Radar Environment Classificator with Clustering Capabilities

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***Abstract*—This paper addresses the problem of training data classification as homogeneous or heterogeneous. To this end, the problem is formulated as a multiple hypothesis test and specific structures for the interference covariance matrix are considered at the design stage. The unknown parameters, under the heterogeneous hypotheses, are estimated by resorting to hidden random variables representing homogeneous clutter classes and to a procedure based upon the expectation maximization algorithm as well as a cyclic estimation procedure. Remarkably, the proposed architectures are capable of estimating the unknown number of statistically homogeneous subsets in the radar window. The performance analysis on simulated data shows that the devised architectures represent an effective means to classify secondary data.**

***Index Terms*—Adaptive detection, Clutter classification, Expectation Maximization, Homogeneous Environment, Heterogeneous Environment, Multiple hypothesis test, Radar.**

I. INTRODUCTION

Adaptive radar detection of targets embedded in unknown interference is a challenging problem in the radar community which has attracted several research efforts [1]–[6]. In a clutter-dominated environment, where the other interference sources are assumed negligible as compared with the clutter, a very spread design model is the homogeneous environment. This model assumes that clutter is stationary over range and time allowing to collect a set of training or secondary data in the vicinity of the cell under test to achieve adaptivity with respect to the unknown clutter parameters [1], [2], [4], [5], [7], [8]. In fact, training data are used to obtain reliable estimates of the Interference Covariance Matrix (ICM) that make the detection schemes adaptive.

However, in practice, the homogeneous assumption may be not perfectly satisfied leading to a severe performance degradation for the architectures designed under this hypothesis [9]. To overcome this problem, a possible solution consists of making the training set homogeneous by means of outliers detection and suppression as shown in [10]–[12]. Another approach relies on a suitable modeling of the heterogeneous environment at the design stage. For instance, in order to account for a different clutter reflectivity between primary and secondary data, the well-known partially homogeneous environment has been introduced [13], [14, and references therein]. This model assumes that interference in primary data shares the same covariance structure of the ICM of the secondary data but a different power level. This model can be further generalized to come up with a "completely-heterogeneous" environment where data are characterized by different power levels [15]–[19].

The above approaches highlight that incorporating a priori information about the environment at the design stage can mitigate the performance degradation due to the model mismatch. Therefore, the problem of environment classification is of primary interest in the radar context since the outcomes of the classification could drive the selection of the most suitable detection and/or estimation procedure [15], [16]. However, a preliminary stage responsible for this task would depend on the ICM estimation quality and, hence, on the volume of available data. In fact, in sample-starved scenarios, a low-quality ICM estimate may lead to unreliable classification results and, as a consequence, to a severe performance depletion of the subsequent detection stages. A remedy for this drawback is the design of ICM estimation procedures that exploit the a priori information about the specific structure of the ICM. Specifically, in this paper, we assume the generic (Hermitian) unstructured case, the persymmetric (or centrohermitian) case, the (real) symmetric case, and the centrosymmetric case (which raises from the joint assumption of a persymmetric ICM structure and a symmetric clutter spectrum). This assumptions properly fit real radar applications. For instance, ICM exhibits a persymmetric structure when symmetrically spaced linear arrays or symmetrically spaced pulse trains are used [20], [21]. Possible symmetry in the clutter spectral characteristics can be also found in the presence of ground clutter, observed by a stationary monostatic radar, often exhibiting a symmetric power spectral density (PSD) centered around the zero-Doppler frequency implying that the resulting ICM is real [22], [23].

With the above remarks in mind, in this paper we borrow the ideas behind [24] to develop new architectures capable of detecting environment heterogeneity and, possibly, clustering data into homogeneous subsets. Unlike [24], we do not assume that the number of statistically homogeneous subsets within the radar reference window is known and estimate it through a penalized likelihood ratio test, which exploits the penalty terms of the Model Order Selection (MOS) rules [25], as the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the Generalized Information Criterion (GIC). More importantly, at the design stage, four different ICM structures are considered in order to take advantage of the a priori information about the operating scenario. The latter aspect represents the main advancement with respect to [24]. Actually, the conceived decision schemes solve a multiple hypothesis test comprising the null hypothesis (representative of the homogeneous environment) and multiple alternative hypotheses that differ in the number of homogeneous clutter subregions. The design is carried out by applying the Latent Variable Model (LVM) and the Expectation Maximization (EM) algorithm in conjunction with cyclic estimation procedures. Finally, the performance analysis conducted on simulated data shows that architectures devised under specific ICM structures can provide excellent detection and classification performance.

The paper is organized as follows[1]. The next section is devoted to the problem formulation. Section III contains the architecture derivations and Section IV provides some numerical examples. Concluding remarks are given in Section V.

II. Problem Formulation

In this section, we formulate the classification problem as a multiple hypothesis test. To this aim, we consider a radar system, equipped with N temporal channels which illuminates an area consisting of K range bins. The signals backscattered by these range cells are suitably conditioned and sampled by the signal-processing unit to form vectors $\boldsymbol{z}_k \in \mathbb{C}^{N\times 1}, k = 1,\ldots,K$, that can be grouped into the data matrix $\boldsymbol{Z} = [\boldsymbol{z}_1,\ldots,\boldsymbol{z}_K] \in \mathbb{C}^{N\times K}$. Now, we are interested in establishing whether or not training data are homogeneous along the range bin dimension in a clutter-dominated environment. In the heterogeneous case, we also estimate the unknown number of homogeneous subsets assuming a maximum number of subsets denoted by $M \geq 2$. Following the lead of [24], partitioning is accomplished by introducing K Independent and Identically Distributed (IID) discrete random variables, c_ks say, which, under H_{m-1} (see below), $m = 2,\ldots,M$, take on values in $\{1,\ldots,m\}$, with unknown probability mass function $P(c_k = l) = \pi_l$, $l = 1,\ldots,m \leq M$, $\sum_{l=1}^m \pi_l = 1$, and such that when $c_k = l$, then $\boldsymbol{z}_k \sim \mathcal{CN}_N(\boldsymbol{0},\boldsymbol{M}_l)$. As for the ICM of these homogeneous subsets, we suppose that the different classes, identified by a common value of c_k, share a common structure of the covariance matrix $\boldsymbol{M}$, but they have different power values $\sigma_l^2, l = 1,\ldots,M$, namely, $\boldsymbol{M}_l = \sigma_l^2 \boldsymbol{M}$. Under these assumptions, we formulate the following multiple hypothesis test

$$\begin{cases} H_0 : \boldsymbol{z}_k \sim \mathcal{CN}_N(\boldsymbol{0},\boldsymbol{M}), & k = 1,\ldots,K, \\ H_{m-1} : \boldsymbol{z}_k \sim \displaystyle\sum_{l=1}^{m} \pi_l \mathcal{CN}_N(\boldsymbol{0},\sigma_l^2\boldsymbol{M}), & k = 1,\ldots,K, \end{cases} \quad (1)$$

where $m = 2,\ldots,M$. As for the covariance structure, we assume that $\boldsymbol{M}$ can be either generic Hermitian, persymmetric such that $\boldsymbol{M}^{-1} = [\boldsymbol{M}^{-1} + \boldsymbol{J}(\boldsymbol{M}^*)^{-1}\boldsymbol{J}]/2$, real symmetric implying that $\Re e\{\boldsymbol{z}_k\}$ and $\Im m\{\boldsymbol{z}_k\}$ are IID, or centrosymmetric such that $\boldsymbol{M}^{-1} = [\boldsymbol{M}^{-1} + \boldsymbol{J}(\boldsymbol{M})^{-1}\boldsymbol{J}]/2 \in \mathbb{R}^{N\times N}$ implying again that $\Re e\{\boldsymbol{z}_k\}$ and $\Im m\{\boldsymbol{z}_k\}$ are IID.

For the next developments, we provide the expression of the Probability Density Function (PDF) of $\boldsymbol{z}_k$ under H_{m-1}, $m = 2,\ldots,M$, i.e., $f_m(\boldsymbol{z}_k;\boldsymbol{\pi}_m,\boldsymbol{\sigma}_m) = \sum_{l=1}^m \pi_l f(\boldsymbol{z}_k|c_k = l;\boldsymbol{M}_l)$, where $\boldsymbol{\pi}_m = [\pi_1,\ldots,\pi_m]^T$, $\boldsymbol{\sigma}_m = [\boldsymbol{\nu}^T(\boldsymbol{M}_1),\ldots,\boldsymbol{\nu}^T(\boldsymbol{M}_m)]^T$, and $f(\boldsymbol{z}_k|c_k = l;\boldsymbol{M}_l) = \exp\{-\mathrm{Tr}\,[\boldsymbol{M}_l^{-1}\boldsymbol{z}_k\boldsymbol{z}_k^\dagger]\}/[\pi^N \det(\boldsymbol{M}_l)]$. Finally, the PDF of $\boldsymbol{z}_k$ under H_0 is given by $f(\boldsymbol{z}_k;\boldsymbol{M}) = \exp\{-\mathrm{Tr}\,[\boldsymbol{M}^{-1}\boldsymbol{z}_k\boldsymbol{z}_k^\dagger]\}/[\pi^N \det(\boldsymbol{M})]$.

III. Architecture Design

In this section, we devise a classification architecture to solve problem (1) exploiting a penalized log-likelihood ratio test [21] whose generic structure is given by

$$\{\Lambda_{\widehat{m}}(\boldsymbol{Z}) - h(\widehat{m})\} \underset{H_0}{\overset{H_{\widehat{m}}}{\gtrless}} \eta \quad (2)$$

where η is a threshold to be set in order to guarantee a given probability of classifying data as heterogeneous when they are homogeneous[2],

$$\widehat{m} = \underset{m=2,\ldots,M}{\arg\max} \left\{\Lambda_m(\boldsymbol{Z}) - h(m)\right\}, \quad (3)$$

and the penalty term, $h(m)$, computed by neglecting the quantities that are invariant under the alternative hypotheses, is [25]

$$h(m) = \begin{cases} 2m, & \text{AIC}, \\ \log(2KN)m, & \text{BIC}, \\ m(1+\rho),\ \rho > 1, & \text{GIC}. \end{cases} \quad (4)$$

Finally, the expression of $\Lambda_m(\boldsymbol{Z})$ is given by

$$\Lambda_m(\boldsymbol{Z}) = \sum_{k=1}^{K} [\log f_m(\boldsymbol{z}_k;\widehat{\boldsymbol{\pi}}_m,\widehat{\boldsymbol{\sigma}}_m) - \log f(\boldsymbol{z}_k;\widehat{\boldsymbol{M}}_0)], \quad (5)$$

[1]In what follows, vectors and matrices are denoted by boldface lower-case and upper-case letters, respectively. The symbols $|\cdot|$, $\mathrm{Tr}\,(\cdot)$, $\det(\cdot)$, $(\cdot)^T$, $(\cdot)^\dagger$, and $(\cdot)^*$ denote modulus value, trace, determinant, transpose, conjugate transpose, and complex conjugate, respectively. $\boldsymbol{\nu}(\cdot)$ denotes a vector-valued function selecting the generally distinct entries of the matrix argument. $\mathbb{C}$ ($\mathbb{R}$) is the set of complex (real) numbers and $\mathbb{C}^{N\times M}$ ($\mathbb{R}^{N\times N}$) is the Euclidean space of $(N\times M)$-dimensional complex (real) matrices. Given $x \in \mathbb{R}$, then $\lfloor x \rfloor$ is the greatest integer less than or equal to x. The real and imaginary parts of $\boldsymbol{x} \in \mathbb{C}^{N\times 1}$ are denoted by $\Re e\{\boldsymbol{x}\}$ and $\Im m\{\boldsymbol{x}\}$, respectively. We denote by $\boldsymbol{J} \in \mathbb{R}^{N\times N}$ a permutation matrix such that $\boldsymbol{J}(l,k) = 1$ if and only if $l + k = N + 1$. Moreover, $\boldsymbol{0}$ is the null vector of proper dimension and $\boldsymbol{I}_N$ stands for the $N\times N$ identity matrix. Finally, we write $\boldsymbol{x} \sim \mathcal{CN}_N(\boldsymbol{m},\boldsymbol{M})$ if $\boldsymbol{x}$ is an N-dimensional complex normal vector with mean $\boldsymbol{m}$ and positive definite covariance matrix $\boldsymbol{M}$.

[2]Notice that the detection threshold depends on the number of alternative hypotheses.

where $\widehat{\boldsymbol{M}}_0$, $\widehat{\boldsymbol{\pi}}_m$, and $\widehat{\boldsymbol{\sigma}}_m$ are the estimates of $\boldsymbol{M}$, under H_0, $\boldsymbol{\pi}_m$, and $\boldsymbol{\sigma}_m$, respectively, obtained as described below.

Under H_0, the estimate of $\boldsymbol{M}$ is accomplished by means of the maximum likelihood approach that leads to the sample covariance matrix over $\boldsymbol{Z}$, whose expression is

$$\widehat{\boldsymbol{M}}_0=\begin{cases}\frac{1}{K}\boldsymbol{Z}\boldsymbol{Z}^\dagger, & \text{Hermitian},\\ \frac{1}{2K}\left(\boldsymbol{Z}\boldsymbol{Z}^\dagger+\boldsymbol{J}\boldsymbol{Z}^*\boldsymbol{Z}^T\boldsymbol{J}\right), & \text{Persymmetric},\\ \frac{1}{K}\Re e\{\boldsymbol{Z}\boldsymbol{Z}^\dagger\}, & \text{Symmetric},\\ \frac{1}{2K}\left(\Re e\{\boldsymbol{Z}\boldsymbol{Z}^\dagger\}+\boldsymbol{J}\Re e\{\boldsymbol{Z}^*\boldsymbol{Z}^T\}\boldsymbol{J}\right), & \text{Centrosymmetric}.\end{cases} \tag{6}$$

On the other hand, under the generic H_{m-1} hypothesis, we resort to the EM algorithm coupled with a cyclic procedure to come up with suitable estimates of $\boldsymbol{M}$, σ_l^2, and $\boldsymbol{\pi}_m$ assuming different symmetries for $\boldsymbol{M}$. Let us recall that the EM estimation procedure consists of the E-step and the M-step [26] that in a iterative way maximize the log-likelihood function using closed-form estimation updates. Now, regardless the symmetry of $\boldsymbol{M}$, the E-step at the $(h-1)$th iteration leads to

$$q_k^{(h-1)}(l)=\frac{f(\boldsymbol{z}_k|c_k=l;\widehat{\boldsymbol{M}}_l^{(h-1)})\widehat{\pi}_l^{(h-1)}}{\sum\limits_{i=1}^{m} f(\boldsymbol{z}_k|c_k=i;\widehat{\boldsymbol{M}}_i^{(h-1)})\widehat{\pi}_i^{(h-1)}}, \tag{7}$$

$l=1,\dots,m$ and $k=1,\dots,K$, where $\widehat{\boldsymbol{M}}_l^{(h-1)}$ (whose expression depends on the specific structure) and $\widehat{\pi}_l^{(h-1)}$ are the available estimates. As for the M-step, it is not difficult to show that the prior updates are given by

$$\widehat{\pi}_l^{(h)}=\frac{1}{K}\sum_{k=1}^{K} q_k^{(h-1)}(l), \quad l=1,\dots,m, \tag{8}$$

that depend on the assumed ICM structure through $q_k^{(h-1)}(l)$. It still remains to come up with the updates for $\boldsymbol{M}$ and σ_l^2. To this end, we have to solve the following problem

$$\max_{\boldsymbol{\sigma}_m}\sum_{k=1}^{K}\sum_{l=1}^{m} q_k^{(h-1)}(l)[-\log\det(\boldsymbol{M}_l)-\mathrm{Tr}\,(\boldsymbol{M}_l^{-1}\boldsymbol{z}_k\boldsymbol{z}_k^\dagger)], \tag{9}$$

that can be specialized according to the considered ICM structures and, hence, leading to different estimation updates. Thus, let us start from the generic Hermitian structure, then, following the lead of [24], we come up with the following cyclic procedure (the iteration index t, $t=1,\dots,t_{max}$, is in addition to the EM iteration index h)

$$(\widehat{\sigma}_l^2)^{(t),(h)}=\sum_{k=1}^{K}\bar{q}_k^{(h-1)}(l)\boldsymbol{z}_k^\dagger[\widehat{\boldsymbol{M}}^{(t-1),(h)}]^{-1}\boldsymbol{z}_k/N, \tag{10}$$

$$\widehat{\boldsymbol{M}}^{(t),(h)}=\frac{1}{K}\sum_{k=1}^{K}\sum_{l=1}^{m} q_k^{(h-1)}(l)\frac{\boldsymbol{Z}_k}{(\widehat{\sigma}_l^2)^{(t),(h)}}, \tag{11}$$

where $\bar{q}_k^{(h-1)}(l)=q_k^{(h-1)}(l)/\sum_{n=1}^{K} q_n^{(h-1)}(l)$, $\boldsymbol{Z}_k=\boldsymbol{z}_k\boldsymbol{z}_k^\dagger$, and $\widehat{\boldsymbol{M}}^{(0),(h)}=\widehat{\boldsymbol{M}}^{(t_{max}),(h-1)}$ for $h>1$. As for the other cases, in the following we provide the final updates and some hints related to the derivations of them due to the limited number of pages. Therefore, in the case of persymmetry, exploiting the equality $\mathrm{Tr}\,[\boldsymbol{M}^{-1}\boldsymbol{Z}\boldsymbol{Z}^\dagger]=\mathrm{Tr}\,[\boldsymbol{M}^{-1}(\boldsymbol{Z}\boldsymbol{Z}^\dagger+\boldsymbol{J}\boldsymbol{Z}^*\boldsymbol{Z}^T\boldsymbol{J})/2]$, it is possible to show that the estimates of σ_l^2 and $\boldsymbol{M}$ become

$$(\widehat{\sigma}_l^2)^{(t),(h)}=\sum_{k=1}^{K}\frac{\bar{q}_k^{(h-1)}(l)}{2N}\mathrm{Tr}\,[(\widehat{\boldsymbol{M}}^{(t-1),(h)})^{-1}\boldsymbol{Z}_{k,J}], \tag{12}$$

$$\widehat{\boldsymbol{M}}^{(t),(h)}=\frac{1}{2K}\sum_{k=1}^{K}\sum_{l=1}^{m} q_k^{(h-1)}(l)\frac{\boldsymbol{Z}_{k,J}}{(\widehat{\sigma}_l^2)^{(t),(h)}}, \tag{13}$$

where $\boldsymbol{Z}_{k,J}=\boldsymbol{Z}_k+\boldsymbol{J}\boldsymbol{Z}_k^*\boldsymbol{J}$. As for the symmetric structure, since $\boldsymbol{M}\in\mathbb{R}^{N\times N}$ it is possible to recast the PDF to account for the independence of the real and imaginary parts of data. As a consequence, the resulting estimates can be written

$$(\widehat{\sigma}_l^2)^{(t),(h)}=\sum_{k=1}^{K}\frac{\bar{q}_k^{(h-1)}(l)}{N}\mathrm{Tr}\,[(\widehat{\boldsymbol{M}}^{(t-1),(h)})^{-1}\Re e\{\boldsymbol{Z}_k\}], \tag{14}$$

$$\widehat{\boldsymbol{M}}^{(t),(h)}=\frac{1}{K}\sum_{k=1}^{K}\sum_{l=1}^{m} q_k^{(h-1)}(l)\frac{\Re e\{\boldsymbol{Z}_k\}}{(\widehat{\sigma}_l^2)^{(t),(h)}}. \tag{15}$$

Finally, in the last case, we assume that $\boldsymbol{M}$ is centrosymmetric. As consequence, the identity $\mathrm{Tr}\,[\boldsymbol{M}^{-1}\Re e\{\boldsymbol{Z}\boldsymbol{Z}^\dagger\}]=\mathrm{Tr}\,[\boldsymbol{M}^{-1}(\Re e\{\boldsymbol{Z}\boldsymbol{Z}^\dagger\}+\boldsymbol{J}\Re e\{\boldsymbol{Z}^*\boldsymbol{Z}^T\}\boldsymbol{J})/2]$ is valid and it yields

$$(\widehat{\sigma}_l^2)^{(t),(h)}=\sum_{k=1}^{K}\frac{\bar{q}_k^{(h-1)}(l)}{2N}\mathrm{Tr}\,[(\widehat{\boldsymbol{M}}^{(t-1),(h)})^{-1}\Re e\{\boldsymbol{Z}_{k,J}\}], \tag{16}$$

$$\widehat{\boldsymbol{M}}^{(t),(h)}=\frac{1}{2K}\sum_{k=1}^{K}\sum_{l=1}^{m} q_k^{(h-1)}(l)\frac{\Re e\{\boldsymbol{Z}_{k,J}\}}{(\widehat{\sigma}_l^2)^{(t),(h)}}. \tag{17}$$

As a final remark, an initial estimate of the unknown parameters is necessary to start the procedures. The details of a possible initialization strategy will be described in the next section.

IV. Performance assessment

The analysis is conducted by means of simulated data and considering the probability of correct classification, P_{cc}, and the probability of correct detection, P_d, as performance metrics. To this end, standard Monte Carlo counting techniques are exploited to estimate the threshold assuring $P_{fa}=10^{-3}$ based on $100/P_{fa}$ runs. Both P_d and P_{cc} are estimated over 10^4 independent trials. As for the ICM, we suppose the prevalence of the clutter contribution assuming an exponentially shaped clutter covariance. As a consequence, the (i,j)th entry of the ICM structure is given by $\nu^{|i-j|}$, where $\nu=0.9$. The clutter power under H_0 is set to 30 dB (assuming as reference $\sigma_{ref}^2=1$).

The iteration number for both the cyclic procedures is equal to 10 (it has been proved in [24] that a small number is sufficient for convergence) and the maximum number of possible classes is set as $M=5$ in order to limit the computational burden. As for the GIC-based rule in (4), we set $\rho=2$.

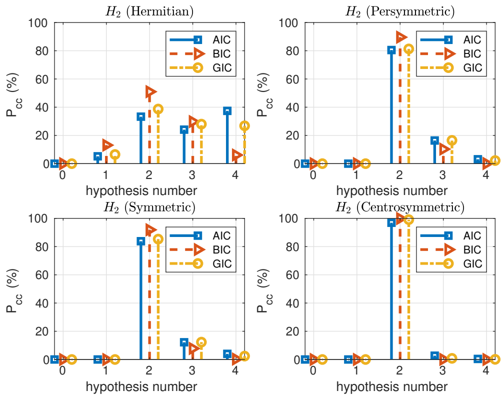

Fig. 1. P_{cc} (%) of the three MOS-based architectures for a simulated heterogeneous scenario with 3 homogeneous subsets using $K_1 = 10$, $K_2 = 10$ and $K_3 = 4$ (H_2).

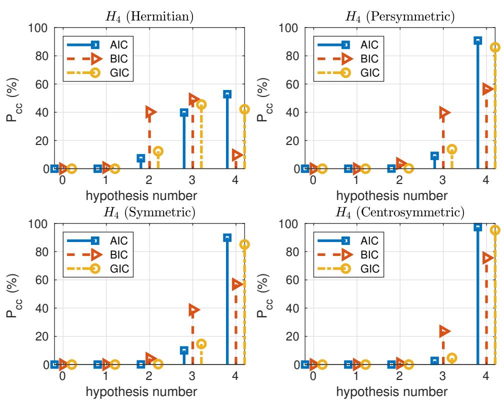

Fig. 2. P_{cc} (%) of the three MOS-based architectures for a simulated heterogeneous scenario with 5 homogeneous subsets using $K_1 = 8$, $K_2 = 6$, $K_3 = 5$, $K_4 = 9$ and $K_5 = 4$ (H_4).

Under each mth hypothesis, we choose equiprobable priors for the initialization of π_ls, namely, $\pi_l = 1/m$, whereas the initial value $\widehat{\boldsymbol{M}}^{(0),(0)}$ is equal to $\widehat{\boldsymbol{M}}_0$. The following initialization procedure for the m clutter power levels is pursued. Specifically, $\forall k = 1, \dots, K$, we compute

$$g(k) = \begin{cases} \frac{1}{N}\mathrm{Tr}\,[\widehat{\boldsymbol{M}}_0^{-1}\boldsymbol{Z}_k], & \text{Hermitian}, \\ \frac{1}{2N}\mathrm{Tr}\,[\widehat{\boldsymbol{M}}_0^{-1}\boldsymbol{Z}_{k,J}], & \text{Persymmetric}, \\ \frac{1}{N}\mathrm{Tr}\,[\widehat{\boldsymbol{M}}_0^{-1}\Re e\{\boldsymbol{Z}_k\}], & \text{Symmetric}, \\ \frac{1}{2N}\mathrm{Tr}\,[\widehat{\boldsymbol{M}}_0^{-1}\Re e\{\boldsymbol{Z}_{k,J}\}], & \text{Centrosymmetric}. \end{cases} \quad (18)$$

Then, we sort the $g(k)$s in ascending order, $\tilde{g}(1) \leq \tilde{g}(2) \leq \ldots \leq \tilde{g}(K)$. Finally, the mean values computed over the $\lfloor K/m \rfloor$ subsets of the ordered powers are used to set the initial value of the clutter power levels, namely,

$$\widehat{\sigma}_l^2 = \frac{1}{\lfloor \frac{K}{m} \rfloor} \sum_{i=(l-1)\lfloor \frac{K}{m} \rfloor + 1}^{l \lfloor \frac{K}{m} \rfloor} \tilde{g}(i), \quad l = 1, \dots, m+1. \quad (19)$$

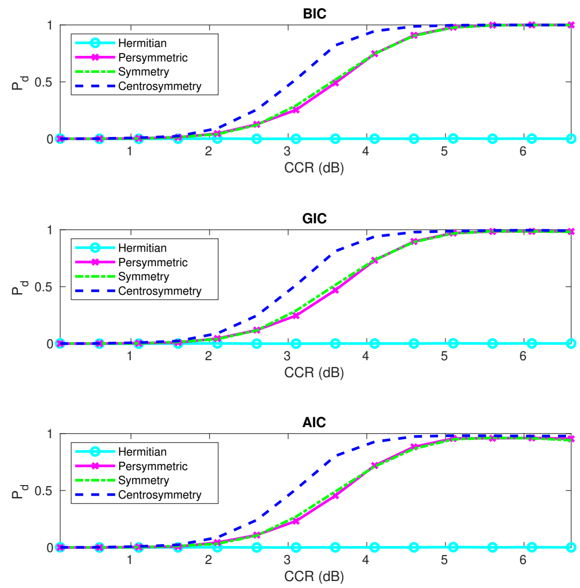

Fig. 3. P_d for varying clutter power ratio: two homogeneous subsets with $K_1 = 10$ and $K_2 = 8$ (σ_1^2 is set to 20 dB).

Summarizing, $\widehat{\boldsymbol{M}}_l^{(0)} = \widehat{\sigma}_l^2 \widehat{\boldsymbol{M}}_0$. All the illustrative examples assume $N = 16$.

In Figures 1 and 2, the P_{cc}s (%) are shown for two heterogeneous scenarios with 3 and 5 homogeneous subsets corresponding to H_2 and H_4, respectively. Under H_2, we assume $K_1 = 10$, $K_2 = 10$, $K_3 = 4$, and [20, 30, 40] dB for the clutter power levels, whereas under H_4, we set $K_1 = 8$, $K_2 = 6$, $K_3 = 5$, $K_4 = 9$, $K_5 = 4$, and clutter power levels [20, 25, 30, 35, 40] dB. The histograms clearly highlight that each MOS-based rule conceived under the centrosymmetric assumption for the ICM structure is superior to the analogous counterpart based upon the other structures at least for the considered parameters. It is also important to notice that the poor performance of the rules based on the most general Hermitian case is due to the lack of an adequate number of data.

Finally, the performance in terms of P_d as a function of the clutter power ratio is shown in Figure 3. For simplicity, we assume that only one clutter edge is present with $K_1 = 10$ and $K_2 = 8$ (i.e., H_1 is in force). The value of σ_1^2 is set to 20 dB and σ_2^2 changes according to the Clutter-to-Clutter Ratio (CCR) defined as σ_2^2/σ_1^2. Inspection of the figure confirms the superior performance of the decision schemes relying on a centrosymmetric ICM structure with a gain of about 0.8 dB at $P_d = 0.9$ over those based upon persymmetric and symmetric structures. Again, due to the low volume of available data, the Hermitian structure does not lead to satisfying performance. In fact, for the considered parameter setting, the curves associated with this case do not achieve $P_d = 0.1$.

V. Conclusions

In this paper, the problem of detecting heterogeneous training data and clustering them according to their unknown statistical properties is solved through the joint exploitation of the LVM, the EM algorithm, and cyclic optimization procedures. In addition, specific structures for the ICM have been considered at the design stage. Remarkably, the proposed architectures can be used to identify homogeneous subregions without any a priori information about their number. The performance analysis on simulated data has shown that the devised architectures represent an effective means to detect heterogeneity and partition data into homogeneous subsets even when their cardinality is low.

Future research tracks may comprise the design of classification architectures also accounting for possible outliers and the (at least asymptotic) statistical characterization of the considered decision schemes.

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