An Analysis for the Performance of the OFDM-IM Systems Impaired by Carrier Frequency Offset

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***Abstract*—For the last years, a great deal of interest has been paid to OFDM systems with index modulation (OFDM-IM) because of its energy efficiency and flexibility in the spectral domain. However, these systems suffer from carrier frequency offset that degrades the system performance. In a recent paper, an attempt was made to evaluate the performance in terms of bit error rate (BER) when the received signal is disturbed by an additive white Gaussian noise and a carrier frequency offset (CFO). Nevertheless, the approximations made in the existing analysis resulted in inaccurate simulation results. In this paper, the approximations we consider seem to be well-suited since the closed form expression of the BER is now consistent with the simulations results based on a maximum-likelihood detector.**

***Index Terms*—OFDM, index modulation, carrier frequency offset, BER**

I. Introduction

THE concept of index modulation (IM) has attracted many researchers' attention due to the advantages it offers. In recent years, IM has been employed in various fields such as mobile communication and RADAR systems [1], [2]. Thus, in communication systems, IM consists in transmitting information bits in addition to the constellation bits in order to activate a subset of transmission entities. The essential idea of introducing IM in the multicarrier OFDM systems is to use the information bits to define the indices of the active subcarriers in order to carry the information bits. Compared with conventional OFDM systems [3], OFDM-IM systems achieve energy efficiency and improve average bit error probability (ABEP). However, OFDM-IM systems are sensitive to carrier frequency offset (CFO), a mismatch in the carrier frequencies generated at the transmitter and the receiver oscillators. The CFO results in inter-carrier interference (ICI), thereby destroying the orthogonality of the OFDM-IM data. The estimation of the bit error rate (BER) of the OFDM-IM system in the presence of CFO is therefore necessary to evaluate the effect of CFO in OFDM-IM systems.

Different types of work have been recently done on OFDM-IM systems with CFO. Thus, a procedure to estimate the CFO in OFDM-IM systems is presented in [4]. In [5], the authors show the effect of the CFO on OFDM-IM systems. Finally in [6], an analytical approach is presented to investigate the performance of the OFDM-IM system in presence of CFO. The assumptions made by the authors in [6] are the following: the ICI due to CFO is divided into two parts, namely the intra-subblock ICI and the inter-subblocks ICI whose variance is approximated as given in [7]. The channel model used for the simulations is the extended vehicular A (EVA). Then, the authors evaluated the BER of OFDM-IM systems in presence of CFO along with an additive white Gaussian measurement noise. Due to the approximations made in this analysis, the closed form expression of the BER that is obtained is not consistent with the results provided by the simulations.

For this reason, we propose to address this problem again in this paper. The proposed analytical approach is an extension of [8] where the BER for OFDM-IM systems for a Rayleigh fading channel is calculated. This extension has the advantage of considering all the effects of ICI in order to get the expression of the BER. More particularly, in the proposed approach, the ICI is approximated by a Gaussian random variable [9]. In addition, the channel model used for the simulations is a Rayleigh fading channel and the maximum-likelihood detector is employed at the receiver. We are going to study whether the closed form BER expression obtained is consistent with the simulation results or not.

The remainder of the paper is organized as follows. Section II presents the system model of OFDM-IM with CFO, while we derive the BER expressions for a Rayleigh fading channel in Section III. In Section IV, the theoretical analysis is validated by means of simulation results. Conclusions are then given.

To end up this introduction, let us recall some notations useful in the following. Lower (upper) bold letters denote column vectors (matrices). Superscripts $(.)^*$ and $(.)^T$ represent the complex conjugate and transpose operators respectively. $\lfloor . \rfloor$ is the floor operation and $||\mathbf{x}||_2$ is the Euclidean norm of vector $\mathbf{x}$. $E(.)$ is the statistical expectation, $\mathbf{I}_d$ is the identity matrix of size d. $\text{Re}(.)$ is the real part and $P(A)$ the probability of the event A and $Q(x) = P(X > x)$ where X is the zero-mean Gaussian random variable with unit variance.

II. System Model of OFDM-IM

Let us consider an OFDM-IM transmission system with N subcarriers. The latter are divided into n_b subblocks with N_b subcarriers in each subblock, *i.e.* $N = n_bN_b$, where the subscript $(.)_b$ refers to the subblock.

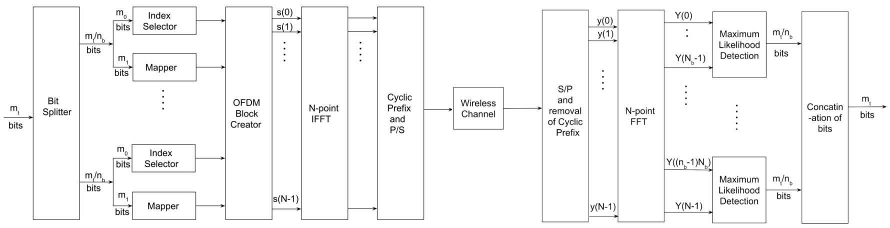

Fig. 1. Transceiver chain for OFDM-IM system

At the OFDM-IM transmitter, only one subcarrier is activated in one subblock. For choosing one active subcarrier out of N_b subcarriers, $m_0 = \lfloor (log_2 N_b) \rfloor$ information bits are required. In addition, the data symbols that are transmitted over the subcarriers belong to a M-ary constellation *i.e.* M-QAM. Therefore, $m_1 = log_2 M$ information bits are required to transmit a symbol belonging to the M-ary constellation, denoted as $\mathbf{S}$, on each active subcarrier. A total of $m_0 + m_1$ information bits are hence transmitted per each subblock per OFDM symbol. Therefore, the total number of bits transmitted per OFDM symbol is $m_t = n_b(m_0 + m_1)$, as shown in figure 1.

The symbols carried over the subcarrier in each subblock form a column vector of size N_b. The β^{th} subblock vector is given by:

$$\mathbf{s}_{\beta,\alpha} = [s((\beta-1)N_b), \dots, s(\beta N_b - 1)]^T \tag{1}$$

where $s(k) \in \{0, \mathbf{S}\}$ for $k = (\beta-1)N_b, ..., \beta N_b - 1$ with $\beta = 1, 2, \dots, n_b$.

The column vector $\mathbf{s}_{\beta,\alpha}$ can be rewritten from the data symbol carried over one subcarrier as follows:

$$\begin{aligned}\mathbf{s}_{\beta,\alpha} &= \left[\mathbf{0}_{1\times\alpha},\ s\left((\beta-1)N_b + \alpha\right),\ \mathbf{0}_{1\times(N_b-\alpha-1)}\right]^T \\ &= \left[\mathbf{0}_{1\times\alpha},\qquad s(\gamma)\qquad, \mathbf{0}_{1\times(N_b-\alpha-1)}\right]^T\end{aligned} \tag{2}$$

where we have introduced for the sake of simplicity:

$$\gamma = (\beta-1)N_b + \alpha \tag{3}$$

$\gamma \in 0, 1, \dots, N-1$ indicates the active subcarrier index whereas $\alpha \in 0, 1, \dots, N_b - 1$ indicates the location of the active subcarrier in the β^{th} subblock.

The vectors defining all the subblocks are then concatenated to form the $N \times 1$ OFDM symbol as follows:

$$\mathbf{s} = [\mathbf{s}_{1,\alpha}^T, \dots, \mathbf{s}_{\mathbf{n_b},\alpha}{}^T]^T \tag{4}$$

Given (2), $\mathbf{s}$ comprises $N - n_b$ zero elements in the proposed system unlike the classical OFDM.

After modulating the signal $\mathbf{s}$ by performing an Inverse Discrete Fourier Transform (IDFT), the cyclic prefix (CP) is added. Then, the signal is assumed to propagate through a Rayleigh fading channel.

At the OFDM-IM receiver, after removing the CP, the signal is demodulated by using the Fast Fourier Transform (FFT). Unlike OFDM, the detector used in OFDM-IM system needs to detect both the index bits and the constellation bits. Therefore, the decoding in OFDM-IM is performed on the subblock basis. To this end, one needs to look at the received β^{th} subblock signal, subject to CFO. It is expressed as:

$$\mathbf{y}_\beta = p_0 \mathbf{H}_\beta \mathbf{s}_{\beta,\alpha} + \mathbf{g}_\beta + \mathbf{w}_\beta \tag{5}$$

where:

- $\mathbf{y}_\beta$ is a column vector of size N_b given by:

$$\mathbf{y}_\beta = [Y((\beta-1)N_b), \dots, Y(\beta N_b - 1)]^T \tag{6}$$

where $Y(\gamma)$ is the received data symbol on the γ^{th} subcarrier.

- The channel is assumed to remain constant over a subblock. The channel matrix for the β^{th} subblock is hence given by:

$$\mathbf{H}_\beta = H((\beta-1)N_b) \times \mathbf{I}_{N_b} = H_\beta \mathbf{I}_{N_b} \tag{7}$$

with $H_\beta = H((\beta-1)N_b))$ the Rayleigh flat fading channel coefficient for the β^{th} subblock. As there are n_b subblocks, the corresponding n_b coefficients are assumed to be independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance.

- The ICI vector is defined by:

$$\mathbf{g}_\beta = [G((\beta-1)N_b), \dots, G(\beta N_b - 1)]^T \tag{8}$$

where $G(\gamma)$ is the intercarrier interference (ICI) for the γ^{th} subcarrier defined as follows:

$$G(\gamma) = \sum_{l=0, l\neq\gamma}^{N-1} p_{l-\gamma} s(l) H(l) \tag{9}$$

where $p_{l-\gamma}$ is the ICI coefficient in the frequency domain given by [10]:

$$p_{l-\gamma} = \frac{\sin(\pi(l-\gamma+\varepsilon))}{N \sin\left(\frac{\pi}{N}(l-\gamma+\varepsilon)\right)} e^{\left(j\pi\left(\frac{N-1}{N}\right)(l-\gamma+\varepsilon)\right)} \tag{10}$$

with ϵ the normalized CFO. It should be noted that p_0 is a specific case of $p_{l-\gamma}$ when $l = \gamma$.

From equation (9), $G(\gamma)$ can be approximated as a Gaussian random variable according to the central limit theorem.

- $\mathbf{w}_\beta = [W((\beta-1)N_b), \dots, W((\beta N_b - 1)]^T$ denotes the independent, additive zero-mean white Gaussian noise (AWGN) vector with variance N_0. The signal-to-noise ratio (SNR) is then defined by $\rho = E_s/N_0$ where E_s denotes the average power of the M-QAM symbol.

Describing the received signal on the γ^{th} subcarrier results in:

$$Y(\gamma) = \begin{cases} p_0 s(\gamma) H(\gamma) + \sum_{l=0, l\neq\gamma}^{N-1} p_{l-\gamma} s(l) H(l) + W(\gamma) \\ \qquad \text{if } \gamma \text{ is active} \\ \sum_{l=0, l\neq\gamma}^{N-1} p_{l-\gamma} s(l) H(l) + W(\gamma) \text{ otherwise} \end{cases} \tag{11}$$

With the received signal, the maximum-likelihood (ML) detector estimates the information bits in this paper. More particularly, for each subblock, the ML detector firstly estimates the index of the active subcarrier and the constellation symbol carried over it [11]. This is done by jointly searching over all possible combinations of the active subcarrier and the constellation symbol. To this end, the channel and the noise variance are assumed to be known at the receiver.

In the next section, we propose to obtain a closed form expression of the BER and compare it with the simulation results we obtain with the above detector.

III. BER ANALYSIS

The BER is defined as the ratio between the number b_e of error bits across all the n_b subblocks, *i.e.* $b_e = \sum_{\beta=1}^{n_b} b_{e,\beta}$ and the total number m_t of transmitted bits:

$$P_b = \frac{b_e}{m_t} = \frac{\sum_{\beta=1}^{n_b} b_{e,\beta}}{m_t} = \frac{\sum_{\beta=1}^{n_b} b_{e,\beta}}{n_b(\log_2 N_b + \log_2 M)} \tag{12}$$

The following events contribute to the erroneous estimation of the transmitted bits at the receiver:

- **Event 1:** incorrect detection of an active subcarrier index and incorrect detection of a M-ary symbol,
- **Event 2:** incorrect detection of an active subcarrier index and correct detection of a M-ary symbol,
- **Event 3:** correct detection of an active subcarrier index and incorrect detection of a M-ary symbol.

To compute P_b, let us focus our attention on $b_{e,\beta}$. To this end, the next subsections deal with the number of bit errors occurred in β^{th} subblock due to event i mentioned above, denoted as $b_{e,\beta}^{(i)}$, with $i = 1, 2, 3$.

A. Upper bound of the number of bit errors in both index and constellation domain, $b_{e,\beta}^{(1)}$:

Given the transmitted β^{th} subblock $\mathbf{s}_{\beta,\alpha}$, let us define $\tilde{\mathbf{s}}_{\beta,\tilde{\alpha}}$ as the incorrectly-detected β^{th} subblock for the incorrectly-detected active subcarrier index $\tilde{\alpha}$. In addition, $\tilde{s}\left(\tilde{\gamma}\right)$ denotes the incorrectly-detected constellation symbol for the subcarrier $\tilde{\gamma} = (\beta - 1)N_b + \tilde{\alpha}$.

Then, let us denote $\left(\mathbf{s}_\alpha \to \mathbf{s}_{\tilde{\alpha}}\right)$ the pairwise error event (PEE) where the active index of the subcarrier is incorrectly detected. Moreover, $\left(s(\gamma) \to \tilde{s}(\tilde{\gamma})/\alpha \neq \tilde{\alpha}\right)$ corresponds to the PEE when the constellation symbol is incorrectly detected provided that the active subcarrier index is detected incorrectly.

Given the PEEs and $\mathbf{H}_\beta$, the upper bound of the number of bit errors in both the index domain and the constellation domain in the β^{th}subblock is given by the following union bound:

$$b_{e,\beta}^{(1)} \leq \frac{1}{N_b} \sum_{\alpha=1}^{N_b} \sum_{\alpha \neq \tilde{\alpha}=1}^{N_b} b_{e_1,\beta}^{(1)}(\alpha, \tilde{\alpha}, s(\gamma), s(\tilde{\gamma})) \times$$
$$P\left(\mathbf{s}_\alpha \to \mathbf{s}_{\tilde{\alpha}}\right) P\left(s\left(\gamma\right) \to \tilde{s}\left(\tilde{\gamma}\right)/\alpha \neq \tilde{\alpha}\right) \tag{13}$$

where $b_{e_1,\beta}^{(1)}(\alpha, \tilde{\alpha}, s(\gamma), s(\tilde{\gamma}))$ is the summation of the error bits in both the index domain and the constellation domain given that PEEs occur. The number of error bits in the index domain corresponds to the hamming distance between α and $\tilde{\alpha}$, denoted by $d_h(\alpha, \tilde{\alpha})$, whereas the number of error bits in the constellation domain is $log_2 M$. Therefore,

$$b_{e_1,\beta}^{(1)}(\alpha, \tilde{\alpha}, s(\gamma), s(\tilde{\gamma})) = d_h(\alpha, \tilde{\alpha}) + log_2 M \tag{14}$$

In addition, PEE $\left(\mathbf{s}_\alpha \to \mathbf{s}_{\tilde{\alpha}}\right)$ occurs when the euclidean distance between the received β^{th} subblock and the incorrectly-detected subblock $\tilde{\mathbf{s}}_{\beta,\tilde{\alpha}}$, is less than the euclidean distance between the received β^{th} subblock and the transmitted subblock $\mathbf{s}_{\beta,\alpha}$. The probability of the PEE can be hence expressed as follows:

$$P\left(\mathbf{s}_\alpha \to \mathbf{s}_{\tilde{\alpha}}\right) = P(||\mathbf{y}_\beta - p_0 \mathbf{H}_\beta \tilde{\mathbf{s}}_{\beta,\tilde{\alpha}}||_2^2 < ||\mathbf{y}_\beta - p_0 \mathbf{H}_\beta \mathbf{s}_{\beta,\alpha}||_2^2) \tag{15}$$

Substituting the expressions of $\mathbf{y}_\beta$, $\mathbf{H}_\beta$, $\mathbf{s}_{\beta,\alpha}$ and $\tilde{\mathbf{s}}_{\beta,\tilde{\alpha}}$ in the above equation, developing and simplifying lead to:

$$P\left(\mathbf{s}_\alpha \to \mathbf{s}_{\tilde{\alpha}}\right) = P(\text{ Re }\{(G(\tilde{\gamma}) + W(\tilde{\gamma}))^* p_0 H_\beta \tilde{s}(\tilde{\gamma}) \tag{16}$$
$$- (G(\gamma) + W(\gamma))^* p_0 H_\beta s(\gamma)\} > |p_0|^2 |H_\beta|^2 E_s)$$

Using the Q function defined at the end of the introduction, the above result can be rewritten this way:

$$P\left(\mathbf{s}_\alpha \to \mathbf{s}_{\tilde{\alpha}}\right) = Q\left(R\right) \tag{17}$$

where:

$$R = \sqrt{\frac{2|p_0|^2 |H_\beta|^2 E_s}{\sigma_\alpha^2 + \sigma_{\tilde{\alpha}}^2 + \sigma_{\alpha\tilde{\alpha}} + \sigma_{\tilde{\alpha}\alpha} + 2N_0}} \tag{18}$$

where σ_α^2 and $\sigma_{\tilde{\alpha}}^2$ are the ICI variances for subcarriers α and $\tilde{\alpha}$ respectively. $\sigma_{\alpha\tilde{\alpha}}$ is the covariance between the ICI for subcarriers α and $\tilde{\alpha}$ whereas $\sigma_{\tilde{\alpha}\alpha}$ is the covariance between the ICI for subcarriers $\tilde{\alpha}$ and α.

Given (17) and (18), the probability that an index is detected incorrectly increases with the ICI variance and the ICI covariance, but decreases with the energy of the M-ary symbol. It should be noted that if there is no CFO, then there will be no terms related to the covariance and the variance of the ICI. Let us now express these variance and covariance so that R can be fully defined:

As $s(\alpha)$ is zero or a M-ary symbol, $E[|s(\alpha)|^2] = \frac{E_s}{N_b}$. Combining this result and (9), the ICI variance of α^{th} subcarrier in β^{th} subblock σ_α^2, can be calculated as follows:

$$\sigma_\alpha^2 = E\left[|G(\gamma)|^2\right] = \sum_{l=0, l\neq\gamma}^{N-1} |p_{l-\gamma}|^2 \frac{E_s}{N_b} \tag{19}$$

Since the ICI of the subcarriers α and $\tilde{\alpha}$ are not independent, the covariance between them is given by:

$$\sigma_{\alpha\tilde{\alpha}} = E[G(\gamma)G(\tilde{\gamma})^*] = \frac{E_s}{N_b} \sum_{l=0, l\neq\gamma\neq\tilde{\gamma}}^{N-1} p_{l-\gamma} p_{l-\tilde{\gamma}}^* \tag{20}$$

Finally, the probability of the PEE $\left(s(\gamma) \to \tilde{s}(\tilde{\gamma})/\alpha \neq \tilde{\alpha}\right)$ depends on the detected active subcarrier index because the transmitted symbol $s(\gamma)$ is estimated from the data symbol carried over the detected active subcarrier index $\tilde{\alpha}$, which is a non information-carrying subcarrier. Therefore, the probability of the above PEE is $\frac{1}{2}$.

Combining this result with (14), (17) (18) and (13) leads to the following inequality:

$$b_{e,\beta}^{(1)} \leq \frac{1}{2N_b} \sum_{\alpha=1}^{N_b} \sum_{\alpha \neq \tilde{\alpha}=1}^{N_b} Q\left(R\right)\left(d_h(\alpha,\tilde{\alpha}) + log_2 M\right) \quad (21)$$

B. Upper bound of the number of bit errors only in the index domain $b_{e,\beta}^{(2)}$:

From the above analysis, the probability that there is no error in the constellation domain provided the active subcarrier index is detected incorrectly is $\frac{1}{2}$. The number of bit errors only in the index domain in the β^{th} subblock hence satisfies:

$$b_{e,\beta}^{(2)} \leq \frac{1}{N_b} \sum_{\alpha=1}^{N_b} \sum_{\alpha \neq \tilde{\alpha}=1}^{N_b} \frac{1}{2} Q\left(R\right) d_h(\alpha,\tilde{\alpha}) \quad (22)$$

C. Upper bound of the number of bit errors only in the constellation domain $b_{e,\beta}^{(3)}$:

Given the transmitted β^{th} subblock $\mathbf{s}_{\beta,\alpha}$, let us define $\tilde{\mathbf{s}}_{\beta,\alpha}$ as the incorrectly-detected β^{th} subblock for the incorrectly-detected constellation symbol $\tilde{s}(\gamma)$.
Let $(s(\gamma) \rightarrow \tilde{s}(\gamma)/\alpha = \tilde{\alpha})$ be a PEE that $s(\gamma)$ is incorrectly detected as $\tilde{s}(\gamma)$ provided the active subcarrier index α is detected correctly.
The probability of the PEE $(s(\gamma) \rightarrow \tilde{s}(\gamma)/\alpha = \tilde{\alpha})$ is the same as the bit error probability of the M-QAM symbol over AWGN channel if there is no ICI component. The presence of ICI leads to the probability of PEE as follows:

$$\begin{aligned} &P\left(s(\gamma) \rightarrow \tilde{s}(\gamma)/\alpha = \tilde{\alpha}\right) \\ &\quad = \sum_i A_i Q\left(\sqrt{\frac{a_i |p_0|^2 |H(\gamma)|^2 E_s}{\sigma_\alpha^2 + N_0}}\right) log_2 M \quad (23)\end{aligned}$$

where the values of i, A_i and a_i can be found in [12]. For example, when BPSK constellation is used, (i, A_i, a_i) =(1,1,2). The probability that the active subcarrier index is detected correctly is upper bounded by considering the joint probability of all the PEEs of the incorrect detection of the active subcarrier index.
With the help of Union bound, the upper bound of the number of bit errors only in the constellation domain in β^{th} subblock, is given as

$$\begin{aligned} &b_{e,\beta}^{(3)} \leq \sum_{\alpha=1}^{N_b} \left(1 - \sum_{\alpha \neq \tilde{\alpha}=1}^{N_b} Q\left(R\right)\right) \\ &\cdot \frac{1}{N_b M} \sum_{s(\gamma) \in S} \sum_{\tilde{s}(\gamma) \in S} P\left(s(\gamma) \rightarrow \tilde{s}(\gamma)/\alpha = \tilde{\alpha}\right) \log_2 M \quad (24)\end{aligned}$$

D. Closed form expression of the upper bound of the BER

Inserting equations (21), (22) and (24) into (12), the generalized expression for the unconditional BER of the OFDM-IM with CFO can be finally obtained in the closed-form as in equation (25). The upper bound of equation (25) is the sum of the upper bounds of $b_{e,\beta}^{(1)}$, $b_{e,\beta}^{(2)}$ and $b_{e,\beta}^{(3)}$. To understand the contribution of each upper bound value in equation (25), each term is numerically evaluated. Based on the evaluation, the value of the upper bound for $b_{e,\beta}^{(3)}$ is less than the upper bounds for $b_{e,\beta}^{(1)}$ and $b_{e,\beta}^{(2)}$. In few simulation scenarios, the upper bound for $b_{e,\beta}^{(3)}$ is negligible than the other two upper bound values, whereas in other scenarios, this term contributes a significant value towards the upper bound of equation (25). However, this term can be ignored for approximate results.
Equation (25) can give a few insights about how the system performance is dependent on the system parameters:
• When the normalized CFO value increases, the attenuation in the desired signal and the ICI increases, resulting in degrading the system performance.
• When keeping the total number of subcarriers constant while increasing the number of subcarriers in a subblock, the number of pairwise error events increases, degrading the system performance.
• When keeping the number of active subcarriers constant while increasing the number of subcarriers in a subblock, ICI decreases. Hence, as the number of subcarriers in a subblock increases, the degradation in the system performance due to CFO decreases.
In the next section, we verify the consistency of the closed-form BER expression with the simulation results.

IV. NUMERICAL EVALUATIONS AND SIMULATIONS

In this section, a simulation study was carried out to evaluate the accuracy of the proposed theoretical analysis for different CFO values. We also compare the proposed theoretical analysis with the one presented in [6] *i.e.,* the theoretical analysis for OFDM-IM with CFO. The simulated BER plots are generated using 10^5 bits. The theoretical plots are generated by averaging the conditioned BER expressions using the Monte-Carlo method, *i.e.* the conditioned BER expressions are averaged over a certain number of channel realizations. 500 independent Rayleigh fading channel realizations are used to generate the theoretical BER plot. All the simulations are carried out using the following parameters. N = 1024, $(N_b, n_b) = (2, 512), (4, 256)$, the modulation scheme used is BPSK, the subcarrier separation, $\Delta f = 15$kHz and the normalized CFO, $\epsilon = 0.01$ and 0.05.
Figures 2 and 3 depict the average BER of the OFDM-IM system with the normalized CFO of 0.01 and 0.05 for $(N_b, n_b) = (2, 512)$ and $(N_b, n_b) = (4, 256)$ respectively. In both figures, when ϵ increases from 0.01 to 0.05, the BER also increases due to increase in ICI. For $\epsilon = 0.05$, an error floor occurs at higher SNRs because BER is mainly due to ICI at higher SNRs.
In figure 2, when the SNR increases, the proposed theoretical analysis for $\epsilon = 0.01$ predicts the simulated BER results. For all the CFO values, the performance of the proposed theoretical analysis is better than the one proposed in [6] because in [6], the covariance of ICI terms is not used. Also weaker approximations are used in the calculation of the BER expressions.

$$P_b \leq \frac{1}{m_t N_b} \sum_{\beta=1}^{n_b} \sum_{\alpha=1}^{N_b} \sum_{\alpha \neq \tilde{\alpha}=1}^{N_b} \frac{1}{2} Q(R) \left(d_h(\alpha, \tilde{\alpha}) + log_2 M\right) + \frac{1}{m_t N_b} \sum_{\beta=1}^{n_b} \sum_{\alpha=1}^{N_b} \sum_{\alpha \neq \tilde{\alpha}=1}^{N_b} \frac{1}{2} Q(R)\, d_h(\alpha, \tilde{\alpha})$$
$$+ \frac{(M-1)}{m_t N_b} \sum_{\beta=1}^{n_b} \sum_{\alpha=1}^{N_b} \left(1 - \sum_{\alpha \neq \tilde{\alpha}=1}^{N_b} Q(R)\right) \sum_i A_i Q\left(\sqrt{\frac{a_i |p_0|^2 |H(\gamma)|^2 E_s}{\sigma_\alpha^2 + N_0}}\right) log_2 M \quad (25)$$

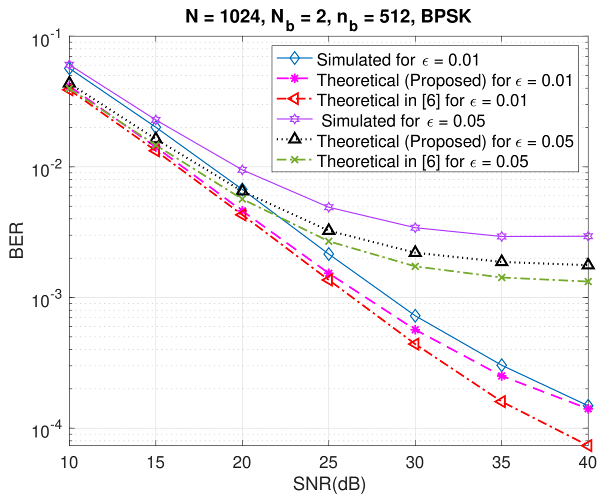

Fig. 2. BER performance of the OFDM-IM system for $(N_b, n_b) = (2, 512)$

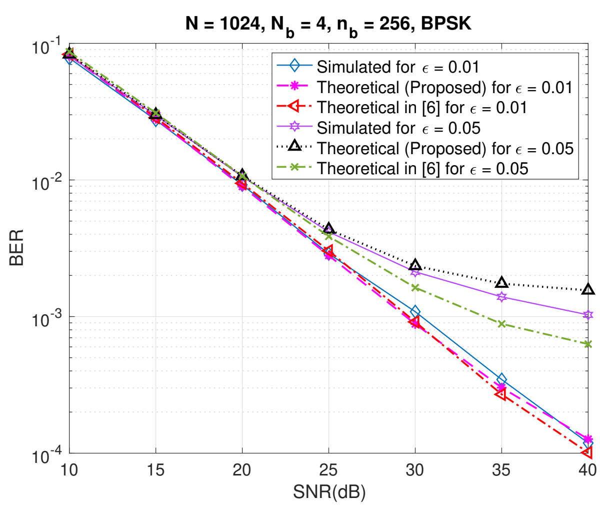

Fig. 3. BER performance of the OFDM-IM system for $(N_b, n_b) = (4, 256)$

In figure 3, the proposed theoretical analysis for $\epsilon = 0.01$ predicts the simulated BER results for all the SNR values. For $\epsilon = 0.05$, at lower SNR, both theoretical analyses predict the simulated BER results. However, at higher SNRs, both theoretical analyses deviate from the simulated BER results. The proposed theoretical analysis performs better for all CFO values than the one presented in [6] due to the approximations used in calculating the BER expression in [6]. Figures 2 and 3 show a deviation in the proposed theoretical results for $\epsilon = 0.05$ from the simulated results at higher SNRs because of the Gaussian approximation of ICI.

V. Conclusions and Perspectives

We propose an approach to derive the BER expression for the OFDM-IM system impaired by CFO. Numerical evaluations of the BER expression match the simulation results better than the existing BER expressions. As a perspective, we plan to take into account other system impairments like sampling frequency offset.

Acknowledgment

The authors would like to thank Prof. Eric Grivel, Bordeaux INP, France, for the help and support in making this work possible. The authors are also thankful to Prime Minister Research Fellowship (PMRF) scheme, Government of India, for funding.

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