Amplitude Detection Over Generalized Fading Models: An Asymptotic Approach

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***Abstract*—Amplitude detection (AD) is a low-complexity scheme for information retrieval, as it only considers the envelop of the received signal. This makes it ideal for low-cost low-powered devices for applications such as the Internet of Things. In this paper, we deal with a basic point-to-point (P2P) system and develop a unified analytical framework for the error performance evaluation of the maximum-likelihood decoder with on-off keying. We consider generalized fading channels and derive closed form asymptotic bounds on the average bit error rate performance. Moreover, we provide the exact diversity order along with a lower and upper bound on the coding gain in terms of the channel parameters. Our results provide important insights into the effects of the fading parameters on the performance of P2P-AD systems.**

***Index Terms*—Nonlinear detection, amplitude detection, maximum-likelihood, diversity, coding gain.**

I. Introduction

Future Massive Internet of Things (m-IoT) envisages a rapid increase in connectivity among a large number of low-cost/low-power devices and sensors [1]. Therefore, designing low-complexity receivers for IoT-enabled wireless networks is crucial. Conventional communication receivers employ linear detection, which requires expensive and power consuming circuitry, such as linear amplifiers and frequency synthesizers [2]. On the other hand, nonlinear detection techniques are of low complexity since they process only the amplitude/phase of the received signal [3]. Despite, this approach affects the performance and loses degrees of freedom, it is more energy and cost efficient and hence ideal for m-IoT applications.

A simple and well known nonlinear detection technique is *amplitude detection* (AD), which only processes the envelop of the received signal [3], [4]. Since AD can only differentiate between symbols with different magnitudes, a suitable modulation scheme is on-off keying (OOK). OOK has increasingly being used in energy-constrained wireless applications such as sensor networks, where efficient design is required to minimize the power consumption and extend the operation lifetime [5]. Furthermore, due to its low-complexity and energy efficiency, OOK is also suitable for wireless energy transfer [6], backscatter communications [7], and optical wireless communications [8]. In [9], a closed form expression for the asymptotic error probability of OOK modulation with envelop detection over additive white Gaussian noise (AWGN) channels is derived. It has been shown that the error probability associated with each OOK symbol contributes equally to the total error probability. In [10], the ratio of asymptotic probability of error for the two symbols is derived over Rician channels in a tabular form. Recently, G. K. Psaltopoulos *et. al.* [11]–[14] studied various fundamental aspects of AD with OOK such as asymptotic error performance, achievable rate and diversity-multiplexing trade-off, for a basic multiple-input multiple-output (MIMO) set-up. Contrary to the widely used conventional non-coherent OOK [4], [10], they propose a coherent OOK system, where channel knowledge is available at the receiver. In addition to the proposed theoretical framework, they have implemented a practical MIMO setup and validated the error performance for OOK modulation over Rayleigh channels.

However, to the best of our knowledge, there is no unified framework that provides simple and tractable bounds on the error performance for the OOK modulation with AD over generalized fading channels. To this end, we consider a point-to-point (P2P) system that employs AD at the receiver and we study the average bit error rate (BER) performance of the maximum likelihood (ML) decoder over various fading models with OOK. We refer to this set-up as P2P-AD system in the rest of this paper.

Specifically, the main contributions presented in this paper are as follows:

- We develop a complete analytical framework for the BER of a P2P-AD system, which takes into account generalized fading models: Rayleigh, Nakagami-n, Nakagami-q (Hoyt), and Nakagami-m.
- We derive asymptotic closed form expressions for the lower and upper bounds on the BER performance for all considered fading models.
- Based on the asymptotic expressions, we derive the exact diversity order along with a lower and upper bound on the coding gain, in terms of the fading parameters.

To the best of authors' knowledge, no previous work has studied the framework discussed in this paper to analyze diversity order and coding gain in the context of nonlinear AD receivers. The developed framework provides a way to acquire important insights into the effects of the fading parameters on the performance of such systems.

The remainder of this paper is organized as follows. In Section II, the system model along with the ML decoder for a P2P-AD system is presented. Section III deals with

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a mathematical framework to study the BER performance for the P2P-AD system with OOK modulation over various fading models. Numerical results are shown and discussed in Section IV, followed by conclusive remarks in Section V.

Notations: $\mathbb{R}^+$ and $\mathbb{C}$ denote the set of positive real numbers and complex numbers, respectively. $\mathcal{CN}$ denote the complex Gaussian random variable. $\Re\{\cdot\}$ and $\Im\{\cdot\}$ represent real part and imaginary part, respectively. $\mathbb{E}[\cdot]$ and $|\cdot|$ stand for the expectation operation and the amplitude function, respectively.

II. System Model

We consider a P2P communication system over a flat-fading channel that employs AD at the receiver. Let $x_s \in \mathbb{R}^+$ be the transmitted symbol from the source, $h \in \mathbb{C}$ be the channel realization, and n be the complex AWGN at the receiver with mean zero and variance σ^2, i.e., $n \sim \mathcal{CN}(0, \sigma^2)$. Then, the received signal can be obtained as

$$y_D = \big|\, h\, x_s + n \,\big|, \tag{1}$$

where $|z| \triangleq \sqrt{\Re(z)^2 + \Im(z)^2}$. An ML symbol-by-symbol decoder is used for detecting the received signal and we assume full channel state information at the receiver [14].

A. ML Decoder

If the symbol x_s is drawn from an equiprobable modulation alphabet set $\mathscr{X}$, then for a given channel realization, the ML decision rule is given by

$$\hat{x}_s = \underset{x_s \in \mathscr{X}}{\arg\max}\; f(y_D | x_s, h), \tag{2}$$

where $f(y_D|x_s, h)$ is the likelihood of the received symbol. For the received signal given in (1), the likelihood function is written as [4, Eqn. (1.4.26)]

$$f(y_D|x_s, h) = \frac{2y_D}{\sigma^2} \exp\left(-\frac{y_D^2 + |x_s\, h|^2}{\sigma^2}\right) I_0\left(\frac{2y_D\, |x_s\, h|}{\sigma^2}\right), \tag{3}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind and zeroth order. The likelihood function depends only on the amplitude of the transmitted symbol (not on the phase) and therefore the ML can only differentiate between symbols with different amplitudes.

B. Asymptotic Analytical Framework

The conditional error probability, i.e., probability of error for a given fading/channel realization of a communication system, depends on the average transmit signal-to-noise ratio (SNR) γ and the random variable β that represents the fading power. If for a given SNR, the conditional error probability of a communication system is represented by $P_e(\beta)$, then the average bit error rate can be evaluated as

$$\bar{P}_e \triangleq \mathbb{E}[P_e(\beta)] = \int_0^\infty P_e(\beta) f_\beta(\beta) d\beta, \tag{4}$$

where $f_\beta(\beta)$ is the PDF of the random variable β. The average BER is dominated by the near-origin behaviour of $f_\beta(\beta)$ at high SNRs (for details see [15]). For most of the channel models, $f_\beta(\beta)$ can be approximated by a single polynomial term as $f_\beta(\beta) = a\beta^t + o(\beta^{t+\epsilon})$ near the origin, where ϵ and a are positive constants. The parameter t quantifies the order of the smoothness of $f_\beta(\beta)$ near the origin. Thus, the asymptotic BER can be obtained as [15]

$$\bar{P}_{e,\infty} \triangleq \int_0^\infty P_e(\beta)\; a\beta^t d\beta, \tag{5}$$

in terms of the near-origin parameters of $f_\beta(\beta)$. We refer to the parameters (t, a) as the *near-origin parameters* of $f_\beta(\beta)$. The parameters (t, a) corresponding to various channel models are provided in [15, Table 1].

Asymptotically, the BER can be approximately written as

$$\bar{P}_{e,\infty} \approx (g\gamma)^{-d} \tag{6}$$

for linear modulations, where g and d are the coding gain and the diversity order, respectively [16]. Thus, by taking into account the expression in (6), the diversity order is obtained as $d = \lim_{\gamma\to\infty} -\frac{\log \bar{P}_{e,\infty}}{\log \gamma}$. The diversity order determines the slope of the BER versus the SNR curve (in a log-log scale), at high SNRs. On the other hand, the coding gain (in decibels) determines the shift of the BER curve in SNR relative to a benchmark BER curve of γ^{-d}.

The above analytical framework is used to study the asymptotic BER performance for the P2P-AD system considered, and to draw insights into the effect of the parameters (t, a) on the coding gain and the diversity order.

III. Error Performance Analysis

In this section, we first analytically characterize the BER performance for the P2P-AD system described in Section II for OOK modulations and then derive simple and tractable asymptotic BER performance bounds. In order to obtain the asymptotic upper bound, we first derive a novel exponential type upper bound on the conditional error probability. We deliberately derive an exponential type upper bound since it satisfies the assumptions used in [15, Proposition 1] and hence asymptotic analysis (see (4)) could be invoked. [1]

A. BER Performance

For a basic OOK scheme, the modulation set is given by $\mathscr{X} = \{0, A\}$, and the transmit SNR can be written as $\gamma \triangleq \frac{A^2}{2\sigma^2}$. Let λ be the optimal detection threshold of the ML decoder that corresponds to the solution of

$$f(y_D|0, h) = f(y_D|A, h), \tag{7}$$

where $f(y_D|\cdot,\cdot)$ is given by (3); then by using (3) in (7), we can write

$$I_0\left(\frac{2\lambda\, |A\, h|}{\sigma^2}\right) = \exp\left(\frac{|A\, h|^2}{\sigma^2}\right). \tag{8}$$

[1] In order to derive the asymptotic lower bound, Q-type lower bound on the conditional error probability is used. The lower bound on the conditional error probability is based on the analysis in [11]. Further, the Q-type error functions satisfies the assumptions given in [15] and hence the asymptotic analysis is feasible.

The exact solution for the equation in (8) can only be evaluated numerically; however, a tight approximation is given in [3, Eqn. (4.15)]

$$\lambda \approx \frac{\sigma}{\sqrt{2}}\sqrt{2+\frac{A^2|h|^2}{2\,\sigma^2}} = \frac{\sigma}{\sqrt{2}}\sqrt{2+\gamma\,|h|^2}. \tag{9}$$

It is noted that λ is a function of γ and the closed form expression for λ at high SNRs ($\gamma \to \infty$) can be obtained as

$$\lambda_\infty = \lim_{\gamma\to\infty} \frac{\sigma}{\sqrt{2}}\sqrt{2+\gamma\,|h|^2} \approx \frac{\sqrt{\gamma}\sigma}{\sqrt{2}}|h| = \frac{A|h|}{2}. \tag{10}$$

From (9) and (10), it follows that $\lambda_\infty \le \lambda$.

Next, for the ML decoder defined in (2), the conditional error probability of the OOK modulation can be written as

$$\begin{aligned} P_e(\beta) &= \frac{1}{2}\left(\mathrm{P}_{\mathrm{e}|0} + \mathrm{P}_{\mathrm{e}|A}\right) \\ &= \frac{1}{2}\Big[\int_\lambda^\infty f(y_D|0,h)\ dy_D + \int_0^\lambda f(y_D|A,h)\ dy_D\Big] \\ &\overset{(a)}{=} \frac{1}{2}\left[\exp\left(-\frac{\lambda^2}{\sigma^2}\right) + 1 - Q_1\left(\frac{\sqrt{2}A|h|}{\sigma}, \frac{\sqrt{2}\lambda}{\sigma}\right)\right], \end{aligned} \tag{11}$$

where $\mathrm{P}_{\mathrm{e|X}}$ is the probability of the error event, when X was transmitted, $Q_1(\cdot,\cdot)$ is the first order Marcum Q-function, and (a) follows from (3) along with the fact that $Q_1\left(0, \frac{\sqrt{2}\lambda}{\sigma}\right) = \exp\left(-\frac{\lambda^2}{\sigma^2}\right)$ from [17, Eqn. 4.22]. Note that due to the Marcum Q-function, it is not straightforward to evaluate the exact average BER by averaging (11) over the random variable β. However, in what follows, we derive simple and tractable bounds on the BER performance.

B. Bounds and Asymptotic Approximations

In what follows, we provide bounds (Proposition 1) on the conditional error probability and then approximations for high SNRs (Lemma 1) on the average error performance.

Proposition 1. *The conditional error probability for the OOK modulation can be bounded as*

$$P_e^{\mathrm{lb}}(\beta) \le P_e(\beta) \le P_e^{\mathrm{ub}}(\beta), \quad \text{where} \tag{12}$$

$$P_e^{\mathrm{ub}}(\beta) = \frac{3}{4}\exp\left(-\frac{\gamma}{2}\,\beta\right) - \frac{1}{4}\exp\left(-\frac{9\gamma}{2}\,\beta\right), \tag{13}$$

$$P_e^{\mathrm{lb}}(\beta) = Q\left(\sqrt{\beta\,\gamma}\right). \tag{14}$$

Proof. See Appendix A. ∎

Next, by using (5), we provide asymptotic bounds on the average BER performance for the OOK modulation in terms of the near-origin parameters (t, a) of the PDF $f_\beta(\beta)$.

Lemma 1. *For a given fading* PDF $f_\beta(\beta)$ *with the near-origin parameters* (t, a)*, the asymptotic average BER performance for the OOK modulation can be bounded as*

$$\bar{P}_{e,\infty}^{\mathrm{lb}} \le \bar{P}_{e,\infty} \le \bar{P}_{e,\infty}^{\mathrm{ub}}, \quad \text{where} \tag{15}$$

$$\bar{P}_{e,\infty}^{\mathrm{ub}} = a\Gamma(t+1)2^{t-1}\left(3-\left(\frac{1}{9}\right)^{t+1}\right)\gamma^{-(t+1)}, \tag{16}$$

$$\bar{P}_{e,\infty}^{\mathrm{lb}} = \frac{2^t a\Gamma\left(t+\frac{3}{2}\right)}{\sqrt{\pi}(t+1)}\gamma^{-(t+1)}. \tag{17}$$

Proof. See Appendix B.[2] ∎

Thus, from Lemma 1, the diversity order corresponding to the upper bound (16) and the lower bound (17), is given by

$$d^{\mathrm{u}} = t+1, \text{and} \quad \mathrm{d}^{\mathrm{l}} = \mathrm{t}+1,$$

respectively. Similarly, the coding gains corresponding to the two bounds are given by

$$\begin{aligned} g^{\mathrm{u}} &= \left(a\Gamma(t+1)2^{t-1}\left(3-\left(\frac{1}{9}\right)^{(t+1)}\right)\right)^{\frac{-1}{t+1}}, \\ g^{\mathrm{l}} &= \left(\frac{2^t a\Gamma\left(t+\frac{3}{2}\right)}{\sqrt{\pi}(t+1)}\right)^{\frac{-1}{t+1}}. \end{aligned} \tag{18}$$

We now obtain the exact diversity order and a closed interval for the coding gain of the OOK modulation.

Theorem 1. *The diversity order and the coding gain of the OOK modulation are given, respectively as*

$$d = t+1, \text{and} \quad \mathrm{g} \in [\mathrm{g}^{\mathrm{u}},\ \mathrm{g}^{\mathrm{l}}], \tag{19}$$

where g^{u} *and* g^{l} *are given by* (18).

Proof. See Appendix C. ∎

The developed mathematical approach is helpful in deriving the exact diversity order and a closed interval for the coding gain for all of the considered fading models which are summarized in [15, Table 1]. For instance, for the case of Rayleigh fading with unity gain, i.e., with PDF $f_\beta(\beta) = \exp(-\beta)$, by using $t = 0$ and $a = 1$ in Theorem 1, we get the diversity order and the coding gains as $d = 1, \quad g \in [0.6923, 2]$.

IV. Numerical Results

Computer simulations are carried out in order to validate the proposed analytical framework. A P2P-AD system with OOK modulation is simulated over Nakagami-m and Nakagami-n channel models. In all simulations, we assume a normalized channel with $\sigma^2 = 1$.

In Fig. 1, we present the BER performance as a function of SNR over Nakagami-m channels with $m = \{0.5, 1, 2\}$. From Fig. 1, we can observe that the upper and lower bounds are parallel along the exact BER simulation curve. We also observe that the asymptotic behaviour of the bounds is captured by the asymptotic upper bound in (16) and the asymptotic lower bound in (17) (e.g., for $m = 1$, the blue

[2] We note that, the upper bound (see (13)) has exponential form. In addition to using (13) for the asymptotic scenario, we can also obtain corresponding average BER (which is valid for all SNRs) in terms of the moment generating function (MGF) corresponding to the fading type, i.e., for a given MGF $\mathcal{M}_\beta(\gamma) = \int_0^\infty f_\beta(\beta)\exp(-\beta\gamma)d\beta$, the upper bound on the BER performance, i.e., $\bar{P}_e \le \mathbb{E}[P_e^{\mathrm{ub}}(\beta)] = \frac{3}{4}\mathcal{M}_\beta\left(\frac{1}{4}\gamma\right) - \frac{1}{4}\mathcal{M}_\beta\left(\frac{9}{4}\gamma\right)$.

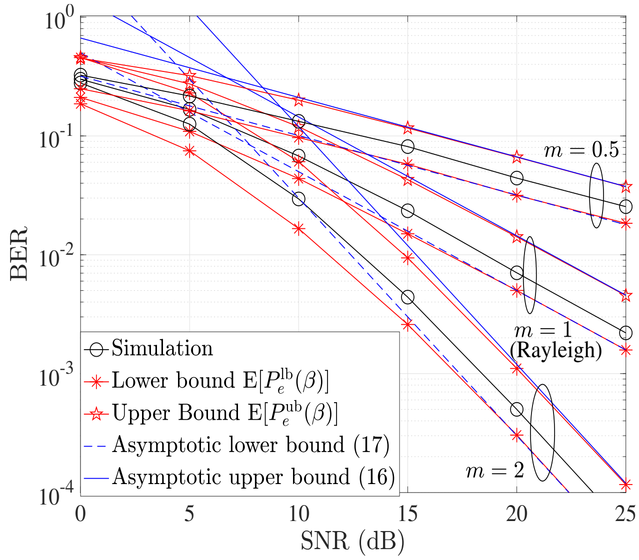

Fig. 1: BER vs SNR for Nakagami-m.

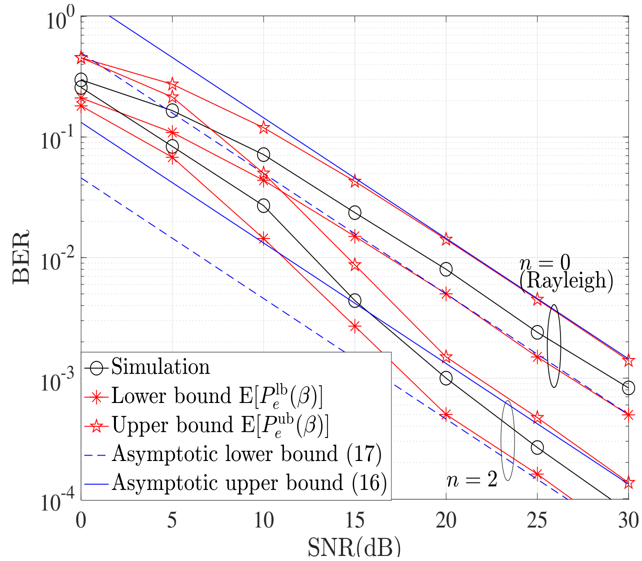

Fig. 2: BER vs SNR for Nakagami-n.

solid line perfectly matches with the solid red line with "$\star$" after $\text{SNR} > 15$ dB). The slope of both the asymptotic bounds (straight lines) is the same, that correctly provides the diversity of the OOK modulation (e.g., $d = 1$ for $m = 1$). Further, the simulated exact BER curve lies in between the two straight lines having the same slope; and therefore the coding gain of the OOK scheme lies in between the coding gains associated with the two asymptotic bounds. We also observe that the asymptotic behaviour of the bounds shows up at relatively high SNRs for larger values of m. The slope of the BER curves/diversity order depends on the value of m and increases as m increases.

Similarly, in Fig. 2 we show the BER performance as a function of SNR over Nakagami-n with $n = \{0, 2\}$. Note that the Nakagami-n channel is the same as the Rician-K channel with the scale factor $K = n^2$. All the key observations made for Nakagami-m channel also hold for the Nakagami-n channels. However, a distingushing remark is that the slope of the BER curves is independent of n, whereas for Nakagami-m channels the slope depends on m. However, the BER curve shifts towards the left as n increases, and therefore the coding gain increases. Note that this observation is also in line with the analytical results derived, as both the upper and the lower bounds on the coding gain given by (18) (with $t = 0$ and $a = (1 + n^2)\exp(-n^2)$), respectively, are increasing functions of n.

V. Conclusions

In this paper, we developed a mathematical framework to study bounds on the BER performance for P2P-AD systems with OOK modulation and also derived the asymptotic behaviour of these bounds as a function of the channel parameters. The BER can be characterized by the diversity order and the coding gain at high SNRs. The developed framework is general and can be used to study the asymptotic performance (i.e., diversity order and coding gain) for a large class of fading channels. Theoretical results are validated via Monte Carlo simulations. We have shown that the asymptotic approximation for each bound perfectly matches with their corresponding bounds at high SNRs.

Appendix A
Proof of Proposition 1

We derive upper and lower bounds on the conditional error probability. We first obtain an upper bound on the conditional error probability by using the fact that $\lambda_\infty < \lambda$ (which provide an upper bound), and then by further using an upper bound on the $Q_1(\cdot,\cdot)$ function. Finally, we present the lower bound based on the analysis in [11].

Upper bound: The conditional error is given by

$$
\begin{aligned}
P_e(\beta) &= \frac{1}{2}\left[\int_{\lambda}^{\infty} f(y_D|0,h)\; dy_D + \int_{0}^{\lambda} f(y_D|A,h)\; dy_D\right] \\
&\overset{(a)}{=} \frac{1}{2}\left[\int_{\lambda_\infty + c(\gamma)}^{\infty} f(y_D|0,h)\; dy_D + \int_{0}^{\lambda_\infty + c(\gamma)} f(y_D|A,h)\; dy_D\right] \\
&= \frac{1}{2}\Bigg[\int_{\lambda_\infty}^{\infty} f(y_D|0,h)\; dy_D + \int_{0}^{\lambda_\infty} f(y_D|A,h)\; dy_D \\
&\qquad - \underbrace{\int_{\lambda_\infty}^{\lambda_\infty + c(\gamma)} f(y_D|0,h) - f(y_D|A,h)\; dy_D}_{\geq 0}\Bigg] \\
&\overset{(b)}{\leq} \frac{1}{2}\left[\int_{\lambda_\infty}^{\infty} f(y_D|0,h)\; dy_D + \int_{0}^{\lambda_\infty} f(y_D|A,h)\; dy_D\right]. \qquad (20)
\end{aligned}
$$

In order to get (a), we use $\lambda_\infty \leq \lambda\ \forall\gamma$ and therefore for a given γ we can write $\lambda = \lambda_\infty + c(\gamma)$, where $c(\gamma)$ is a non-negative function of γ; (b) follows from the fact that $f(y_D|0,h) \geq f(y_D|A,h)$, $y_D \leq \lambda$ and equality holds when

$y_D = \lambda$. Thus, by using expressions for $f(y_D|0,h)$ and $f(y_D|A,h)$ in (20), we have

$$P_e(\beta) \leq \frac{1}{2}\left(\exp\left(-\frac{\lambda_\infty^2}{\sigma^2}\right) + 1 - Q_1\left(\frac{\sqrt{2}}{\sigma}A|h|, \frac{\sqrt{2}\lambda_\infty}{\sigma}\right)\right).$$

Next, by using (10), we substitute $\lambda_\infty = \frac{A|h|}{2}$ in the above equation, and we have

$$\begin{aligned} P_e(\beta) &\leq \frac{1}{2}\left(\exp\left(-\frac{A^2|h|^2}{4\sigma^2}\right) + 1 - Q_1\left(\frac{\sqrt{2}}{\sigma}A|h|, \frac{A|h|}{\sqrt{2}\sigma}\right)\right) \\ &\overset{(a)}{\leq} \frac{1}{2}\left(\exp\left(-\frac{A^2|h|^2}{4\sigma^2}\right) + \frac{1}{2}\left[\exp\left(-\frac{A^2|h|^2}{4\sigma^2}\right)\right.\right. \\ &\qquad\qquad \left.\left. + \exp\left(-\frac{9A^2|h|^2}{4\sigma^2}\right)\right]\right) \\ &\overset{(b)}{=} \frac{3}{4}\exp\left(-\frac{\beta\gamma}{2}\right) - \frac{1}{4}\exp\left(-\frac{9\beta\gamma}{2}\right), \end{aligned} \tag{21}$$

where (a) follows from the inequality [17, Eqn. (4)]

$$Q_1(p,q) = 1 - \frac{1}{2}\left(\exp\left(-\frac{(p-q)^2}{2}\right) - \exp\left(-\frac{(p+q)^2}{2}\right)\right),$$

where $p > q \geq 0$, with $p = \frac{\sqrt{2}}{\sigma}A|h|$ and $q = \frac{A\,|h|}{\sqrt{2}\,\sigma}$. Then, by using $\beta = |h|^2$ and $\gamma = \frac{A^2}{2\sigma^2}$ in (a) we get (b).

Lower bound: The ML error performance of the system with output as $r \triangleq |hx_s| + n$ provides a lower bound on the error performance of the P2P-AD system under study, i.e., $y_D = |hx_s + n|$ [11]. The error performance of the system model $r = |hx_s| + n$ is well studied (this is a conventional AWGN channel for a given channel realization); and the corresponding conditional error probability with OOK scheme is given by $Q\sqrt{\frac{|hA|^2}{2\sigma^2}}$. Thus, we can write

$$Q(\sqrt{\beta\gamma}) \leq P_e(\beta), \tag{22}$$

which corresponds to a lower bound on $P_e(\beta)$.

Thus, from (21) and (22), we obtain an upper and lower bounds on the conditional error probability. This completes the proof.

Appendix B
Proof of Lemma 1

In order to prove Lemma 1, we derive the asymptotic upper and lower bounds by averaging the corresponding conditional error functions over the random variable β. We utilize the fact that the BER depends only on the the near-origin parameters (t, a) of $f_\beta(\beta)$ at high SNRs.

Asymptotic upper bound: With $P_e^{\text{ub}}(\beta) = \frac{3}{4}\exp\left(-\frac{\gamma\beta}{2}\right) - \frac{1}{4}\exp\left(-\frac{9\gamma\beta}{2}\right)$, and $f_\beta(\beta) \approx a\beta^t$ near the origin, the asymptotic upper bound is given by $\bar{P}_{e,\infty}^{\text{ub}} = \int_0^\infty P_e^{\text{ub}}(\beta) \times a\beta^t d\beta$, which after integration simplifies to

$$\bar{P}_{e,\infty}^{\text{ub}} = a\Gamma(t+1)2^{t-1}\left(3 - \left(\frac{1}{9}\right)^{t+1}\right)\gamma^{-(t+1)}, \tag{23}$$

where $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx$.

Asymptotic lower bound: From [15, Proposition 1], we have

$$\lim_{\gamma\to\infty} \mathbb{E}\left[Q\left(\sqrt{k\beta\,\gamma}\right)\right] = \frac{2^t a\Gamma\left(t+\frac{3}{2}\right)}{\sqrt{\pi}(t+1)}(k\gamma)^{-(t+1)}. \tag{24}$$

Thus, the expression for $\bar{P}_{e,\infty}^{\text{lb}} = \lim_{\gamma\to\infty} \mathbb{E}\left[Q(\sqrt{\beta\,\gamma})\right]$ is obtained by replacing k with 1 in (24). Hence, we have $\bar{P}_{e,\infty}^{\text{lb}} = \frac{2^t a\Gamma\left(t+\frac{3}{2}\right)}{\sqrt{\pi}(t+1)}\gamma^{-(t+1)}$, and the proof is completed.

Appendix C
Proof of Theorem 1

Asymptotically, the BER performance for the OOK modulation is bounded as $\bar{P}_{e,\infty}^{\text{lb}} \leq \bar{P}_{e,\infty} \leq \bar{P}_{e,\infty}^{\text{ub}}$. By substituting each asymptotic BER expression with its corresponding expression in terms of coding gain and diversity order, we get $(g^{\text{l}}\gamma)^{-d^{\text{l}}} \leq (g\gamma)^{-d} \leq (g^{\text{u}}\gamma)^{-d^{\text{u}}}$. It is simple to deduce that $d^{\text{u}} \leq d \leq d^{\text{l}}$. And if $d^{\text{u}} = d^{\text{l}} = d$, then $g^{\text{u}} \leq g \leq g^{\text{l}}$. This completes the proof.

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