HB-DTW: Hyperdimensional Bayesian Dynamic Time Warping for Non-uniform Doppler

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***Abstract*—Acoustic signaling systems often suffer from severe distortions from motion induced Doppler. Therefore, estimation and compensation of such time warping have been considered indispensable blocks in various applications including underwater acoustic communications. However, conventional matched filtering based methods often fail to provide robust tracking of time warping trajectories, especially when non-uniform Doppler effects reside in multi-path arrivals separately. In this paper, we propose HB-DTW, a hyperdimensional generalization of standard dynamic time warping (DTW) algorithms, to precisely estimate non-uniform time varying Doppler in acoustic multipath channels. The proposed algorithm exploits a Bayesian approach to map multidimensional temporal scaling, while guaranteeing polynomial time complexity in the length of the signal. Simulation results on synthetic channels with non-uniform Doppler effects are demonstrated to evaluate the proposed method.**

***Index Terms*—Doppler Estimation, Dynamic Time Warping, Acoustic Communications.**

I. Introduction

Estimation of Doppler effects has been considered indispensable for successful implementation of many acoustic signal processing applications including underwater acoustic communication. Computationally expensive ambiguity-function based methods have shown good performance in estimating slowly varying uniform Doppler. However, for fast-fading and highly reverberant environments such as in shallow water acoustics, such matched filter based methods often fail, and no robust approach to the estimation of non-uniformly time varying Doppler along each of the different propagation paths has been found.

Meanwhile, Dynamic Time warping (DTW) has been widely used to find a minimum distance matching between two temporal or spatial sequences with non-linear distortions in their indices for various applications including speech recognition [1], medical image processing [2], radar detection [3], and gesture recognition [4]. However, in spite of its proven performance, DTW has been rarely adopted for the receiver front end processing of acoustic communication systems; this algorithm is not suitable for mapping channel outputs from multi-path arrivals. Figure 1 describes the situation where an autonomous underwater vehicle (AUV) communicates with the tethered receiver in shallow water. As can be seen from the figure, each of the propagation paths not only introduces delay, but also different levels of motion induced Doppler.

In this paper, we propose hyperdimensional Bayesian dynamic time warping (HB-DTW), a generalized form of standard DTW algorithms, which is designed to precisely estimate nonuniform time varying Doppler in multipath channels. Optimality of the proposed algorithm is supported by preliminary achievements in Bayesian estimation and Dynamic Programming problems, while computational complexity is guaranteed to be of polynomial time, when there exist a finite number of paths with non-uniform Doppler.

This paper is organized as follows. In section II, we mathematically formulate non-uniform Doppler estimation problems in multi-path settings. Section III summarizes preliminary results of Bayesian approaches on DTW algorithms, and shows how these approaches can be applied on Doppler estimation in a single-path scenario. In section IV, HB-DTW is proposed to precisely estimate non-uniform time varying Doppler in multi-path channels described in section II. In section V, the performance of the proposed approach is evaluated with simulation results on multi-path fading channels.

II. Problem Statement

Let us assume an acoustic signal $x(t)$ is sent and received between subsonically moving objects in a free space filled with an ideal acoustically dispersionless fluid. Then, a sampled version of the recorded output $y(t)$ with sampling period T, $y[\cdot]$, can be expressed as follows:

$$y[n] = h\, x(w[n]), \tag{1}$$

where $w[\cdot]$ is a monotonically increasing sequence which represents the time-warped mapping due to Doppler, and h refers a time varying gain.

Let us assume $w[\cdot]$ as a random vector on positive real numbers, whose increments Δw are given as i.i.d. random variables , i.e.,

$$w[n] = w[n-1] + \Delta w, \tag{2}$$

and $P_T(\Delta w) : [0, 2T] \mapsto [0, 1]$ is a probability denstiy function of Δw. Note that finite support assumption of $[0, 2T]$ comes from the subsonic constraint [5] and assumptions on the sampling rate relative to the acoustic propagation velocity in the medium.

In multi-path settings like the one described in Figure 1, each propagation paths can be modeled as if transmitted from mirrored phantom sources moving in different directions. Therefore, each of these paths introduce different levels of time-varying Doppler, which we will call paths with non-uniform

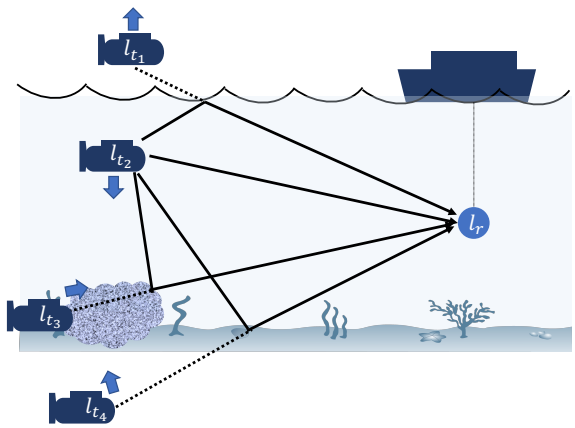

Fig. 1: In shallow water environments, because of reflection and refraction, acoustic signals propagate through multi-paths with non-uniform Doppler shifts due to differences in the motion of mirrored transmitting sources.

Doppler in our following discussion, and the resulting received signal $y[\cdot]$ can be written

$$y[n] = \sum_i h_i x(w_i[n]) + v[n] \tag{3}$$

with the existence of an ambient noise $v[n]$, where h_i's and w_i's each refer to gains, and Doppler from different paths.

In this paper, we will discuss *maximum a posteriori* (MAP) estimation of non-uniform Doppler w_i's in a Bayesian setting, i.e., seek $\hat{\boldsymbol{w}} = (\hat{w}_1, \hat{w}_2, \hat{w}_3, ...)$ such that

$$\hat{\boldsymbol{w}} = \arg\max_{\boldsymbol{w}} p(\boldsymbol{w}|y[\cdot], x(\cdot)). \tag{4}$$

III. Bayesian Formulation of Dynamic Time Warping for Doppler

A. Dynamic Time Warping

Dynamic Time warping is a well-known algorithm, commonly used in speech and image processing [6]. The goal of this algorithm is to find best mapping between two given finite length sequences x and y with length L_x and L_y, which minimizes the Euclidean distance after index-remapping, i.e., we seek two time warping sequences of the same length, w_x and w_y, which minimizes $d(x, y; w_x, w_y) = \sum_{n=1}^{M} \|x[w_x[n]] - y[w_y[n]]\|^2$, and resulting minimum d is called the *DTW distance*.

A *warping path* $\boldsymbol{q} = \{q_1, q_2, ..., q_M\}$ is defined as a sequence of pairs w_x and w_y in that $q_i = (w_x[i], w_y[i]) \in [1 : L_x] \times [1 : L_y]$ for $1 \leq i \leq M$. In addition, time warping sequences w_x and w_y satisfy the following criteria [7]:

- Boundary condition: start and end points of x and y must be matched, i.e., $w_x[1] = x[1]$, $w_y[1] = y[1]$, $w_x[M] = x[L_x]$, and $w_y[M] = y[L_y]$
- Monotonicity condition: two sequences w_x and w_y must be non-decreasing.
- Step size condition: increments of the warping path, $\Delta_i = q_{i+1} - q_i$, can only have values among $(0, 1)$, $(1, 0)$, $(1, 1)$

Now, let us define *DTW matrices* $\Gamma \in \mathbb{R}_+^{L_x \times L_y}$, whose element satisfy $\gamma(i, j) = \min_{\boldsymbol{q}: q_{end}=(i,j)} d(x_1^i, y_1^j; \boldsymbol{q})$, which refers the DTW distance between two sequences x_1^i and y_1^j. The elements γ can be computed by recursively updating the following Bellman equation:

$$\begin{aligned}\gamma(i, j) =& \|x[i] - y[j]\|^2 + \\ & \min\{\gamma(i-1, j-1), \gamma(i, j-1), \gamma(i-1, j)\},\end{aligned} \tag{5}$$

where $\gamma(1, 1)$ is commonly initialized to be zero. The top right element of a DTW matrix is equivalent to the DTW distance between x and y, i.e., $\min d(x, y; w_x, w_y) = \gamma(L_x, L_y)$. The associated computational complexity is of $\mathcal{O}(L_x L_y)$, while brute-force searching algorithms are of $\mathcal{O}(3^{L_x+L_y})$. Also, in each Bellman update, optimal warping transitions are memorized, and can be used to trace back to obtain the corresponding optimal time warping path, which is also called a *DTW warping path*.

B. Bayesian Formulation

Traditional DTW problems can be interpreted as special cases of Bayesian approaches to time warping sequence estimation problems. In [8], a unified interpretation of Hidden Markov Models and Dynamic Time Warping was first proposed. Also, these Bayesian methods have been more elaborately formulated by algorithms called enhanced DTW (EDTW) [9].

Let us assume *a priori* probability on warping paths $\boldsymbol{q} = \{q_1, \ldots, q_M\} = \{(w_x[1], w_y[1]), \ldots (w_x[M], w_y[M])\}$ as a Markov chain. For example, time-homogeneous Markov chain $\boldsymbol{q}$ can be represented with transition probability

$$p(q_{i+1}|q_i) = \begin{cases} \theta_1 & \text{if } \Delta_i = (1, 0), \\ \theta_2 & \text{if } \Delta_i = (0, 1), \\ 1 - \theta_1 - \theta_2 & \text{if } \Delta_i = (1, 1). \end{cases} \tag{6}$$

Then, we define *a posteriori quality measure*, $f(y, x|\boldsymbol{q})$ as

$$f(y, x|\boldsymbol{q}) = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi}\sigma} exp\left(- \frac{|y[w_y[i]] - x[w_x[i]]|^2}{2\sigma^2} \right), \tag{7}$$

and *path quality measure* $f(\boldsymbol{q}, y, x)$ as

$$f(\boldsymbol{q}, y, x) = f(y, x|\boldsymbol{q}) p(\boldsymbol{q}), \tag{8}$$

where M refers the length of $\boldsymbol{q}$. When x and y are given, our goal is to find a warping path $\hat{\boldsymbol{q}}$ which maximizes path quality measure, or equivalently,

$$\begin{aligned}\hat{\boldsymbol{q}} &= \arg\max_{\boldsymbol{q}} f(\boldsymbol{q}, y, x) \\ &= \arg\max_{\boldsymbol{q}} f(y, x|\boldsymbol{q}) p(\boldsymbol{q})\end{aligned} \tag{9}$$

This can also be easily solved by recursively updating Bellman equations of the following:

$$\begin{aligned}\gamma(i, j) =& \frac{\|x[i] - y[j]\|^2}{2\sigma^2} + \\ & \min\{\gamma(i-1, j-1) - \log(1 - \theta_1 - \theta_2)), \\ & \gamma(i, j-1) - \log(\theta_2), \gamma(i-1, j) - \log(\theta_1)\},\end{aligned} \tag{10}$$

which is nothing but a penalized version of Eq.(5), and we can obtain the traditional DTW algorithm as the case with equal priors.

Also, *model quality measure* $f_{\boldsymbol{\theta}}(y, x) = \sum_{\boldsymbol{q} \in \boldsymbol{\mathcal{Q}}} f(\boldsymbol{q}, y, x)$ evaluates the expected performance of the algorithms on parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, \sigma)$. Therefore, parameters $\boldsymbol{\theta}$ can be

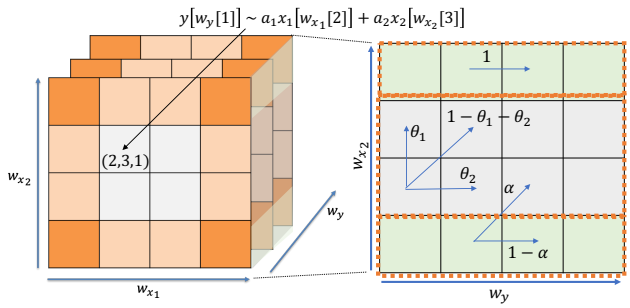

Fig. 2: Schematic of Hyperdimensional Bayesian DTW (HB-DTW) for two paths (left). Transition probabilities of q_i's on different regions are described on (right).

chosen via optimizing this model quality measure. Detailed procedures for parameter selections will be discussed in the next section.

Claim: Assume $x(t)$ is a band-limited signal and $y[\cdot]$ is a sampled version of the recorded output with sampling period T, which is given by a single path version of Eq.(3), $y[n] = x(w[n]) + v[n]$, and $w[\cdot]$ is defined in Eq.(2).

Then, $\forall \epsilon > 0$, there exist some sampling period T and parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, \sigma)$, that satisfies the following, when $\hat{\boldsymbol{q}} = \{(\hat{w}_x[1], \hat{w}_y[1]), \ldots\}$ is given by solving Eq.(9), and $\hat{w} = \arg\max_w p(w|y[\cdot], x(\cdot))$,

$$\frac{1}{M}\sum_{i=1}^{M} \|\hat{w}_x[i] - \hat{w}[\hat{w}_y[i]]\|^2 < \epsilon, \tag{11}$$

where M denotes the length of $\hat{\boldsymbol{q}}$.

Sketch of Proof: We outline a proof of this claim using an approach based on the central limit theorem (CLT). Assume two different i.i.d. random sequences with finite first and second moments. Then, two series, built from cumulative sums of these sequences will eventually converge to the same Gaussian distribution with the exact first and second moments. ■

This claim tells us that, as long as the subsonic constraint is met, a solution to the time warping problem in Eq.(9) well approximates MAP estimation of Doppler shifts in Eq.(4) for bandlimited signals in a 'single-path' scenario.

IV. Hyperdimensional Bayesian DTW for Non-uniform Multipath Doppler shifts

In the previous section, we have explained how time distorted signals can be aligned via DTW in a single path scenario. However, our main focus stands on N-path situations where each path experiences statistically different Doppler induced time warping, i.e.,

$$y[w_y[n]] = \sum_{i=1}^{N} h_i x[w_{x_i}[n]] + v[n]. \tag{12}$$

For example, let us consider a two-paths case. In standard DTW methods, $y[w_y[n]]$ is matched to $x[w_x[n]]$ in DTW matrices as previously explained. However for this scenario, $y[w_y[n]]$ should be matched to $h_1x[w_{x_1}[n]] + h_2x[w_{x_2}[n]]$. Or, we can equivalently postulate that $y[w_y[n]]$, $x[w_{x_1}[n]]$, and $x[w_{x_2}[n]]$ simultaneously match among others on the element $(w_{x_1}[n], w_{x_2}[n], w_y[n])$ of $L_{x_1} \times L_{x_2} \times L_{x_3}$ tensor illustrated in Figure 2.

Similarly as in the previous section, we can define a posteriori, path, and model quality measures as $f_{\boldsymbol{\theta}}(y, x|\boldsymbol{q})$, $f_{\boldsymbol{\theta}}(y, x, \boldsymbol{q})$, and $f_{\boldsymbol{\theta}}(y, x)$, where $\boldsymbol{q} = \{q_1, q_2, ...\}$ and $q_i = (w_{x_1}[i], \ldots, w_{x_N}[i], w_y[i])$. An a posteriori quality measure for HB-DTW, $f(y, x|\boldsymbol{q})$ is defined as

$$f(y, x|\boldsymbol{q}) = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi}\sigma} exp\left(- \frac{|y[w_y[i]] - \sum_{j=1}^{N} h_j x[w_{x_j}[i]]|^2}{2\sigma^2} \right). \tag{13}$$

Also a corresponding path transition prior $p(q_{i+1}|q_i)$ is defined as

$$p(q_{i+1}|q_i) = \prod_{j=1}^{N} p(q_{i+1}^j|q_i^j), \tag{14}$$

where $q_i^j = (w_{x_j}[i], w_y[i])$, and each $p(q_{i+1}^j|q_i^j)$ is given for $\Delta_i^j = q_{i+1}^j - q_i^j$ as described in Eq.(15) (note that Eq.(15) is after the conclusions due to space limitations).

In streaming scenarios, the signal almost always includes null regions around the informational sequences. However, boundary condition for DTW algorithms is not satisfied in this case. In [10], SPRING, a simple but efficient solution for streamed versions of DTW algorithms, was proposed. Instead of relying on brute-force searching methods to find matching boundaries between x and y, an additional 'null' symbol can be inserted at the beginning and the end of $x[\cdot]$, i.e, $x = [0, x_{original}[\cdot], 0]$.

In our formulation, we allocate new transition probabilities on states q_i's with these null indices. For $q_i = (1 \text{ or } L_x, w_y[i])$, path transitions don't necessarily mean time distortions; rather, transition events mean either path arrivals or ends in transmitted signals. Especially, each of jth path arrival is modeled as $geometric(\alpha^j)$, as illustrated in Eq.(15).

Eq.(14) assumes that each of the paths are statistically independent. This need not hold in general, but we sacrifice model complexity when considering correlated paths; while available transition paths are as many as $2^N - 1$, independence enables the use of a transition prior $p(q_{i+1}|q_i)$ to be modeled by only $3N$ parameters. Then, in HB-DTW, path quality maximization in Eq.(9) can be solved by recursively updating the Bellman equation:

$$\gamma(q_i) = \frac{|y[w_y[i]] - \sum_{j=1}^{N} h_j x[w_{x_j}[i]]|^2}{2\sigma^2} + \min_{q' \in \{q' : q' = q_i - [\sum_{k \in \mathcal{K}} e_k, 0]\}} \{\gamma(q') - \log(p(q_i|q')\}, \tag{16}$$

where e_k is the kth standard basis vector in N-dimensional Cartesian coordinates, and $\mathcal{K}$ is a set of every subsets of $[1, N]$ except for the null-set. Resulting $\gamma(q_i)$ forms a *DTW tensor*, $\Gamma \in \mathbb{R}_+^{L_{x_1} \times \ldots \times L_y}$, similar to the Bellman updates form a DTW matrix in a standard DTW. Therefore, computational complexity is of $\mathcal{O}(L_y \prod_{i=1}^{N} L_{x_i})$.

In general transmission scenarios, environmental parameters that affect statistical properties of the model represented by Eq.(3) are not known. However, while maintaining the same order of complexity, the Expectation-maximization (EM) algorithm [11] can be used to update parameters, maximizing the

model quality $f_{\boldsymbol{\theta}}(y,x)$, given observations of x and y, i.e.,

$$\hat{\boldsymbol{\theta}} = \arg\max_{\theta} \sum_{\boldsymbol{q}\in\mathcal{Q}} f_{\boldsymbol{\theta}}(y,x,\boldsymbol{q}), \tag{17}$$

where $\mathcal{Q}$ refers a set of all available warping paths. EM recursions are conducted through the following steps:

1) **Initialization**. Set $\boldsymbol{\theta}$ with an initial guess. For example, we allocate random probability values on transition priors, and coarsely measured noise variance σ.
2) **Expectation**. Compute the expected value of the log of the path quality measure on $\boldsymbol{\phi}$, with respect to the conditional probability of $\boldsymbol{q}$, whose parameters are given by $\boldsymbol{\theta}$, given x and y:

$$\begin{aligned} &E_{(\boldsymbol{q}|y,x),\boldsymbol{\theta}}(\log f_{\boldsymbol{\phi}}(y,x,\boldsymbol{q})) \\ &= \sum_{\boldsymbol{q}\in\mathcal{Q}} \frac{f_{\boldsymbol{\theta}}(y,x,\boldsymbol{q})}{f_{\boldsymbol{\theta}}(y,x)} \log\big(f_{\boldsymbol{\phi}}(y,x,\boldsymbol{q})\big) \\ &= \frac{1}{f_{\boldsymbol{\theta}}(y,x)} \sum_{\boldsymbol{q}\in\mathcal{Q}} f_{\boldsymbol{\theta}}(y,x,\boldsymbol{q}) \log f_{\boldsymbol{\phi}}(y,x|\boldsymbol{q}) \\ &\quad + \frac{1}{f_{\boldsymbol{\theta}}(y,x)} \sum_{\boldsymbol{q}\in\mathcal{Q}} f_{\boldsymbol{\theta}}(y,x,\boldsymbol{q}) \log p_{\boldsymbol{\phi}}(\boldsymbol{q}) \end{aligned} \tag{18}$$

3) **Maximization**. Update $\boldsymbol{\theta}$ with new $\boldsymbol{\theta}'$ s.t.,

$$\begin{aligned} \boldsymbol{\theta}' = &\arg\max_{\boldsymbol{\phi}} \sum_{\boldsymbol{q}\in\mathcal{Q}} f_{\boldsymbol{\theta}}(y,x,\boldsymbol{q}) \log f_{\boldsymbol{\phi}}(y,x|\boldsymbol{q}) \\ &+ \sum_{\boldsymbol{q}\in\mathcal{Q}} f_{\boldsymbol{\theta}}(y,x,\boldsymbol{q}) \log p_{\boldsymbol{\phi}}(\boldsymbol{q}). \end{aligned} \tag{19}$$

Then, $\theta_i^{j\prime}$ is given as

$$\theta_i^{j\prime} = \frac{K(\theta_i^j)}{\sum_{i=1}^{3} K(\theta_i^j)}, \tag{20}$$

where $K(\theta_i^j) = \sum_{\boldsymbol{q}\in\mathcal{Q}} f_{\boldsymbol{\theta}}(y,x,\boldsymbol{q}) L(\theta_i^j;\boldsymbol{q})$. Here, $L(\theta_i^j;\boldsymbol{q})$ denotes the number of corresponding transitions with θ_i^j in the warping path $\boldsymbol{q}\in\mathcal{Q}$. This can be interpreted as a ratio of expected appearances of the corresponding transitions in the model; the more this transition appears in preferable warping paths, the larger the corresponding transition probability.

Meanwhile, $K(\theta_i^j)$ can be rewritten

$$K(\theta_i^j) = \sum_{s\in(S)} \sum_{s':p(q_{s'}^j|q_s^j)=\theta_i^j} p(s'|s)\kappa_f(s)\kappa_b(s'), \tag{21}$$

where $s, s' \in [1:L_{x_1}] \times \ldots \times [q:L_y]$ refers the location of DTW tensor. κ_f and κ_s each refers

$$\kappa_f(s) = \sum_{\boldsymbol{q_f}\in\mathcal{Q}_1^s} f_{\boldsymbol{\theta}}(y,x,\boldsymbol{q_f}), \tag{22}$$

and

$$\kappa_b(s) = \sum_{\boldsymbol{q_b}\in\mathcal{Q}_s^{q_M}} f_{\boldsymbol{\theta}}(y,x,\boldsymbol{q_b}), \tag{23}$$

where $\mathcal{Q}_i^j$ refers to every warping path which connects the point $i\in\mathcal{S}$ to the point $j\in\mathcal{S}$ in DTW tensors. Then, $K(\theta_i^j)$ can be efficiently computed via the forward-backward algorithms as follows:

Forward Algorithm:

$$\begin{aligned} \kappa_f(q_i) = &\exp\left(-\frac{|y[w_y[i]] - \sum_{j=1}^{N} h_j x[w_{x_j}[i]]|^2}{2\sigma^2}\right) \\ &\cdot \sum_{q'\in\{q':q'=q_i-[\sum_{k\in\mathcal{K}} e_k,0]\}} \{\kappa_f(q')\}, \end{aligned} \tag{24}$$

Backward Algorithm:

$$\begin{aligned} \kappa_b(q_i) = &\exp\left(-\frac{|y[w_y[i]] - \sum_{j=1}^{N} h_j x[w_{x_j}[i]]|^2}{2\sigma^2}\right) \\ &\cdot \sum_{q'\in\{q':q'=q_i+[\sum_{k\in\mathcal{K}} e_k,0]\}} \{\kappa_b(q')\}. \end{aligned} \tag{25}$$

4) **Repeat** step 2 and 3 until convergence.

V. Simulation Results

In this section, we evaluate the performance of HB-DTW for non-uniform time warping path estimation problems in a multi-path scenario illustrated in Figure 3.

Here, the signal x is a 500 kHz (T is 2 μs) sampled Gaussian pulse with 30 kHz bandwidth modulated at 50 kHz center frequency. The signal y is a recorded output of the channel, where two paths with different time-varying Doppler exist; path gains for these arrivals are 1 and 0.5. Two warping paths are generated via Eq.(4.15), where $P(\Delta_{w_{x_1}}) \sim$ uniform$(0.5T, 1.1T)$ and $P(\Delta_{w_{x_2}}) \sim$ uniform$(0.9T, 1.8T)$. Before running HB-DTW on numerically generated signals, model parameters were updated after 10 EM iterations. To evaluate the proposed method by comparison, a non-Bayesian implementation of the hyperdimensional DTW was also conducted, which will be denoted 'Conventional' in our following discussion.

Estimation results from HB-DTW are shown in Figure 4. As can be seen from Figure 4c and d, both HB-DTW and Conventional algorithms succeeded to estimate $\hat{y}$, a denoised version of y, which is regenerated from x and estimated time warpings $\hat{w}_{x_1}$ and $\hat{w}_{x_2}$. However, the non-Bayesian algorithm

$$p(q_{i+1}^j|q_i^j) = \begin{cases} \theta_1^j & \text{if } \Delta_i^j = (1,0),\ w_{x_j}[i] \neq 1, \text{ and } w_{x_j}[i] \neq L_x, \\ \theta_2^j & \text{if } \Delta_i^j = (0,1),\ w_{x_j}[i] \neq 1, \text{ and } w_{x_j}[i] \neq L_x, \\ 1-\theta_1^j-\theta_2^j & \text{if } \Delta_i^j = (1,1),\ w_{x_j}[i] \neq 1, \text{ and } w_{x_j}[i] \neq L_x, \\ \alpha^j & \text{if } \Delta_i^j = (1,1) \text{ and } w_{x_j}[i] = 1, \\ 1-\alpha^j & \text{if } \Delta_i^j = (0,1) \text{ and } w_{x_j}[i] = 1, \\ 1 & \text{if } \Delta_i^j = (0,1) \text{ and } w_{x_j}[i] = L_x, \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

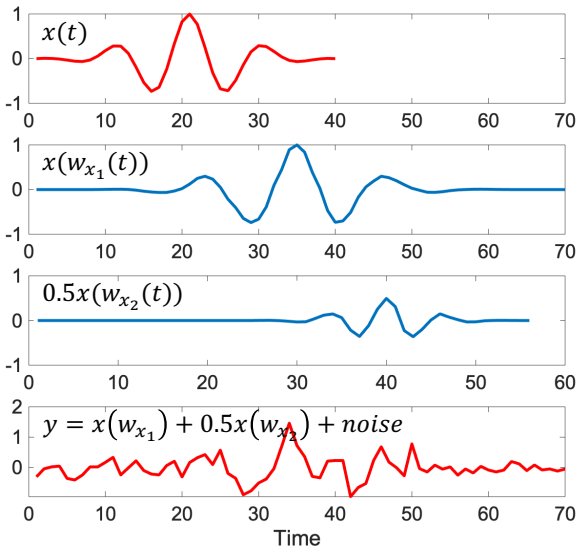

Fig. 3: Multi-path output $y(t)$ (fourth) is a sum of two time-distorted recording of input $x(t)$ (first) through synthetically generated time warping functions w_{x_1} (second) and w_{x_2} (third). Path gains are each selected to 1 and 0.5.

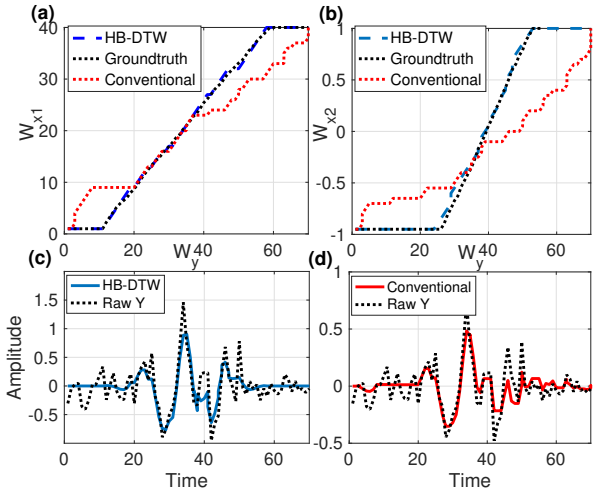

Fig. 4: Comparisons between HB-DTW and Conventional method without Bayesian approaches upon warping path estimation results for signals generated in Figure 3: Estimated warping paths for (a) the first and (b) the second path arrivals, and regenerated y from time warping estimates from (c) HB-DTW and (d) Conventional method.

failed to approximate time warping trajectories, while HB-DTW successfully tracked the groundtruth path. The reason standard DTW fails estimating warping paths, in spite of impressive denoising performance, is that its objective is not targeted to find w_{x_i}'s but designed to minimize Euclidean DTW distance between y and $\hat{y}$; this is equivalent to the least square estimation of $\hat{y}$, while the proposed HB-DTW well approximates the solution of the original problem in Eq.(4).

VI. Conclusion

In this paper, HB-DTW, a hyperdimensioanl DTW method that exploits Bayesian modeling of time distortion, was proposed. We showed that the proposed method can be used to robustly estimate non-uniform time distortions of multi-path arrivals. This was achieved by adding additional dimensions on conventional DTW algorithms, which represent different path arrivals with non-uniform time distortions. In future work, we will deepen the analysis on hyperdimensional time warping problems by focusing on more general scenarios with non-stationary environments. Also, experimental evaluation of the proposed method will be followed with field measured data from experimental underwater acoustic communication systems.

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