An Exponential Time Algorithm for Multilevel Quantization in Distributed Detection

Gökhan Gül, Michael Baßler
Fraunhofer Institute for Microengineering and Microsystems (IMM)
Carl-Zeiss-Str. 18-20, 55129 Mainz, Germany
{Goekhan.Guel, Michael.Bassler}@imm.fraunhofer.de

***Abstract*—A multilevel quantization algorithm is proposed for distributed sensor networks where each sensor transmits a summary of its observation to the fusion center and the fusion center makes the final decision. The proposed scheme comprises a person-by-person optimum quantization at each sensor and an optimum fusion rule at the fusion center. The complexity of the algorithm is exponential yet practical for small to moderate networks. The rationale behind the proposed scheme is that from small to moderate networks, the naive algorithm is computationally impractical while practically applicable algorithms experience performance losses. Experimental results indicate that the proposed scheme is able to fill in this gap and provide near optimal solutions.**

***Index Terms*—Distributed detection, quantization, cognitive radio, wireless sensor networks, signal detection**

I. Introduction

Detection of events in a distributed manner has been studied in various applications such as cognitive radio or industrial monitoring [1]. In parallel sensor networks, each sensor makes an observation about a certain phenomenon and transmits it to the fusion center for a final decision. Transmitting observations or their likelihood ratios (LRs) to the fusion center under a certain statistical model -called centralized detection- is optimal, if the sensors are statistically independent, since LRs are sufficient statistics and maximize the detection performance at the fusion center [2]. Such a transmission is impractical for wireless channels due to bandwidth and energy limitations [3]. Therefore, quantized versions of observations are transmitted to the fusion center -called decentralized detection- subject to the constraint that the loss of detection performance due to quantization is insignificant [4].

Decentralized detection was first studied in [5] for binary sensor quantization without considering the design of data fusion algorithms. This work was later extended by [6], [7] generalizing the binary quantization to multilevel quantization. In order to be able to improve the performance further and reduce the computational cost, pseudo objective functions based on distances between probability distributions are used for optimization. In [8] the J-divergence, in [9] the deflection distance, and in a more recent work [10], both the J-divergence and Bhattacharyya distances are considered.

Among all distances, only the Chernoff Information (CI) has the asymptotic optimality property [11] and maximizing the CI amounts to minimizing the upper bound on the true objective function [12]. Based on this idea, the CI was first studied in [11] for multilevel quantization. Scalable solutions were obtained in [13] by locally maximizing the CI without providing algorithmic solutions. In [14] Chernoff information and deflection distance were used for the locally optimum quantization of independent and identically distributed sensors. This work was later extended by the same authors to independent but not necessarily identically distributed sensor observations [15]. In both [14] and [15] explicit algorithmic solutions were presented.

Since the optimality of CI is invalid for any finite number of sensors, it is desirable to consider the true objective function with practical algorithmic solutions. In [16], a fast multilevel quantization algorithm is proposed considering a Gaussian approximation to the overall test statistic. Both [15] and [16] optimize the performance by using person-by-person optimum (PBPO) quantization at the sensors and asymptotic optimization at the fusion center. In [17] exact error probability is considered for the fusion center together with PBPO for the sensors but the condition is exact only for binary quantization and the performance starts to degrade for non-binary and non-identical sensors. Therefore, for small to moderate number of sensors and for non-identically distributed sensor observations, the performance of available practically applicable schemes are open to improvement.

In this paper a multilevel quantization algorithm is proposed to fill in this gap. The proposed scheme quantizes the sensor observations via considering PBPO optimization at the sensors and an optimal fusion at the fusion center. The algorithm is iterative and achieves near optimal solutions both for identically as well as non-identically distributed sensors. Numerical results indicate its superiority over [14], [15], which is known to perform better than [16] and [17] for non-identical sensors.

The rest of this paper is organized as follows. In Section II, the decentralized detection problem is introduced. In Section III, the proposed algorithm is derived and its computational complexity is analyzed. In Section IV, the performance of the proposed algorithm is evaluated and compared to [14], [15]. Finally in Section V, the paper is concluded.

II. Decentralized Detection

Consider a distributed detection network with K decision makers $\phi_1, \ldots, \phi_K$ and a fusion center γ as illustrated by Figure 1. Each sensor ϕ_k makes an observation $y_k \in \Omega_k$ from a certain phenomenon, where Ω_k is an interval, and gives a

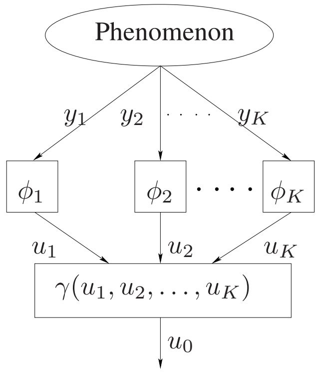

Fig. 1. Distributed detection network with K decision makers, each represented by the decision rule ϕ, and a fusion center associated with the fusion rule γ.

multilevel decision $u_k \in \{0, \ldots, N_k - 1\}$. The phenomenon is modeled by a binary hypothesis test

$$\begin{aligned} \mathcal{H}_0 &: Y_k \sim F_0^k, \\ \mathcal{H}_1 &: Y_k \sim F_1^k, \end{aligned} \tag{1}$$

where the random variables (r.v.s) Y_k corresponding to the observations y_k are mutually independent and follow the probability distribution function F_0^k (or F_1^k), which has the density function f_0^k (or f_1^k), conditioned on the hypothesis $\mathcal{H}_0$ (or $\mathcal{H}_1$). The fusion center receives multilevel decisions from all sensors and gives a binary decision u_0.

Optimum quantization, which minimizes the error probability of the fusion center is known to be the monotone likelihood ratio test [2]. Assuming that the likelihood ratio function $l_k = f_1^k / f_0^k$ is strictly monotone, thresholding can be done directly over the observations. Thus, the decisions can be obtained by

$$\phi_k(y_k) = u_k^{i_k} \quad \text{if} \quad \lambda_k^{i_k - 1} \leqslant y_k < \lambda_k^{i_k}, \tag{2}$$

where $\lambda_k^{i_k}$ denotes the thresholds, $k \in \{1, \ldots, K\}$ denotes the indices of sensors and $i_k \in \{1, \ldots, N_k\}$ denotes the indices of the multilevel decision u_k for the kth sensor. The upper and lower thresholds are given by $\lambda_k^0 := \inf \Omega_k$ and $\lambda_k^{N_k} := \sup \Omega_k$, leaving $N_k - 1$ unknown thresholds to be determined per sensor. From (1) and (2) the probability mass functions of the decisions conditioned on the hypothesis $\mathcal{H}_m$, $m \in \{0, 1\}$, can be found by

$$p_m^k(u_k^{i_k}) = F_m[\lambda_k^{i_k - 1} \leqslant l_k(Y_k) < \lambda_k^{i_k}]. \tag{3}$$

Let p_0 and p_1 denote the joint probability mass functions of the random variables U_k, corresponding to the multilevel decisions u_k, conditioned on the hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. Furthermore, let the transmitted decisions u_k be reformed by the fusion center optimally [2], [9] as

$$u_k := \log \frac{p_1^k(u_k)}{p_0^k(u_k)}.$$

Then, the optimum test at the fusion center can be obtained by

$$\log \frac{p_1(u_1, \ldots, u_K)}{p_0(u_1, \ldots, u_K)} = \sum_{k=1}^{K} \log \frac{p_1^k(u_k)}{p_0^k(u_k)} = \sum_{k=1}^{K} u_k \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda_0, \tag{4}$$

where $\lambda_0 \in \mathbb{R}$ is a suitable threshold. Since test statistic in (4) corresponds to the summation of K random variables U_k, the probability mass function of the sum can be obtained by K-fold convolution of the marginal mass functions as

$$g_m(z) = \sum_{i_1=1}^{N_1} \cdots \sum_{i_K=1}^{N_K} p_m^1(u_1^{i_1}) \cdots p_m^K(u_K^{i_K}) \delta\Big(\sum_{k=1}^{K} u_k^{i_k} - z\Big), \tag{5}$$

where δ is the dirac delta function.

III. Optimization of The Sensor Network

In this section an iterative algorithm will be derived, which is capable of quantizing both identically as well as non-identically distributed sensor observations.

A. *Derivation of the Algorithm*

Let all non-trivial thresholds of the sensors be represented by the set of parameters

$$\boldsymbol{\lambda} = \{\lambda_k^{i_k} : i_k \in \{1, \ldots, N_k - 1\}, k \in \{1, \ldots, K\}\},$$

Then, considering (5) the minimum error probability can explicitly be written as

$$P_E = \min_{\boldsymbol{\lambda}} \sum_{i_1=1}^{N_1} \cdots \sum_{i_K=1}^{N_K} \min\left(\prod_{k=1}^{K} p_0^k(u_k^{i_k}), \prod_{k=1}^{K} p_1^k(u_k^{i_k})\right), \tag{6}$$

where p_m^k are dependent on $\lambda_k^{i_k}$ through (3). An exact calculation of (6) is of exponential complexity even for identically distributed sensor observations, since there are counterexamples showing that identical sensor decisions are not always optimum for identically distributed sensors [18]. In order to simplify the problem consider the following PBPO assumption.

Assumption III.1. *All thresholds except for the threshold $\lambda_k^{i_k}$ are known in solving the optimization problem* (6).

Using Assumption III.1, except for $p_m^k(u_k^{i_k})$, $p_m^k(u_k^{i_k+1})$, all other terms are some constants. Hence, (6) can be written as

$$\begin{aligned} P_E = \min_{\lambda_k^{i_k}} & \sum_{i_1=1}^{N_1} \cdots \sum_{i_{k-1}=1}^{N_{k-1}} \sum_{i_{k+1}=1}^{N_{k+1}} \cdots \\ & \sum_{i_K=1}^{N_K} \min\left(p_0^k(u_k^{i_k}) \prod_{\substack{1 \leqslant t \leqslant K \\ t \neq k}} p_0^t(u_t^{i_t}),\, p_1^k(u_k^{i_k}) \prod_{\substack{1 \leqslant t \leqslant K \\ t \neq k}} p_1^t(u_t^{i_t})\right) \\ & + \min\left(p_0^k(u_k^{i_k+1}) \prod_{\substack{1 \leqslant t \leqslant K \\ t \neq k}} p_0^t(u_t^{i_t}),\, p_1^k(u_k^{i_k+1}) \prod_{\substack{1 \leqslant t \leqslant K \\ t \neq k}} p_1^t(u_t^{i_t})\right) \\ & + C_k, \end{aligned} \tag{7}$$

where C_k is a constant. For simplicity, Equation (7) can be reformulated using a bijective mapping running over the index n as follows

$$n : \{i_1, \ldots, i_{k-1}, i_{k+1}, \ldots, i_K\} \mapsto \{1, \ldots, N\}, \tag{8}$$

where

$$M_k = \prod_{\substack{1\leqslant t\leqslant K\\ t\neq k}} N_t. \tag{9}$$

For example, if $N_k = 3$ for all k and $K = 3$ we have $\{1,1\} \rightarrow 1$, $\{1,2\} \rightarrow 2$, $\{1,3\} \rightarrow 3$, $\{2,1\} \rightarrow 4, \ldots, \{3,3\} \rightarrow 9$. Accordingly (7) can be written as

$$\begin{aligned} P_E =& \min_{\lambda_k^{i_k}} \sum_{n=1}^{M_k} \min\left(p_0^k(u_k^{i_k})c_0^k(n), p_1^k(u_k^{i_k})c_1^k(n)\right) \\ &+ \min\left(p_0^k(u_k^{i_k+1})c_0^k(n), p_1^k(u_k^{i_k+1})c_1^k(n)\right) + C_k, \end{aligned} \tag{10}$$

where c_0^k and c_1^k are some known vectors, which can be calculated as

$$\begin{aligned} c_0^k(n) &= \sum_{i_1=1}^{N_1} \cdots \sum_{i_{k-1}=1}^{N_{k-1}} \sum_{i_{k+1}=1}^{N_{k+1}} \sum_{i_K=1}^{N_K} \prod_{\substack{1\leqslant t\leqslant K\\ t\neq k}} p_0^t(u_t^{i_t}), \\ c_1^k(n) &= \sum_{i_1=1}^{N_1} \cdots \sum_{i_{k-1}=1}^{N_{k-1}} \sum_{i_{k+1}=1}^{N_{k+1}} \sum_{i_K=1}^{N_K} \prod_{\substack{1\leqslant t\leqslant K\\ t\neq k}} p_1^t(u_t^{i_t}), \end{aligned} \tag{11}$$

using the index mapping stated by (8). Let $|\cdot|$ denote the absolute value function and let us consider the identity given by

$$\min(a,b) = \frac{a+b}{2} - \frac{|a-b|}{2}. \tag{12}$$

Accordingly, by using (12) one can write (10) as

$$\begin{aligned} P_E =& \min_{\lambda_k^{i_k}} \sum_{n=1}^{M_k} \frac{p_0^k(u_k^{i_k})c_0^k(n) + p_1^k(u_k^{i_k})c_1^k(n)}{2} \\ &- \frac{|p_0^k(u_k^{i_k})c_0^k(n) - p_1^k(u_k^{i_k})c_1^k(n)|}{2} \\ &+ \frac{p_0^k(u_k^{i_k+1})c_0^k(n) + p_1^k(u_k^{i_k+1})c_1^k(n)}{2} \\ &- \frac{|p_0^k(u_k^{i_k+1})c_0^k(n) - p_1^k(u_k^{i_k+1})c_1^k(n)|}{2} + C_k. \end{aligned} \tag{13}$$

Exchanging the sum and min terms, the minimum can be obtained by solving

$$\begin{aligned} \frac{dP_E}{d\lambda_k^{i_k}} =& \frac{1}{2} \sum_{n=1}^{M_k} \frac{d}{d\lambda_k^{i_k}} \Big[(c_0^k(n)(p_0^k(u_k^{i_k}) + p_0^k(u_k^{i_k+1})) \\ &+ c_1^k(n)(p_1^k(u_k^{i_k}) + p_1^k(u_k^{i_k+1})) \\ &- |p_0^k(u_k^{i_k})c_0^k(n) - p_1^k(u_k^{i_k})c_1^k(n)| \\ &- |p_0^k(u_k^{i_k+1})c_0^k(n) - p_1^k(u_k^{i_k+1})c_1^k(n)| + C_k \Big] = 0, \end{aligned}$$

which is equivalent to

$$\begin{aligned} &\frac{1}{2} \sum_{n=1}^{M_k} \left(f_0^k(\lambda_k^{i_k})c_0^k(n) - f_1^k(\lambda_k^{i_k})c_1^k(n)\right) \\ &\Bigg(\frac{p_0^k(u_k^{i_k+1})c_0^k(i) - p_1^k(u_k^{i_k+1})c_1^k(n)}{|p_0^k(u_k^{i_k+1})c_0^k(n) - p_1^k(u_k^{i_k+1})c_1^k(n)|} \\ &- \frac{p_0^k(u_k^{i_k})c_0^k(n) - p_1^k(u_k^{i_k})c_1^k(n)}{|p_0^k(u_k^{i_k})c_0^k(n) - p_1^k(u_k^{i_k})c_1^k(n)|} \Bigg) = 0, \end{aligned} \tag{14}$$

by making use of the relations given by

$$\begin{aligned} \frac{dp_0^k(u_k^{i_k})}{d\lambda_k^{i_k}} &= f_0^k(\lambda_k^{i_k}), & \frac{dp_1^k(u_k^{i_k})}{d\lambda_k^{i_k}} &= f_1^k(\lambda_k^{i_k}), \\ \frac{dp_0^k(u_k^{i_{k+1}})}{d\lambda_k^{i_k}} &= -f_0^k(\lambda_k^{i_k}), & \frac{dp_1^k(u_k^{i_{k+1}})}{d\lambda_k^{i_k}} &= -f_1^k(\lambda_k^{i_k}), \end{aligned}$$

and

$$\frac{d|r(n;\lambda_k^{i_k})|}{d\lambda_k^{i_k}} = \frac{r(n;\lambda_k^{i_k})}{|r(n;\lambda_k^{i_k})|} \frac{dr(n;\lambda_k^{i_k})}{d\lambda_k^{i_k}},$$

where r is a real differentiable function. Let us consider the sets

$$\begin{aligned} \mathcal{N}_1 =& \{n : c_0^k(n)p_0^k(u_k^{i_k}) > c_1^k(n)p_1^k(u_k^{i_k}) \\ &\wedge c_0^k(n)p_0^k(u_k^{i_k+1}) < c_1^k(n)p_1^k(u_k^{i_k+1})\}, \\ \mathcal{N}_2 =& \{n : c_0^k(n)p_0^k(u_k^{i_k}) < c_1^k(n)p_1^k(u_k^{i_k}) \\ &\wedge c_0^k(n)p_0^k(u_k^{i_k+1}) > c_1^k(n)p_1^k(u_k^{i_k+1})\}, \\ \mathcal{N}_3 =& \{n : c_0^k(n)p_0^k(u_k^{i_k}) < c_1^k(n)p_1^k(u_k^{i_k}) \\ &\wedge c_0^k(n)p_0^k(u_k^{i_k+1}) = c_1^k(n)p_1^k(u_k^{i_k+1})\}, \\ \mathcal{N}_4 =& \{n : c_0^k(n)p_0^k(u_k^{i_k}) > c_1^k(n)p_1^k(u_k^{i_k}) \\ &\wedge c_0^k(n)p_0^k(u_k^{i_k+1}) = c_1^k(n)p_1^k(u_k^{i_k+1})\}, \\ \mathcal{N}_5 =& \{n : c_0^k(n)p_0^k(u_k^{i_k}) = c_1^k(n)p_1^k(u_k^{i_k}) \\ &\wedge c_0^k(n)p_0^k(u_k^{i_k+1}) < c_1^k(n)p_1^k(u_k^{i_k+1})\}, \\ \mathcal{N}_6 =& \{n : c_0^k(n)p_0^k(u_k^{i_k}) = c_1^k(n)p_1^k(u_k^{i_k}) \\ &\wedge c_0^k(n)p_0^k(u_k^{i_k+1}) > c_1^k(n)p_1^k(u_k^{i_k+1})\}. \end{aligned} \tag{15}$$

Then, Equation (14) can be rewritten as

$$\begin{aligned} &- \sum_{n\in\mathcal{N}_1} f_0^k(\lambda_k^{i_k})c_0^k(n) - f_1^k(\lambda_k^{i_k})c_1^k(n) \\ &+ \sum_{n\in\mathcal{N}_2} f_0^k(\lambda_k^{i_k})c_0^k(n) - f_1^k(\lambda_k^{i_k})c_1^k(n) \\ &+ \frac{1}{2} \sum_{n\in\mathcal{N}_3\cap\mathcal{N}_6} f_0^k(\lambda_k^{i_k})c_0^k(n) - f_1^k(\lambda_k^{i_k})c_1^k(n) \\ &- \frac{1}{2} \sum_{n\in\mathcal{N}_4\cap\mathcal{N}_5} f_0^k(\lambda_k^{i_k})c_0^k(n) - f_1^k(\lambda_k^{i_k})c_1^k(n) = 0, \end{aligned} \tag{16}$$

which is equivalent to

$$l_k(\lambda_k^{i_k}) = \frac{f_1^k(\lambda_k^{i_k})}{f_0^k(\lambda_k^{i_k})} = \frac{2d_1 - 2d_2 - d_3 + d_4}{2d_5 - 2d_6 - d_7 + d_8}, \tag{17}$$

where

$$\begin{aligned} d_1 &= \sum_{n\in\mathcal{N}_1} c_0^k(n), & d_2 &= \sum_{n\in\mathcal{N}_2} c_0^k(n), \\ d_3 &= \sum_{n\in\mathcal{N}_3\cap\mathcal{N}_6} c_0^k(n), & d_4 &= \sum_{n\in\mathcal{N}_4\cap\mathcal{N}_5} c_0^k(n), \\ d_5 &= \sum_{n\in\mathcal{N}_1} c_1^k(n), & d_6 &= \sum_{n\in\mathcal{N}_2} c_1^k(n), \\ d_7 &= \sum_{n\in\mathcal{N}_3\cap\mathcal{N}_6} c_1^k(n), & d_8 &= \sum_{n\in\mathcal{N}_4\cap\mathcal{N}_5} c_1^k(n). \end{aligned} \tag{18}$$

Algorithm 1 Iterative multilevel observation quantizer

Input: f_0, f_1, and initialized $\lambda_k^{i_k}$, p_0^k, p_1^k
Output: Updated thresholds $\lambda_k^{i_k}$ and probabilities p_0^k, p_1^k

do
  for each $k \in \{1, \ldots, K\}$ **do**
    Find c_0^k and c_1^k using (11)
    for each $i_k \in \{1, \ldots, N_k\}$ **do**
      Find the index sets $\mathcal{N}_1$-$\mathcal{N}_6$ using (15)
      Update thresholds $\lambda_k^{i_k}$ by using (19)
      Update probabilities p_0^k, p_1^k by using (3)
    end for
  end for
while $\forall \lambda_{k,i_k}$ converges or P_E doesn't decrease anymore

Assuming that the inverse function of l_k exists and is denoted by l_k^{-1}, the threshold in (17) can eventually be computed as

$$\lambda_k^{i_k} = l_k^{-1}\left(\frac{2d_1 - 2d_2 - d_3 + d_4}{2d_5 - 2d_6 - d_7 + d_8}\right). \tag{19}$$

Remark III.1. *Without loss of generality, d_3, d_4, d_7 and d_8 may be assumed zero for continuous valued observations since $\mathcal{N}_3$, $\mathcal{N}_4$, $\mathcal{N}_5$, $\mathcal{N}_6$ are almost surely empty sets due the equality conditions.*

B. Complexity Analysis

The proposed scheme which iteratively computes the optimum quantization thresholds is given by Algorithm 1. For each k, calculating c_0^k and c_1^k requires $\mathcal{O}(KM_k)$ computations, where $\mathcal{O}(\cdot)$ is the standard Landau notation. Given c_0^k and c_1^k, calculating $\mathcal{N}_1$-$\mathcal{N}_6$ for each i_k and k requires $\mathcal{O}(KM_k)$ operations. Since calculating $\lambda_k^{i_k}$ by finding d_1-d_8 requires no more than $\mathcal{O}(KM_k)$ computations and finding the probabilities p_0^k, p_1^k is independent of K, the overall computational complexity of Algorithm 1 is $\mathcal{O}(K^2M)$, which is equivalent to $\mathcal{O}(K^2N^K)$ if $N = N_k$ for all k, where $M = \sum_k M_k$. Remember that the naive solution for the same problem requires $\mathcal{O}(d^{NK+1}(N^K + K))$ computations, where d is the total number of discretization points of the domain of Y_k [2]. Although it is still exponential, the complexity of the proposed scheme is exponentially faster than the naive solution. Hence, it makes a solution from small to moderate number of sensor networks feasible with ordinary computers.

IV. Numerical Results

In this section, performance of the proposed algorithm is evaluated over identically as well as non-identically distributed independent sensor observations.

A. Identically distributed observations

Consider the uniform vs. linear (UL) and Gaussian vs. Gaussian (GG) hypothesis testing problems,

$$\mathcal{H}_m : Y_k \sim f_m(y) = \frac{1}{2} y^m \mathbf{1}_{\{0 \leqslant y \leqslant 2\}}(y), \qquad m \in \{0,1\},$$
$$\mathcal{H}_m : Y_k \sim f_m(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(y-m)^2}{2\sigma^2}}, \qquad m \in \{0,1\},$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. In the simulations $\lambda_k^{i_k}$ are initialized uniformly on $[\lambda_k^0, \lambda_k^{N_k} - t]$ for some suitable t and the minimum P_E is picked over a fixed number of iterations, e.g. 100. Figures 2 and 3 illustrate the results of the UL- and GG-problems, respectively, with 1-, 2- and 3-bit quantizations for various total number of sensors. In all cases the proposed scheme provides lower error probabilities than its Chernoff distance based counterpart. Although the difference in detection performance is very minor for 3-bit quantization, it is recognizable for 2-bit quantization and a considerable difference can be observed if 1-bit quantization is of interest. For 2- and 3-bit quantization, the difference between two methods tend to decrease as the number of sensors increases.

B. Non-identically distributed observations

Consider the hypothesis testing problem with non-identically χ^2 distributed random variables,

$$\mathcal{H}_m : Y_k \sim f_m^k(y_k) = \frac{y_k^{\frac{W}{2}-1} e^{-\frac{y_k}{2(\psi_k+1)^m}}}{2^{\frac{W}{2}} (\psi_k + 1)^{\frac{mW}{2}} \Gamma\left(\frac{W}{2}\right)}, \ m \in \{0,1\},$$

where W is the number of samples collected by each sensor, ψ_k is the signal-to-noise ratio (SNR) and Γ is the gamma function. These distributions arise from a signal detection problem, where each detector is an energy detector over a static channel model facing a presumably different SNR. The details of this problem can be found in [15, p. 42]. We assume that each sensor collects $W = 2$ samples and the SNR range of $[0, 8]$dB is divided uniformly to the total number of sensors in the network and assigned to each sensors as

$$\gamma_k = \{0, 8, 0.89, 7.11, 1.78, 6.22, 2.67, 5.33, 3.56, 4.44\}. \tag{20}$$

Figure 4 illustrates the thresholds of $K = 10$ sensors for 2-bit quantization (3 thresholds) in comparison to that of the Chernoff distance based quantizer [15]. The thresholds may vary considerably for higher SNRs while they are very similar for low SNRs. In the next simulation the minimum error probabilities of the sensor networks with various number of sensors and with 1-, 2- and 3-bit quantizations have been computed and compared with that of the Chernoff distance based quantizer. The results have been depicted in Figure 5. We can observe that the proposed scheme offers lower error probabilities than the Chernoff distance based quantizer for all cases and the difference between the error probabilities decreases with the total number of sensors in the network.

V. Conclusion

An algorithm was proposed for the optimization of sensor networks consisting of a finite number of sensors and a fusion center. The algorithm is capable of quantizing both identically as well as non-identically distributed independent sensor observations. The proposed scheme has an exponential time complexity, yet exponentially faster than the naive solution, which makes it practically applicable for small to moderate sensor networks. The motivation behind the proposed scheme is that the existing algorithms are either computationally

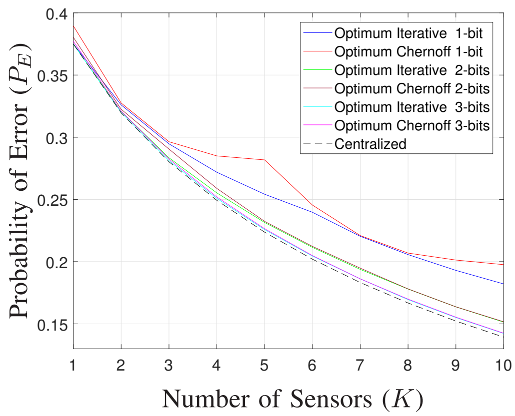

Fig. 2. Minimum error probability for the UL problem.

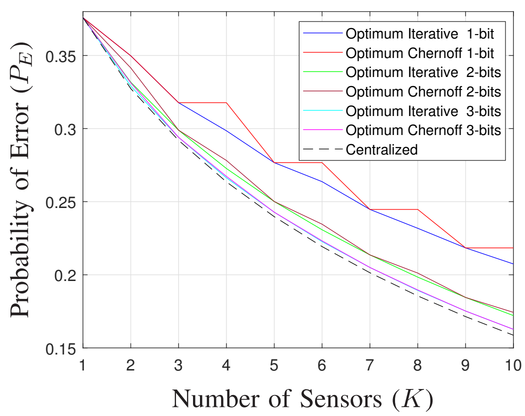

Fig. 3. Minimum error probability for the GG problem.

intractable or their performance is not near optimal for small to moderate number of sensors. Additionally, unlike other schemes no equation solving is necessary. Numerical results indicate that the proposed scheme is superior in performance in comparison to the state-of-the-art. Moreover, the method can easily be extended to density functions which have non-monotone likelihood ratios by using generalized inverse functions.

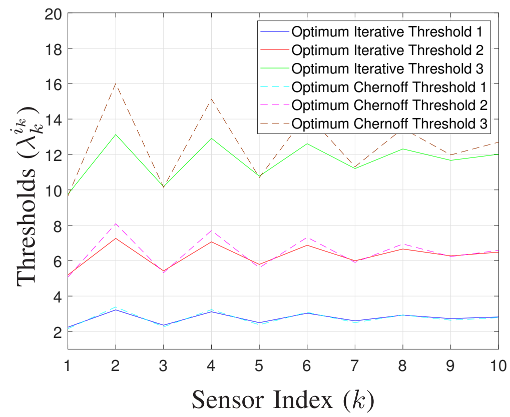

Fig. 4. Thresholds corresponding to each sensor for $K = 10$ sensors.

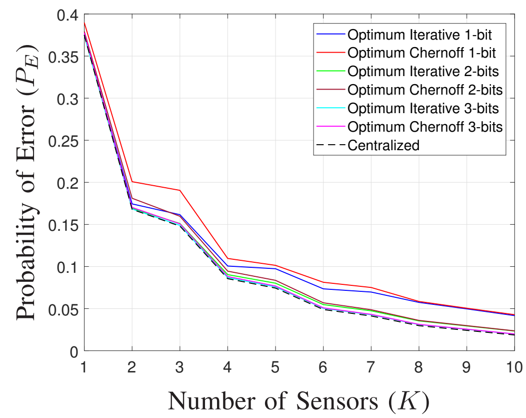

Fig. 5. Minimum error probability versus the number of sensors.

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