A FAST RECURSIVE ALGORITHM FOR MULTIPLE BRIDGED KNIFE-EDGE DIFFRACTION

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ABSTRACT

Multiple bridged knife-edge diffraction estimation can be seen as a generalization of the multiple knife-edge diffraction one which can be found in many applications of wireless communications. The considered model is formed by bridging the spaces among knife-edges with reflecting planes. So far, the series-based standard solution for this problem suffers from high computational complexity, thus limiting its use in practice. We, thus, propose a fast recursive algorithm to tackle its computational burden. To illustrate the effectiveness of the proposed algorithm, we compare our results with the state-of-the-art algorithms. Numerical results show that the running time of the proposed algorithm is much faster than that of the standard solution while benefiting from similar accuracy.

Index Terms— Multiple bridged knife-edges diffraction, reflection coefficients, irregular terrain, channel modeling, recursive algorithm.

1. INTRODUCTION

We consider the problem of computing multiple bridged knife edge diffraction (MBKED) that aims to estimate the diffraction attenuation of radio waves over a terrain of interest. This problem, introduced in [1], is of great importance and can be used in various applications, for example, radio-wave propagation modeling over point-to-point terrestrial paths in radar [2,3] or wireless sensor network [4], physical model for wireless channel [5], area coverage and its applications [6], millimeter wave propagation for 5G communications [7], to name a few.

The key advantage of using multiple bridged knife edge model is that it covers many useful other models used for a specific terrain. Generally, the irregular terrain can be approximated by several important geometric forms such as single knife-edge (SKE), multiple knife-edges [5] (MKE) (e.g., used for modeling mountains), flat-topped obstacles with slanted sides [8] (e.g. used for modeling buildings in a city), and the single rounded obstacle [3] (e.g. used for modeling hills). In this sens, those geometric forms are special cases of MBKE.

Related works: For multiple knife edge diffraction (MKED), this topic has a long history and there are many proposed methods in the literature, for example, the Epstein/Peterson method [9], the Deygout method [10], the Giovanelli method [11], the Vogler method [12] and the uniform theory of diffraction (UTD) [5]. Among them, the Vogler method is known to be the best accurate method [5]. A review and comparison of those methods can, for example, be found in [5, 13]. For MBKED, Fresnel-Kirchhoff theory is used to derive a general diffraction attenuation formula in [1], where small wavelength and small diffraction angles are assumed. However, as stated in [14] that, the accuracy of result is difficult to assure in evaluating a numerical integration to infinity of an oscillating function. To resolve this problem, a series solution based on the Vogler method is presented in [14]. It was shown by simulation that, the solution is accurate in some standard scenario tests. The algorithm for MBKED presented in [14], nevertheless, has very high computational complexity in a similar way to the original Vogler algorithm, thus limiting its use for time-sensitive applications.

To tackle the computational burden in [14], we present here a fast algorithm for MBKED estimation. Due to the presence of the reflecting planes, the problem becomes more complicated and challenging than multiple knife edge diffraction one. The key idea of our proposed algorithm is based on a recursive form which allows to avoid repeated calculations of the original form in [14]. Our analysis based on number of computed integrals reveals that the proposed algorithm has lower complexity than the algorithm in [14]. Moreover, since MBKED can be considered as a generalized form of MKED, our algorithm can, thus, be seen as a generalized version of the recursive Vogler algorithm [12, 15]. To illustrate the main idea, we introduce a case study of three bridged knife edges ($N = 3$) and then present a general formula for $N \geq 3$. The simulation results show that the proposed recursive algorithm is much faster than that of the algorithm presented in [14] while having the identical accuracy, thus bringing a step forward for an efficient and fast diffraction estimation.

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2. MULTIPLE BRIDGED KNIFE-EDGE DIFFRACTION

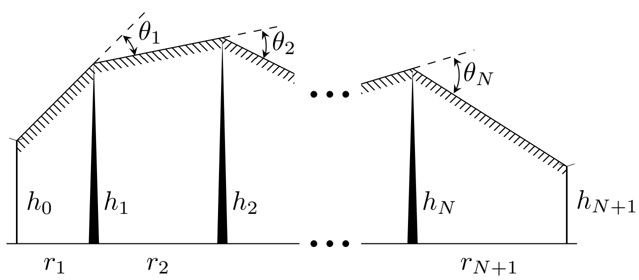

Fig. 1: Geometry of multiple bridged knife-edge.

In this section, a background on the multiple bridged knife edge diffraction problem and the series solution in [14] will be presented. Recall that the model can be obtained from multiple knife edge one by bridging the spaces among knife-edges with reflecting planes. The geometry of N bridged knife edge problem is described as follows (see Figure 1): h_0 and h_{N+1} are the transmitter and receiver heights respectively; We denote the knife-edge heights to a reference surface as $\{h_n\}_{n=1}^{N}$, diffraction angles as $\{\theta_n\}_{n=1}^{N}$, and $N+1$ separation distances among knife-edges as $\{r_n\}_{n=1}^{N+1}$. Let $\mathrm{i} = \sqrt{-1}$ and k be the complex number and the wave number respectively. Following the derivation of [14], the diffraction attenuation for a perfect reflection (i.e., reflection coefficients are assumed to be -1), A_N, can be represented in the following formula

$$\begin{aligned} A_N =&(1/\pi)^{N/2} C_N \exp\left(\sigma_N - \sigma_N'\right) \\ &\times \int_{\beta_1}^{\infty} du_1 \cdots \int_{\beta_N}^{\infty} du_N \sum_{q_1=1-\lambda_1}^{\mu_1} \cdots \sum_{q_{N-1}=1-\lambda_{N-1}}^{\mu_{N-1}} (-1)^s \\ &\times \exp(2f) \exp\left(-\sum_{n=1}^{N} u_n^2\right) \end{aligned} \tag{1}$$

where

$$C_N = \begin{cases} 1 \ \text{ for } N=1 \\ \left[\frac{\left(\sum_{n=1}^{N+1} r_n\right)\prod_{n=1}^{N} r_n}{\prod_{n=1}^{N}(r_n + r_{n+1})}\right]^{1/2} & , N \geq 2 \end{cases}, \tag{2}$$

$$\sigma_N = \sum_{n=1}^{N} \beta_n^2, \tag{3}$$

$$\sigma_N' = \mathrm{i}k \sum_{n=0}^{N} \frac{\left(h_{n+1} - h_n\right)^2}{2r_n}, n = 0, \cdots, N, \tag{4}$$

$$s = \sum_{n=1}^{N-1} q_n, \tag{5}$$

$$f = \begin{cases} 0 \ \text{ for } N=1 \\ \sum_{n=1}^{N-1} \alpha_n (-1)^{q_n} \left(u_n - \beta_n\right)\left(u_{n+1} - \beta_{n+1}\right) & , N \geq 2, \end{cases} \tag{6}$$

$$\alpha_n = \left[\frac{r_n r_{n+1}}{\left(r_n + r_{n+1}\right)\left(r_{n+1} + r_{n+2}\right)}\right]^{1/2}, \ 1 \leq n \leq N-1, \tag{7}$$

$$\beta_n = \theta_n \left[\frac{\mathrm{i}k r_n r_{n+1}}{2\left(r_n + r_{n+1}\right)}\right]^{1/2}, \ 1 \leq n \leq N. \tag{8}$$

Here, λ_n and μ_n are set to one for the initial MBKED problem. If the nth reflecting bridge is omitted, we will assign μ_n to zero. Moreover, as mentioned in [14], λ_n will be set to zero occasionally if the Babinet's principle[1] [16] is applied. Note that we can achieve the MKED formula as in [17], if all reflecting bridges are eliminated, thus explaining why MBKED can be considered as a generalization of MKED.

By applying the following expansion of $\exp(2f)$

$$\begin{aligned} \exp(2f) =& \sum_{p=0}^{\infty} 2^p \sum_{p_2=0}^{p_1} \cdots \sum_{p_{N-1}}^{p_{N-2}} (-1)^{s'} \\ &\times \prod_{n=1}^{N} \frac{\left[\alpha_n \left(u_n - \beta_n\right)\left(u_{n+1} - \beta_{n+1}\right)\right]^{p_n - p_{n+1}}}{(p_n - p_{n+1})!}, \end{aligned} \tag{9}$$

where the following notation is used: $p_1 \equiv p, p_N \equiv 0, p_{N+1} \equiv 0, \alpha_N \equiv 1$, and

$$s' = \sum_{n=1}^{N-1} \left(p_n - p_{n+1}\right) q_n \tag{10}$$

and using the repeated integrals of the complementary error function, defined as

$$I(m, \beta) \triangleq \frac{2}{m!\sqrt{\pi}} \int_{\beta}^{\infty} (u - \beta)^m \exp\left(-u^2\right) du, \tag{11}$$

a series solution is obtained as in [14]

$$\begin{aligned} A_N =& 2^{N'-N} C_N \sum_{p=0}^{\infty} 2^p \sum_{p_2=0}^{p_1=p} \cdots \sum_{p_{N-1}=0}^{p_{N-2}} \varepsilon \\ &\times \prod_{n=1}^{N} \frac{(p_{n-1} - p_{n+1})! \alpha_n^{p_n - p_{n+1}} I\left(p_{n-1} - p_{n+1}, \beta_n\right)}{(p_n - p_{n+1})!} \end{aligned} \tag{12}$$

where N' is number of n values, $n = 1, \cdots, N$, for which $\lambda_n = \mu_n = 1$; the value of ε is given by the following condition: if there is at least one n so that $\lambda_n = \mu_n = 1$ and

[1] Following the Babinet's principle, we can convert a diffraction problem involving thin diffracting screens into two new easier problems to solve.

$p_n - p_{n+1}$ is even, then $\varepsilon = 0$. Otherwise, $\varepsilon = (-1)^q$ with q is the number of values of n for which $\lambda_n = 0$ and $p_n - p_{n+1}$ is even.

3. PROPOSED FAST RECURSIVE ALGORITHM

Here, we present the main idea and outline steps to evaluate equation (1) of the proposed solution. We emphasize that the proposed solution here is more general than the one presented in [15] due to the presence of relecting surfaces. Our objective is to compute each integral corresponding to each variable u_n independently instead of the N-fold integral. To this end, we first use the Maclaurin series expansion of $\exp(2f)$, leading to computation of $f^m, m \geq 1$. Note that, f in (6) is the sum of a combined expressions of variables u_n including $N-1$ terms. Thus, we use the binomial theorem for the case of $N = 3$ and the multinomial theorem in a recursive form for general case of $N \geq 3$ (see Lemma 1 in [15]) to represent f^m and separate the expression related to each u_n. By exploiting this recursive form, we avoid the repeated calculations and thus reduce the over all computational complexity.

3.1. Case study of $N = 3$

To illustrate the main steps used to derive the proposed algorithm, we provide a case study of $N = 3$. We can rewrite equation (1) using the Maclaurin series expansion of $\exp(2f)$ as follows

$$A_3 = \frac{1}{2^3} C_3 \exp\left(\sigma_3 - \sigma_3'\right) \sum_{m=0}^{\infty} I_m \tag{13}$$

where

$$I_m = \left(\frac{2^m}{m!}\right)\left(\frac{2}{\sqrt{\pi}}\right)^3 \int_{\beta_1}^{\infty}\int_{\beta_2}^{\infty}\int_{\beta_3}^{\infty} \times \sum_{q_1=1-\lambda_1}^{\mu_1} \sum_{q_2=1-\lambda_2}^{\mu_2} (-1)^s f^m \exp\left(-\sum_{n=1}^{3} u_n^2\right) du_1 du_2 du_3 \tag{14}$$

Moreover, we can express f^m, for $N = 3$, using the binomial theorem as

$$f^m = \sum_{m_1=0}^{m} \frac{m!}{(m-m_1)!m_1!} (-1)^{q_1(m-m_1)+q_2 m_1} \times \alpha_1^{m-m_1} (u_1-\beta_1)^{m-m_1} (u_2-\beta_2)^m \alpha_2^{m_1} (u_3-\beta_3)^{m_1} \tag{15}$$

Let us denote

$$\tilde{C}(2, m_1, m) = m! \alpha_2^{m_1} I(m, \beta_2) I(m_1, \beta_3) \tag{16}$$

Substituting (15) into (14) yields

$$\begin{aligned}
I_m &= 2^m \sum_{q_1=1-\lambda_1}^{\mu_1} \sum_{q_2=1-\lambda_2}^{\mu_2} (-1)^s \\
&\quad \times \sum_{m_1=0}^{m} \frac{1}{(m-m_1)} (-1)^{q_1(m-m_1)+q_2 m_1} \\
&\quad \times \alpha_1^{m-m_1} \frac{2}{\sqrt{\pi}} \int_{\beta_1}^{\infty} (u_1-\beta_1)^{m-m_1} du_1 \tilde{C}(2, m_1, m) \\
&= 2^m \sum_{q_1=1-\lambda_1}^{\mu_1} \sum_{q_2=1-\lambda_2}^{\mu_2} (-1)^s \sum_{m_1=0}^{m} (-1)^{q_1(m-m_1)+q_2 m_1} \\
&\quad \times \alpha_1^{m-m_1} \left[\frac{2}{\sqrt{\pi}} \int_{\beta_1}^{\infty} \frac{(u_1-\beta_1)^{m-m_1}}{(m-m_1)!} du_1\right] \tilde{C}(2, m_1, m) \\
&= 2^m \sum_{q_1=1-\lambda_1}^{\mu_1} \sum_{q_2=1-\lambda_2}^{\mu_2} \sum_{m_1=0}^{m} (-1)^{q_1(m-m_1+1)+q_2(m_1+1)} \\
&\quad \times \alpha_1^{m-m_1} I(m-m_1, \beta_1) \tilde{C}(2, m_1, m)
\end{aligned} \tag{17}$$

Observe that $q_i \in \{0, 1\}, i = 1, 2$, depends on the geometry of the considered terrain, which is controlled by parameters λ_i and μ_i. Let us consider the case of 'full' MBK[2]. Thus, we can rewrite I_m in (17) by factorizing coefficients related to variables q_1 and q_2 as

$$I_m = 2^m \sum_{m_1=0}^{m} \left(1 - (-1)^{m-m_1}\right) \times \alpha_1^{m-m_1} I(m-m_1, \beta_1) C(2, m_1, m) \tag{18}$$

where

$$C(2, m_1, m) = \left(1 - (-1)^{m_1}\right) \tilde{C}(2, m_1, m). \tag{19}$$

We show that this case study is a special case of Theorem 1 presented in the next section.

3.2. General case of $N \geq 3$

We state the following theorem to estimate MBKED for general case of $N \geq 3$. Due to limited space, we omit the proof in this manuscript.

Theorem 1. *(Recursive computation of A_N whereas bridging terrain is presented among the knife edges, but not on either side of them): Let*

$$\begin{aligned}
&C(N-1, m_{N-2}, m_{N-3}) \\
&= \left(1 - (-1)^{m_{N-2}}\right) (m_{N-3})! \alpha_{N-1}^{m_{N-2}} I(m_{N-3}, \beta_{N-1}) \\
&\quad \times I(m_{N-2}, \beta_N).
\end{aligned} \tag{20}$$

[2] That is, $\lambda_i = \mu_i = 1$, corresponding to two reflecting bridges, neither bridge from transmitter to the first knife-edge nor bridge from the last knife-edge to the receiver and $q_i = \{0, 1\}, i = 1, 2$.

Then, given the following notation

$$i = m_{N-L}, j = m_{N-L-1}, k = m_{N-L-2}$$
$$2 \le L \le N-2, N \ge 4$$

and the recursive relationship,

$$\begin{aligned} & C(N-L, j, k) \\ & = \sum_{i=0}^{j} \left(1-(-1)^{j-i}\right) \\ & \quad \times \frac{(k-i)!}{(j-i)!} \alpha_{N-L}^{j-i} I(k-i, \beta_{N-L}) C(N-L+1, i, j), \end{aligned} \tag{21}$$

the diffraction attenuation of MBKE, A_N, is given by

$$A_N = \frac{1}{2^N} C_N \exp(\sigma_N - \sigma'_N) \sum_{m=0}^{\infty} I_m$$

where I_m is computed recursively as follows

$$\begin{aligned} I_m = & 2^m \sum_{m_1=0}^{m} \left(1-(-1)^{m-m_1}\right) \\ & \times \alpha_1^{m-m_1} I(m-m_1, \beta_1) C(2, m_1, m). \end{aligned} \tag{22}$$

It is worth mentioning that, compared to the recursive Vogler algorithm [12], the proposed theorem is different in terms of the coefficients of (20), (21) and (22) (i.e., $1-(-1)^{m_{N-2}}$, $1-(-1)^{j-i}$, and $1-(-1)^{m-m_1}$ respectively) taking into account of reflecting surfaces. Thus, we can expect that our proposed algorithm has the same computational complexity as the recursive Vogler one.

3.3. Computational complexity

We provide here result for number of computed integrals as a computational complexity index (with $N \ge 3$) since it contributes dominant part to overall complexity of mentioned algorithms. Due to limited space, a detail derivation and analysis are omitted. Let M be a truncated value of the index m.

- Following (12), the number of computed integrals for the series based algorithm is of $N \prod_{n=1}^{N-1} \frac{(M+n)}{n}$.
- Following the formula used in Theorem 1, the number of computed integrals for the proposed algorithm is of $3 \prod_{n=1}^{N-1} \frac{(M+n)}{n} + \prod_{n=1}^{N-2} \frac{(M+n)}{n} + \cdots + \prod_{n=1}^{2} \frac{(M+n)}{n}$.

It is straightforward to see that number of computed integrals for the proposed algorithm is smaller than that of the series based algorithm [14], thus proving the effectiveness of our proposed algorithm.

4. NUMERICAL RESULTS

In this section, we present numerical results for illustrating the effectiveness of the proposed algorithm. Then, we compare the accuracy and relative running time our proposed algorithm with the series-based algorithm in [14] and the recursive Vogler algorithm presented in [12]. Beside the complexity analysis presented in Section 3.3, we use the *CPU execution time* (in second) as an additional index to have *rough complexity assessment* of the algorithms.

We used an example of five bridged knife-edges (i.e., $N = 5$) described in [14] as follows: we have a propagation path with a distance of 60 km. There are three fixed knife-edges, h_1, h_3, and h_5 ($h_1 = h_3 = h_5 = 100$ m) at distances of 10 km, 30 km and 50 km from the transmitter respectively. Two knife-edges with variable heights, h_2 and h_4, are placed at the exact middle of two fixed knife-edge. Bridging terrain is presented between knife-edges but not either side of them. The operation frequency is set at 100 MHz. The transmitter and receiver heights are of 100 m above the reference plane. We consider two cases: i) h_2 is fixed at 100 m *below* the reference plane (i.e., -100 m) and h_4 varies; ii) both h_2 and h_4 vary.

In terms of precision, it can be observed in Fig. 2 that the accuracy of proposed recursive algorithm is identical to that of the algorithm in [14]. Due to the appearance of reflecting planes, the diffraction attenuation values increase and change smoother than that of a standard multiple knife-edge.

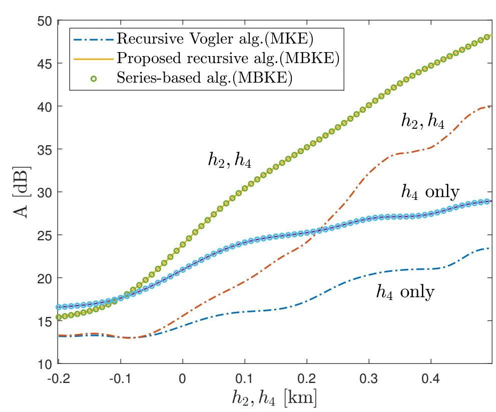

Fig. 2: Illustration of the accuracy: the diffraction attenuation of $N = 5$ knife-edges are estimated. We consider two scenarios: with and without reflecting bridges. While the former is used to compare to the series-based algorithm presented in [14] in terms of accuracy, the later is to assess the running time. The results from our proposed fast algorithm and the series-based algorithm in [14] are identical.

In terms of running time, our proposed recursive algorithm is around two times faster than the series-based algorithm in [14] as illustrated in Fig. 3. Indeed, the recursive structure is more efficient to be implemented, since it reduces redundant computation in some sub-calculations. Furthermore, our proposed algorithm has a close performance to the recursive Vogler algorithm due to their similar structure.

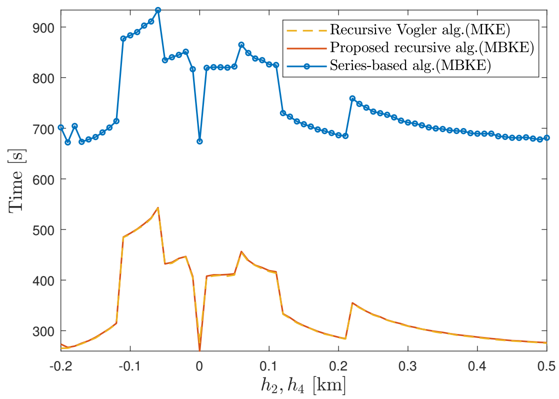

Fig. 3: The running time of three algorithms as a function of h_2 and h_4: our proposed algorithm, the series based algorithm [14] and the recursive Vogler algorithm [12] are compared.

5. CONCLUSION

We presented a fast recursive algorithm for the estimating multiple bridged knife-edge diffraction accurately. The results show that our proposed algorithm can provide faster solution than the series based algorithm introduced in [14], while having a similar accuracy. The proposed algorithm can be considered as a generalization of the well-known recursive Vogler algorithm for the multiple knife-edge problem. When a trade-off between the accuracy and the computational complexity is desired, a deep learning approach used for MKE problem [18] can also be applied for MBKE.

6. REFERENCES

[1] J. H. Whitteker, "Fresnel-Kirchhoff theory applied to terrain diffraction problems," *Radio Science*, vol. 25, no. 05, pp. 837–851, 1990.

[2] O. Ozgun, "New software tool (GO+UTD) for visualization of wave propagation [testing ourselves]," *IEEE Antennas and Propagation Magazine*, vol. 58, no. 3, pp. 91–103, 2016.

[3] ITU, *Propagation par diffraction*. Recommendation ITU-R P.526-14, 2008.

[4] T. Alwajeeh, P. Combeau, and L. Aveneau, "An efficient ray-tracing based model dedicated to wireless sensor network simulators for smart cities environments," *IEEE Access*, vol. 8, pp. 206 528–206 547, 2020.

[5] S. R. Saunders and A. Aragón-Zavala, *Antennas and propagation for wireless communication systems*. John Wiley & Sons, 2007.

[6] J. H. Whitteker, "Physical optics and field-strength predictions for wireless systems," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 3, pp. 515–522, 2002.

[7] T. S. Rappaport, G. R. MacCartney, S. Sun, H. Yan, and S. Deng, "Small-scale, local area, and transitional millimeter wave propagation for 5G communications," *IEEE Trans. Antennas Propag.*, vol. 65, no. 12, pp. 6474–6490, Dec 2017.

[8] X. Zhao and P. Vainikainen, "Multipath propagation study combining terrain diffraction and reflection," *IEEE Transactions on Antennas and Propagation*, vol. 49, no. 8, pp. 1204–1209, 2001.

[9] J. Epstein and D. W. Peterson, "An experimental study of wave propagation at 850 mc," *Proceedings of the IRE*, vol. 41, no. 5, pp. 595–611, May 1953.

[10] J. Deygout, "Multiple knife-edge diffraction of microwaves," *IEEE Transactions on Antennas and Propagation*, vol. 14, no. 4, pp. 480–489, July 1966.

[11] C. L. Giovaneli, "An analysis of simplified solutions for multiple knife-edge diffraction," *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 3, pp. 297–301, March 1984.

[12] L. E. Vogler, "An attenuation function for multiple knife-edge diffraction," *Radio Science*, vol. 17, no. 06, pp. 1541–1546, Nov 1982.

[13] C. Tzaras and S. R. Saunders, "Comparison of multiple-diffraction models for digital broadcasting coverage prediction," *IEEE Transactions on Broadcasting*, vol. 46, no. 3, pp. 221–226, Sep. 2000.

[14] J. H. Whitteker, "A series solution for diffraction over terrain modeled as multiple bridged knife edges," *Radio Science*, vol. 28, no. 04, pp. 487–500, July 1993.

[15] V.-D. Nguyen, H. Phan, A. Mansour, A. Coatanhay, and T. Marsault, "On the proof of recursive Vogler algorithm for multiple knife-edge diffraction," *IEEE Transactions on Antennas and Propagation*, 2020.

[16] J. A. Kong, *Electromagnetic Wave Theory*. Wiley-Interscience, 1990.

[17] L. E. Vogler, "Further investigations of the multiple knife-edge attenuation function," *NASA STI/Recon Technical Report N*, vol. 84, 1983.

[18] V.-D. Nguyen, H. Phan, A. Mansour, and A. Coatanhay, "VoglerNet: multiple knife-edge diffraction using deep neural network," in *2020 14th European Conference on Antennas and Propagation (EuCAP)*, 2020, pp. 1–5.