf{H}_{mk}, \quad (1)$$

where β_{mk} represents the large-scale fading and $\mathbf{H}_{mk}$ is the small-scale fading matrix, whose elements are assumed to be independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables (RVs) with zero mean and variance one i.e. $\mathcal{CN}(0,1)$. For this work, uplink payload data transmission is ignored and we focus

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entirely on the downlink transmission; therefore, the coherence interval consists of only uplink training and downlink payload data transmission. These phases of the coherence interval are detailed in the following sections.

A. Uplink Training

During the uplink training phase, all pilot sequences from their respective users are sent to the APs. Let τ be the length of training duration per coherence interval T, and $\boldsymbol{\Phi}_k$ be the pilot matrix from the kth user of size $\tau \times N$. We assume that all pilot sequences assigned for all user antennas are real and pairwisely orthogonal as in [7], i.e, $\boldsymbol{\Phi}_k^H \boldsymbol{\Phi}_k = \boldsymbol{\Phi}_k^T \boldsymbol{\Phi}_k = \mathbf{I}_N$, and $\boldsymbol{\Phi}_k^H \boldsymbol{\Phi}_{k'} = \boldsymbol{\Phi}_k^T \boldsymbol{\Phi}_{k'} = \mathbf{0}$ for $k \neq k'$. This requires $\tau \geq KN$. The received pilot signal at the mth AP is

$$\mathbf{Y}_m = \sum_{k=1}^{K} \sqrt{\tau \rho_p} \mathbf{G}_{mk} \boldsymbol{\Phi}_k^H + \mathbf{W}_m, \tag{2}$$

where ρ_p is the normalized signal-to-noise ratio of each pilot symbol and $\mathbf{W}_m \in \mathbb{C}^{L \times \tau}$ is the matrix of additive noise. The elements of $\mathbf{W}_m$ are assumed to be $\mathcal{CN}(0,1)$ i.i.d. RVs.

We first focus on the IQI at the APs because we want to study the fundamental limit of cell-free massive MIMO in terms of IQI. In addition, we use the asymmetrical IQI model.[1] The corresponding received pilot signal at the mth AP under AP IQI can be expressed as

$$\mathbf{Y}_{imb,m} = \mathbf{K}_1 \mathbf{Y}_m + \mathbf{K}_2 \mathbf{Y}_m^*, \tag{3}$$

where the received IQI coefficients of the mth AP, $\mathbf{K}_1$ and $\mathbf{K}_2$, are $L \times L$ diagonal matrices with diagonal elements $K_{1,l} = \frac{1}{2}\left(1 + g_l e^{-j\theta_l}\right)$ and $K_{2,l} = \frac{1}{2}\left(1 - g_l e^{j\theta_l}\right)$, where g_l and θ_l denote the amplitude and phase mismatch respectively at the lth AP antenna.

To estimate the channel $\mathbf{G}_{mk}$, the AP m first projects the received pilot sequence $\mathbf{Y}_{imb,m}$ onto $\boldsymbol{\Phi}_k$ as $\mathbf{Y}_{imb,mk} = \mathbf{Y}_{imb,m} \boldsymbol{\Phi}_k$. Then, it uses the MMSE estimation technique. The received pilot sequence at AP m after the projection is

$$\mathbf{Y}_{imb,mk} = \sqrt{\tau \rho_p} \mathbf{K}_1 \mathbf{G}_{mk} + \sqrt{\tau \rho_p} \mathbf{K}_2 \mathbf{G}_{mk}^* + \tilde{\mathbf{W}}_{mk}, \tag{4}$$

where $\tilde{\mathbf{W}}_{mk} = \mathbf{K}_1 \mathbf{W}_{mk} + \mathbf{K}_2 \mathbf{W}_{mk}^*$, with $\mathbf{W}_{mk} = \mathbf{W}_m \boldsymbol{\Phi}_k$. We now stack all columns of $\mathbf{Y}_{imb,mk}$ on top of each other with the vectorization operation $\mathrm{vec}(\cdot)$,

$$\begin{aligned} \mathrm{vec}\left(\mathbf{Y}_{imb,mk}\right) = {} & \sqrt{\tau \rho_p}\left(\mathbf{I}_N^T \otimes \mathbf{K}_1\right) \mathrm{vec}\left(\mathbf{G}_{mk}\right) \\ & + \sqrt{\tau \rho_p}\left(\mathbf{I}_N^T \otimes \mathbf{K}_2\right) \mathrm{vec}\left(\mathbf{G}_{mk}^*\right) + \mathrm{vec}\left(\tilde{\mathbf{W}}_{mk}\right). \end{aligned} \tag{5}$$

Denote by $\hat{\mathbf{G}}_{mk}$ the MMSE estimate of $\mathbf{G}_{mk}$ given $\mathbf{Y}_{imb,mk}$. Then, from [12], we obtain

$$\begin{aligned} \mathrm{vec}\left(\hat{\mathbf{G}}_{mk}\right) = {} & \left(\beta_{mk} \sqrt{\tau \rho_p} \bar{\mathbf{K}}_1^H\right) \\ & \times \left(\tau \rho_p \bar{\mathbf{K}}_1 \beta_{mk} \bar{\mathbf{K}}_1^H + \mathbf{C}_{\tilde{\mathbf{n}}_{mk}}\right)^{-1} \mathrm{vec}\left(\mathbf{Y}_{imb,mk}\right), \end{aligned} \tag{6}$$

where $\bar{\mathbf{K}}_1 = \mathbf{I}_N \otimes \mathbf{K}_1$, $\tilde{\mathbf{n}}_{mk} = \sqrt{\tau \rho_p}\left(\mathbf{I}_N^T \otimes \mathbf{K}_2\right) \mathrm{vec}\left(\mathbf{G}_{mk}^*\right) + \mathrm{vec}\left(\tilde{\mathbf{W}}_{mk}\right)$, and $\mathbf{C}_{\tilde{\mathbf{n}}_{mk}}$ is the covariance matrix of $\tilde{\mathbf{n}}_{mk}$.

[1] IQI can be modeled by a symmetric model as well [8]. This model can be easily obtained from the asymmetrical model by a linear transformation [6].

Using the derivation in Section VI-A, we obtain the MMSE estimate of the channel matrix $\mathbf{G}_{mk}$ as follows:

$$\hat{\mathbf{G}}_{mk} = \boldsymbol{\Omega}_{mk} \mathbf{Y}_{imb,mk}, \tag{7}$$

where $\boldsymbol{\Omega}_{mk}$ is an $L \times L$ diagonal matrix whose lth diagonal element is

$$\omega_{mkl} = \frac{\beta_{mk} \sqrt{\tau \rho_p} K_{1,l}^*}{\left(\tau \rho_p \beta_{mk} + 1\right)\left(\left|K_{1,l}\right|^2 + \left|K_{2,l}\right|^2\right)}. \tag{8}$$

We now have that the (l,n)th element of $\hat{\mathbf{G}}_{mk}$ has zero mean and variance γ_{mkl}, where

$$\gamma_{mkl} = \beta_{mk} \sqrt{\tau \rho_p} K_{1,l} \omega_{mkl}. \tag{9}$$

B. Downlink Payload Data Transmission

Once the BS has acquired the channel information, the channel estimates can be utilized by each AP to precode the intended symbols, before transmitting to each user. If IQI is not considered, the $L \times 1$ transmitted signal from AP m is

$$\mathbf{x}_m = \sqrt{\rho_d} \sum_{k=1}^{K} \mathbf{W}_{mk} \mathbf{s}_k, \tag{10}$$

where $\mathbf{s}_k$ is an $N \times 1$ vector of symbols intended for the kth user which satisfies $\mathbb{E}\left\{\mathbf{s}_k \mathbf{s}_k^H\right\} = \mathbf{I}_N$, ρ_d is the normalized transmit power of each data symbol, and $\mathbf{W}_{mk}$ is an $L \times N$ precoding matrix. $\mathbb{E}\{\cdot\}$ stands for the expectation. The power constraint at each AP can be expressed as

$$\mathbb{E}\left\{\left\|\mathbf{x}_m\right\|^2\right\} = \rho_d. \tag{11}$$

However, in this paper we consider the more practical model with IQI, hence, the IQI impaired transmitted signal is modeled as

$$\mathbf{x}_{imb,m} = \sqrt{\rho_d} \sum_{k=1}^{K} \mathbf{A}_1 \mathbf{W}_{mk} \mathbf{s}_k + \sqrt{\rho_d} \sum_{k=1}^{K} \mathbf{A}_2 \mathbf{W}_{mk}^* \mathbf{s}_k^*, \tag{12}$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ represent the transmit IQI. They both are diagonal matrices whose diagonal elements are $A_{1,l} = \frac{1}{2}\left(1 + f_l e^{-j\alpha_l}\right)$ and $A_{2,l} = \frac{1}{2}\left(1 - f_l e^{j\alpha_l}\right)$, where f_l and α_l denote the amplitude and phase mismatch respectively. The signal received at the kth user is

$$\mathbf{r}_k = \sum_{m=1}^{M} \mathbf{G}_{mk}^H \mathbf{x}_{imb,m} + \mathbf{n}_k, \tag{13}$$

where $\mathbf{n}_k$ is the noise vector at user k, assumed to have elements that are i.i.d. $\mathcal{CN}(0,1)$ RVs.

In this paper, we will focus the investigation on maximum-ratio (MR) processing. MR processing scheme has been widely used in cell-free massive MIMO research because it is computationally simple and can be implemented in a distributed manner [2]. With MR processing, the precoding matrix $\mathbf{W}_{mk}$ is given by

$$\mathbf{W}_{mk} = \frac{\hat{\mathbf{G}}_{mk}}{\sqrt{K\, \mathbb{E}\left\{\mathrm{tr}\left(\hat{\mathbf{G}}_{mk} \hat{\mathbf{G}}_{mk}^H\right)\right\}}} = \frac{\hat{\mathbf{G}}_{mk}}{\sqrt{KN\left(\sum_{l=1}^{L} \gamma_{mkl}\right)}}. \tag{14}$$

The normalization factor in (14) is chosen to satisfy the constraint (11).

III. Performance Analysis

A. Asymptotic Analysis

In this section, we provide some insights into the performance of cell-free massive MIMO under IQI, when M is very large, while K, L, and N are kept fixed. By using Tchebyshev's theorem [13], as $M \to \infty$, we have

$$\frac{1}{M}\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk} - \frac{1}{M}\mathbf{\Xi}_{1,mk} \xrightarrow{P} 0, \tag{15}$$

where $\xrightarrow{P}$ represents convergence in probability, where

$$\mathbf{\Xi}_{1,mk} = \sum_{m=1}^{M}\mathbb{E}\left\{\mathbf{G}_{mk}^H\mathbf{A}_1\frac{\hat{\mathbf{G}}_{mk}}{\sqrt{KN\sum_{l=1}^{L}\gamma_{mkl}}}\right\}. \tag{16}$$

Substituting (7) into (16) and using the fact that $\mathbb{E}\left\{\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{K}_2\mathbf{G}_{mk}^*\right\} = \mathbf{0}$, $\mathbb{E}\left\{\mathbf{G}_{mk}^H\mathbf{A}_1\tilde{\mathbf{W}}_{mk}^*\right\} = \mathbf{0}$, we obtain

$$\begin{aligned}\mathbf{\Xi}_{1,mk} &= \sum_{m=1}^{M}\mathbb{E}\left\{\mathbf{G}_{mk}^H\mathbf{A}_1\frac{\mathbf{\Omega}_{mk}\sqrt{\tau\rho_p}\mathbf{K}_1\mathbf{G}_{mk}}{\sqrt{KN\sum_{l=1}^{L}\gamma_{mkl}}}\right\}\\ &= \sum_{m=1}^{M}\frac{\sum_{l=1}^{L}\beta_{mk}A_{1,l}\omega_{mkl}K_{1,l}\sqrt{\tau\rho_p}}{\sqrt{KN\sum_{l=1}^{L}\gamma_{mkl}}}\\ &= \sum_{m=1}^{M}\frac{\sum_{l=1}^{L}A_{1,l}\gamma_{mkl}}{\sqrt{KN\sum_{l=1}^{L}\gamma_{mkl}}}\mathbf{I}_N,\end{aligned} \tag{17}$$

where in the last equality, we have used (9). Similarly, as $M \to \infty$, for $k \neq k'$, we have

$$\frac{1}{M}\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_2\mathbf{W}_{mk}^* - \frac{1}{M}\mathbf{\Xi}_{2,mk} \xrightarrow{P} \mathbf{0}, \tag{18}$$

$$\frac{1}{M}\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk'} \xrightarrow{P} \mathbf{0}, \tag{19}$$

$$\frac{1}{M}\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_2\mathbf{W}_{mk'}^* \xrightarrow{P} \mathbf{0}, \tag{20}$$

where

$$\mathbf{\Xi}_{2,mk} = \sum_{m=1}^{M}\frac{\sum_{l=1}^{L}A_{2,l}\gamma_{mkl}K_{2,l}^*/K_{1,l}^*}{\sqrt{KN\sum_{l=1}^{L}\gamma_{mkl}}}\mathbf{I}_N. \tag{21}$$

From (15), (18), (19), and (20), as $M \to \infty$, we obtain

$$\frac{\mathbf{r}_k}{M} - \frac{1}{M}\left(\sqrt{\rho_d}\mathbf{\Xi}_{1,mk}\mathbf{s}_k + \sqrt{\rho_d}\mathbf{\Xi}_{2,mk}\mathbf{s}_k^*\right) \xrightarrow{P} \mathbf{0}. \tag{22}$$

We can see from the above result that the received signal (normalized by M) includes only the desired signal plus interference due to the IQI. This implies that when $M \to \infty$, the inter-user interference and noise disappear. However, the effect of IQ imbalance persists.

B. Achievable SE

In this section, we derive the achievable SE for finite M. The signal received at the kth user (13) can be rewritten as

$$\begin{aligned}\mathbf{r}_k &= \sqrt{\rho_d}\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk}\mathbf{s}_k\\ &+ \sqrt{\rho_d}\sum_{k'\neq k}^{K}\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk'}\mathbf{s}_{k'}\\ &+ \sqrt{\rho_d}\sum_{m=1}^{M}\sum_{k'=1}^{K}\mathbf{G}_{mk}^H\mathbf{A}_2\mathbf{W}_{mk'}^*\mathbf{s}_{k'}^* + \mathbf{n}_k.\end{aligned} \tag{23}$$

By using (23) together with the fact that the users know only the statistical property of the channels, we obtain the following achievable SE.

Theorem 1: The achievable SE of the kth user in a cell-free massive MIMO system under IQI in bit/s/Hz is

$$\begin{aligned}R_k &= (T-\tau)/T\\ &\times \log_2\left|\mathbf{I}_N + \mathbf{C}_{s_k,r_k}\left(\mathbf{C}_{r_k,r_k} - \mathbf{C}_{r_k,s_k}\mathbf{C}_{s_k,r_k}\right)^{-1}\mathbf{C}_{r_k,s_k}\right|,\end{aligned} \tag{24}$$

where T is the coherence interval in symbols,

$$\mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k} = \sqrt{\rho_d}\,\mathbb{E}\left\{\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk}\right\}^H, \tag{25}$$

$$\begin{aligned}\mathbf{C}_{\mathbf{r}_k,\mathbf{r}_k} &= \rho_d\,\mathbb{E}\left\{\sum_{k'=1}^{K}\Upsilon_{kk',1}\Upsilon_{kk',1}^H\right\}\\ &+ \rho_d\,\mathbb{E}\left\{\sum_{k'=1}^{K}\Upsilon_{kk',2}\Upsilon_{kk',2}^H\right\} + \mathbf{I}_N,\end{aligned} \tag{26}$$

$$\mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k} = \mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}^H, \tag{27}$$

and $\Upsilon_{kk',1} \triangleq \sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk'}$, $\Upsilon_{kk',2} \triangleq \sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_2\mathbf{W}_{mk'}^*$.

Proof: See Appendix VI-B. □

IV. Numerical Results

In this section, numerical results are generated to investigate the performance of the cell-free massive MIMO under the IQ-impaired effect. This is performed through an analysis of the sum SE which is defined as $\mathrm{SE} = \sum_{k=1}^{K}R_k$. The uplink training duration is set equal to the number of data streams $\tau = KN$ while the coherence interval is $T = 200$. We choose $\beta_{mk} = 1$ for all m, k, $\rho_d = 10$ dB, and $\rho_p = 0$ dB. Additionally, the IQI coefficients at the AP antenna for both the uplink training and the downlink transmission phases are set as equal, i.e., $f_l = g_l$ and $\alpha_l = \theta_l$, for $l = 1, \ldots, L$.

Fig. 1 presents the impact that increased IQI has on the performance of cell-free massive MIMO with varying numbers of AP antennas. Since the phase mismatch is chosen as $\theta_l = \alpha_l = 0°$, the amplitude $g_l = 1$ corresponds to perfect IQ matching. As expected, when g_l reduces from 1 to 0.2, the sum SE reduces. The IQI shows significant effect on cell-free massive MIMO only when g_l is below 0.6. Additionally, the

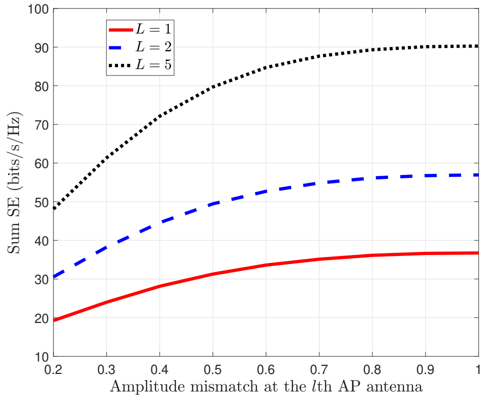

Fig. 1. Sum SE versus the amplitude mismatch with phase mismatch $\theta_l = \alpha_l = 0°$, at $N = 2$, $K = 20$ and $M = 50$.

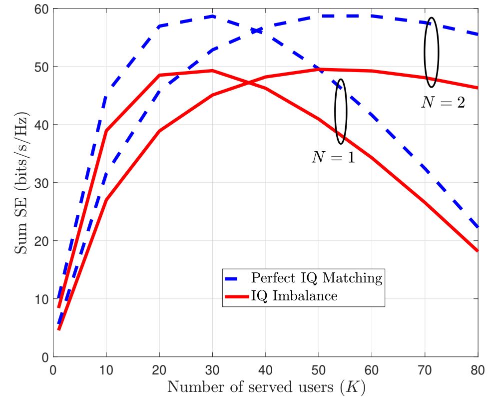

Fig. 3. Sum SE versus the number of served users K, for perfect and imperfect IQI, at $L = 2$ and $M = 50$.

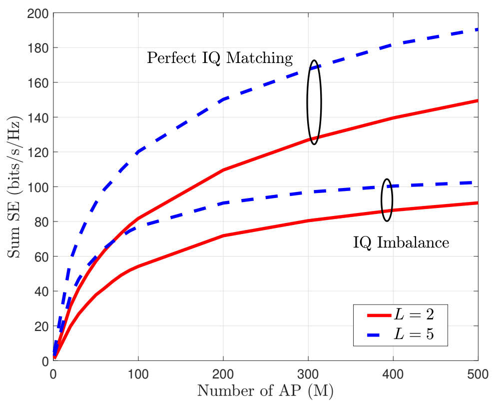

Fig. 2. Sum SE versus the number of AP M, for perfect and imperfect IQI, at $N = 2$ and $K = 20$.

performance gains from increasing the number of AP antennas can also be observed; these gains remain relatively consistent across the changing IQI values.

The impact that the number of APs have on both, a perfectly IQ matched and an IQ impaired system, is evaluated in Fig. 2. The system under IQI has parameters $g_l = 0.3$ and $\theta_l = 15°$. In Fig. 2, the performance gains from an increased number of APs are displayed across all operating conditions. The sum SE of a perfectly IQ matched system is seen to increase without bound, however, the IQ impaired system is shown to be severely limited despite the increase in the number of APs. Specifically, the sum SE of the IQ imbalanced system begins to converge towards a limit, therefore, the performance is bounded. This supports our theoretical analysis on the achievable SE in Section III-A, in particular, as $M \to \infty$ the IQI will remain.

Fig. 3 investigates the performance of an increased number of users for perfect and impaired IQ matching. The IQI, parameters are $g_l = \alpha_l = 0.5$ and $\theta_l = \alpha_l = 15°$ and the system setup was repeated for user antennas $N = 1$ and $N = 2$. The results show that the performance will reach a maximum point. This is because the channel estimation overhead KN will limit the gains produced by the number of users and user antennas. The relative loss of performance due to the IQI is shown to remain mostly constant across all the operating values.

V. Conclusion

We have investigated the impact of IQI in cell-free massive MIMO with multiple-antenna users. The achievable SE was derived taking into account the imperfect channel knowledge at the APs and no small-scale fading knowledge at the users. The results revealed the impact of IQ impaired hardware on the cell-free system to be mostly uniform, across a range of operating conditions. Using both analytical and numerical analysis, the presence of IQI was shown to remain as the number of APs increases large. More work on this topic is needed to develop an advanced IQI compensation technique to neutralize this issue.

VI. Appendix

A. Derivation of (7)

From the definition of $\tilde{\mathbf{N}}_{mk}$ in (6), we have

$$\begin{aligned}\mathbf{C}_{\tilde{\mathbf{n}}_{mk}} &= \mathbb{E}\left\{\tilde{\mathbf{n}}_{mk}\tilde{\mathbf{n}}_{mk}^H\right\} \\ &= \tau\rho_p\beta_{mk}\left(\mathbf{I}_N \otimes \left(\mathbf{K}_2\mathbf{K}_2^H\right)\right) + \mathbf{J}_{mk}, \end{aligned} \tag{28}$$

where

$$\begin{aligned}\mathbf{J}_{mk} &= \mathbb{E}\left\{\text{vec}\left(\tilde{\mathbf{W}}_{mk}\right)\left(\text{vec}\left(\tilde{\mathbf{W}}_{mk}\right)\right)^H\right\} \\ &= \mathbb{E}\left\{\text{vec}(\mathbf{K}_1\mathbf{W}_{mk}+\mathbf{K}_2\mathbf{W}_{mk}^*)(\text{vec}(\mathbf{K}_1\mathbf{W}_{mk}+\mathbf{K}_2\mathbf{W}_{mk}^*))^H\right\} \\ &= \mathbf{I}_N \otimes \left(\mathbf{K}_1\mathbf{K}_1^H\right) + \mathbf{I}_N \otimes \left(\mathbf{K}_2\mathbf{K}_2^H\right). \end{aligned} \tag{29}$$

The above derivation utilizes $\text{vec}\left(\mathbf{K}_1\mathbf{W}_{mk}\right) = \left(\mathbf{I}_N \otimes \mathbf{K}_1\right)\text{vec}\left(\mathbf{W}_{mk}\right)$ and $\text{vec}\left(\mathbf{K}_2\mathbf{W}_{mk}^*\right) = \left(\mathbf{I}_N \otimes \mathbf{K}_2\right)\text{vec}\left(\mathbf{W}_{mk}^*\right)$. Substituting (29) into (28), we obtain

$$\begin{aligned}\mathbf{C}_{\tilde{\mathbf{n}}_{mk}} &= \tau\rho_p\beta_{mk}\left(\mathbf{I}_N \otimes \left(\mathbf{K}_2\mathbf{K}_2^H\right)\right) \\ &\quad + \mathbf{I}_N \otimes \left(\mathbf{K}_1\mathbf{K}_1^H\right) + \mathbf{I}_N \otimes \left(\mathbf{K}_2\mathbf{K}_2^H\right). \end{aligned} \tag{30}$$

Substituting (30) into (6) obtains

$$\begin{aligned}
&\mathrm{vec}\left(\hat{\mathbf{G}}_{mk}\right) = \beta_{mk}\sqrt{\tau\rho_p}\left(\mathbf{I}_N \otimes \mathbf{K}_1^H\right) \\
&\quad \times \Big(\mathbf{I}_N \otimes \big(\tau\rho_p\beta_{mk}\mathbf{K}_1\mathbf{K}_1^H + \tau\rho_p\beta_{mk}\mathbf{K}_2\mathbf{K}_2^H \\
&\quad + \mathbf{K}_1\mathbf{K}_1^H + \mathbf{K}_2\mathbf{K}_2^H\big)\Big)^{-1} \mathrm{vec}\left(\mathbf{Y}_{imb,mk}\right) \\
&= \beta_{mk}\sqrt{\tau\rho_p}\mathbf{I}_N \otimes \Big[\mathbf{K}_1^H \\
&\quad \times \Big(\tau\rho_p\beta_{mk}\mathbf{K}_1\mathbf{K}_1^H + \tau\rho_p\beta_{mk}\mathbf{K}_2\mathbf{K}_2^H \\
&\quad + \mathbf{K}_1\mathbf{K}_1^H + \mathbf{K}_2\mathbf{K}_2^H\Big)^{-1}\Big] \mathrm{vec}\left(\mathbf{Y}_{imb,mk}\right). \qquad (31)
\end{aligned}$$

From (31) and the identity $\mathrm{vec}(\mathbf{ABC}) = \mathbf{C}^T \otimes \mathbf{A}\mathrm{vec}(\mathbf{B})$, (7) can be derived.

B. Proof of Theorem 1

To find an achievable SE of the kth user, we start with the mutual information of $\mathbf{r}_k$ and $\mathbf{s}_k$ as

$$\begin{aligned}
I\left(\mathbf{r}_k, \mathbf{s}_k\right) &= h\left(\mathbf{s}_k\right) - h\left(\mathbf{s}_k|\mathbf{r}_k\right) \\
&= \log_2\left|\pi e \mathbf{I}_N\right| - h\left(\mathbf{s}_k - \hat{\mathbf{s}}_k|\mathbf{r}_k\right) \qquad (32)
\end{aligned}$$

where $h(\cdot)$ is the differential entropy, $\hat{\mathbf{s}}_k$ is the linear MMSE estimate of $\mathbf{s}_k$ given $\mathbf{r}_k$. Since the condition reduces the entropy, (13) becomes

$$\begin{aligned}
I\left(\mathbf{r}_k, \mathbf{s}_k\right) &\geq \log_2\left|\pi e \mathbf{I}_N\right| - h\left(\mathbf{s}_k - \hat{\mathbf{s}}_k\right) \\
&\geq \log_2\left|\pi e \mathbf{I}_N\right| - \log_2\left|\pi e\, \mathbb{E}\left\{\left\|\mathbf{s}_k - \hat{\mathbf{s}}_k\right\|^2\right\}\right| \qquad (33)
\end{aligned}$$

where the last inequality follows the fact that the differential entropy of a RV is maximized when the RV is Gaussian with the same variance. In (33), $\mathbb{E}\left\{\left\|\mathbf{s}_k - \hat{\mathbf{s}}_k\right\|^2\right\}$ is the MSE of the linear MMSE which is given by [12, Eq. (12.8)]

$$\mathbb{E}\left\{\left\|\mathbf{s}_k - \hat{\mathbf{s}}_k\right\|^2\right\} = \mathbf{C}_{\mathbf{s}_k,\mathbf{s}_k} - \mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}\mathbf{C}_{\mathbf{r}_k,\mathbf{r}_k}^{-1}\mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k}, \qquad (34)$$

where

$$\mathbf{C}_{\mathbf{s}_k,\mathbf{s}_k} = \mathbb{E}\left\{\mathbf{s}_k\mathbf{s}_k^H\right\} = \mathbf{I}_N, \qquad (35)$$

$$\mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k} = \mathbb{E}\left\{\mathbf{s}_k\mathbf{r}_k^H\right\} = \sqrt{\rho_d}\,\mathbb{E}\left\{\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk}\right\}^H, \qquad (36)$$

$$\begin{aligned}
&\mathbf{C}_{\mathbf{r}_k,\mathbf{r}_k} = \mathbb{E}\left\{\mathbf{r}_k\mathbf{r}_k^H\right\} \\
&= \rho_d\mathbb{E}\left\{\sum_{k'=1}^{K}\left(\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk'}\right)\left(\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk'}\right)^H\right\} \\
&+\rho_d\,\mathbb{E}\left\{\sum_{k'=1}^{K}\left(\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_2\mathbf{W}_{mk'}^*\right)\left(\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_2\mathbf{W}_{mk'}^*\right)^H\right\} \\
&+\mathbf{I}_N, \qquad (37)
\end{aligned}$$

and

$$\mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k} = \mathbb{E}\left\{\mathbf{r}_k\mathbf{s}_k^H\right\} = \sqrt{\rho_d}\,\mathbb{E}\left\{\sum_{m=1}^{M}\mathbf{G}_{mk}^H\mathbf{A}_1\mathbf{W}_{mk}\right\}. \qquad (38)$$

Substituting (34) into (33), we obtain

$$\begin{aligned}
I\left(\mathbf{r}_k, \mathbf{s}_k\right) &\geq \log_2\left|\pi e \mathbf{I}_N\right| \\
&\quad - \log_2\left|\pi e\left(\mathbf{C}_{\mathbf{s}_k,\mathbf{s}_k} - \mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}\mathbf{C}_{\mathbf{r}_k,\mathbf{r}_k}^{-1}\mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k}\right)\right| \\
&\geq \log_2\left(\frac{1}{\left|\mathbf{C}_{\mathbf{s}_k,\mathbf{s}_k} - \mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}\mathbf{C}_{\mathbf{r}_k,\mathbf{r}_k}^{-1}\mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k}\right|}\right) \\
&\geq \log_2\left|\left(\mathbf{I}_N - \mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}\mathbf{C}_{\mathbf{r}_k,\mathbf{r}_k}^{-1}\mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k}\right)^{-1}\right|. \qquad (39)
\end{aligned}$$

By using the matrix inversion lemma, we obtain

$$\begin{aligned}
&\left(\mathbf{I}_N - \mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}\mathbf{C}_{\mathbf{r}_k,\mathbf{r}_k}^{-1}\mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k}\right)^{-1} \\
&= \mathbf{I}_N + \mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}\left(\mathbf{C}_{\mathbf{r}_k,\mathbf{r}_k} - \mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k}\mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}\right)^{-1}\mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k}. \qquad (40)
\end{aligned}$$

Therefore,

$$\begin{aligned}
&I\left(\mathbf{r}_k, \mathbf{s}_k\right) \\
&\geq \log_2\left|\left(\mathbf{I}_N + \mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}\left(\mathbf{C}_{\mathbf{r}_k,\mathbf{r}_k} - \mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k}\mathbf{C}_{\mathbf{s}_k,\mathbf{r}_k}\right)^{-1}\mathbf{C}_{\mathbf{r}_k,\mathbf{s}_k}\right)^{-1}\right|. \qquad (41)
\end{aligned}$$

From (41), we can arrive at the desired result in Theorem 1.

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