Using Interpolated FIR Technique for Digital Crossover Filters Design

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***Abstract*—Digital filters design is an important aspect of the audio processing. They are used in several applications and the key point is not only the achieved final audio quality but also their computational complexity. In this context, the use of Interpolated Finite Impulse Response (IFIR) filters is proposed for the optimized implementation of a digital crossover network. In particular, the structure can be viewed as an analysis filter-bank that can be optimized introducing IFIR filters that allow to reach very narrow transition bands, with low computational complexity and linear phase, avoiding ripple between adjacent bands. Starting from a real-time implementation of the proposed crossover network, several experiments have been conducted to prove the effectiveness of the proposed approach in terms of computational complexity and obtained audio quality.**

***Index Terms*—Interpolated FIR, Digital Filters Design, Digital Crossover Network**

I. Introduction

Digital audio filters design is an important topic of audio processing. They have an extensive role in several applications such as sound synthesis, audio effects, spatial sound, sound enhancement and microphone and/or speaker signals processing. The design of these filters is mainly focused on the perceived audio quality but another important aspect is the computational complexity they can achieve since they are usually involved in real time applications. In this context, a valuable methodology for audio filters design is the interpolated FIR (IFIR). This technique firstly developed by Neuvo et al. [1] is capable of deriving narrow band filters with less complexity relative to conventional filter design methods [2]. For this characteristic, it is suitable for those applications in which the input signal has to be divided in subband without loss of information, using a structure similar to a filter-bank. An example of application is the use of IFIR technique to build an uniform filter-bank for the development of a graphic equalizer, as reported in [3]. Among all the possible audio applications, also crossover networks can be viewed as an analysis filter-bank and are promising candidates for the use of this technique. More in details, digital crossover filters allow to divide the input signal in subband to correctly feed each selected loudspeaker avoiding distortion and drivers damages [4]. An important aspect is that the split of the original signal should be performed preserving its quality and integrity not at the expense of the computational complexity. The requirements for conventional high quality loudspeaker crossover networks are reported in [5] and they can be resumed in the following four points: (a) flatness in the magnitude of the combined outputs, (b) adequate steep cut-off rates of the individual filters in their stop bands, (c) acceptable polar response for the combined output, taking into account the physical separation of the drivers and (d) acceptable phase response for the combined output, the most desirable characteristic being phase linearity. In the technical literature, both FIR and IIR implementations have been presented separately. Their respective strengths mainly depend on FIR and IIR filter characteristics. FIR realizations can approximate desired frequency responses arbitrarily well for sufficiently long impulse responses, they are easily designed to achieve linear phase, and are always stable. However, the number of filter coefficients required for sharp-cutoff filters is generally quite large leading to high computational costs and unacceptable delay. On the other hand, IIR filters are characterized by the fact that arbitrary magnitude characteristics can readily be approximated, and designs are generally very efficient (small number of poles and zeros), especially for sharp-cutoff filters. In spite of this, no exact linear phase designs are possible and IIR are not guaranteed to be stable when implemented. In [6]–[8], FIR realizations have been investigated, in which the high frequency channel is the complementary of the low frequency one and the filtering process can be carried out through frequency domain methods (i.e., overlap and save). In [9], a technique for designing digital linear-phase FIR crossover systems, based on the principle of vector space projections, is proposed. In [10] the crossover network has been designed exploiting a genetic algorithm, while in [11], B-spline functions are used to design a multi-way crossover network. In [12], a procedure to develop an optimal multirate filter structure based on frequency sampling method in the weighted least mean squared sense is presented.

Concerning IIR implementations, Bessel and Linkwitz-Riley filters [13], [14] are often used for crossover network implementation due to the achievable high roll-off with low computational complexity. However, many other IIR based approaches can be found in literature, e.g., a family of time delay derived crossover is proposed in [15], while a tree structure for a multi-way crossover is proposed in [16], improved in [17], successively. In [18], Bessel polynomials are used to derive a crossover network, while in [19], [20] a crossover networks derived from an elliptic prototype is

reported. Finally, a crossover implementation derived from Bessel filters and Bernstein polynomials is described in [16]. In general, the IIR filters guarantee a low computational complexity, however they present a non-linear phase.
Starting from the state of the art, considering these scenarios, a possible solution to obtain an efficient and linear phase crossover network is presented in this paper exploiting IFIR filter technique. An innovative implementation of an N-way digital crossover network is proposed and deeply analyzed to show its effectiveness in terms of satisfied requirements and computational complexity in comparison with the state of the art.

The paper is organized as follows. Sec. II briefly describes the IFIR filter theory. Sec. III reports the innovative digital multi-way crossover network. Sec. IV analyzes the obtained results for the crossover network in terms of time/frequency analysis and computational complexity. Finally conclusions are written in Sec. V.

II. IFIR Filters Design

The general idea of the IFIR theory is the possibility to develop FIR filters with very strict specifications, guaranteeing a reduced computational cost and linear phase. The IFIR filters are composed of a cascade of two FIR filters [21], as shown in Fig. 1, and the overall frequency response of the IFIR structure is computed as follows:

$$H_{\text{IFIR}}(z) = F(z^L)G(z). \quad (1)$$

The first FIR is designed from the model filter $F(z)$ applying an upsampling by a factor L, while the second FIR $G(z)$ is called interpolator which is designed to attenuate the unwanted copies of $F(z)$, due to the interpolation procedure. In fact, the cut-off frequency of the model filter $F(z)$ is L times greater than the cut-off frequency of the desired filter, so $F(z)$ can be designed using a lower order N. The interpolation procedure consists of adding $L-1$ zeros after each sample of the impulse response of $F(z)$. The upsampling allows to obtain the desired cut-off frequency and generates unwanted copies, which are deleted by the interpolator $G(z)$. In this paper, for the sake of simplicity, the filters are designed with the windowing method using the Kaiser window with a shape parameter $\beta = 10$ [22], but other design techniques can be found in the literature [23]–[25]. The design of the filter $F(z)$ is achieved considering a cut-off frequency of $f_c^{\text{F}} = Lf_c$, where f_c is the cut-off frequency of the desired low-pass filter and L is the interpolation factor. In the proposed system, the order M of the filter $F(z)$ must be an even value, because the filter delay, that is $M/2$ must be an integer value to allow the time synchronization of the crossover outputs. For this reason, M is chosen as the even number closest to the optimum order M_{opt}, as follows:

$$M = 2M_{\text{opt}} - 2\left\lfloor \frac{M_{\text{opt}}}{2} \right\rceil, \quad (2)$$

and M_{opt} is calculated by the following Eq. given by [22]:

$$M_{\text{opt}} = \left\lfloor \frac{A-8}{2.285\Delta\omega} \right\rceil, \quad (3)$$

where the brackets $\lfloor\cdot\rceil$ denote the rounding to the closest integer value, A is the stop-band attenuation and $\Delta\omega$ is the width of the transition band, that is imposed to be twice the cut-off frequency of the model filter f_c^F, e.g.,

$$\Delta\omega = \frac{4\pi f_c^F}{F_s} = \frac{4\pi L f_c}{F_s}, \quad (4)$$

where F_s is the sampling frequency. To further reduce the computational complexity and the memory allocation, in this paper the filter $G(z)$ is imposed equal to the filter $F(z)$, e.g.,

$$G(z) = F(z). \quad (5)$$

Eq. (5) can be applied when the filter $F(z)$ is designed in order to eliminate the images of the interpolated version $F(z^L)$. Empirical tests proved that this characteristic is achieved when the equation $2Lf_c = F_s/L - 2f_c$ is satisfied, that means computing the interpolation factor as

$$L = \left\lfloor \frac{-f_c + \sqrt{f_c^2 + 2f_cF_s}}{2f_c} \right\rceil. \quad (6)$$

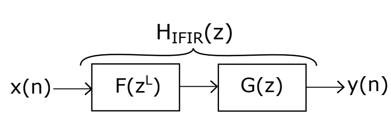

Fig. 1. Cascade of two FIR filters which represents the IFIR implementation.

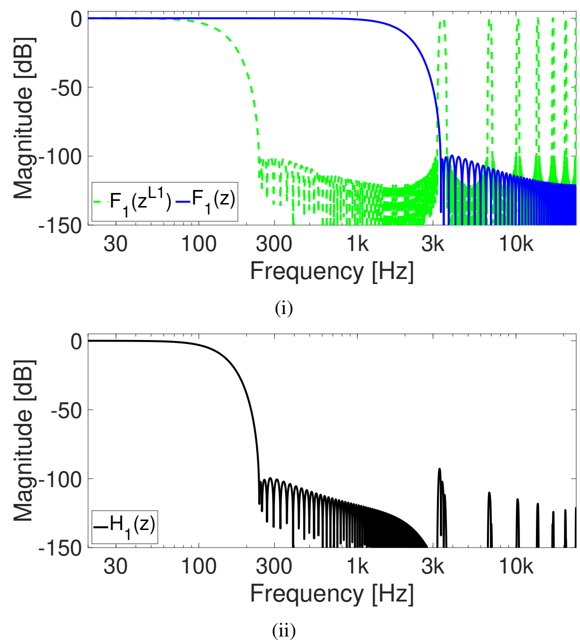

Fig. 2. Design of the first band of the proposed crossover network based on IFIR filters. The filter $F(z)$ is designed with the Kaiser window considering a shape factor of $\beta = 10$, a cut-off frequency of $f_c = 120$ Hz, a stop-band attenuation of $A = 100$ dB, an interpolation factor of $L = 14$ and a filter order of $M = 92$. Fig. (i) shows the model filter and the interpolated filter and Fig. (ii) shows the resulting IFIR filter.

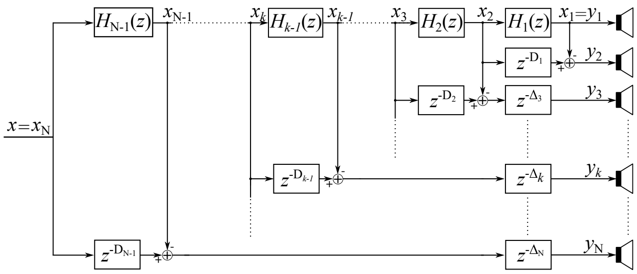

Fig. 3. Scheme of the proposed N-way crossover network. $H_1(z)$, $H_2(z)$, ..., $H_{N-1}(z)$ are the IFIR basis low-pass filters with cut-off frequencies of f_{c_1}, f_{c_2}, ..., $f_{c_{N-1}}$, respectively.

This method allows to obtain a reduction in the number of multipliers and additions needed in the implementation of FIR filtering. In fact, the number of taps of the cascade of the two filters is less than a standard implementation of FIR filter with the same specifications. The number of multiplications Σ of the total proposed IFIR filtering is computed as follows:

$$\Sigma = 2M + 2, \tag{7}$$

where M is the order of $F(z)$. Fig. 2 shows the design of the low-pass IFIR filter. In this case, the cut-off frequency is set to $f_c = 120$ Hz, the stop-band attenuation is $A = 100$ dB and the sampling frequency is $F_s = 48$ kHz. Following Eq.s (6)-(4), the resulting interpolation factor is $L = 14$ and the filter order is $M = 92$, corresponding to a filter length of 93 samples (obtained as $M + 1$). An usual FIR filter designed using the Kaiser window with the same specifications would require a length of 1283 samples, so it demands for a much higher computational cost than the IFIR. Although the reduction in computation, the delay D introduced by the IFIR filtering is comparable to one introduced by the FIR and is computed as follows:

$$D = \frac{ML + M}{2}. \tag{8}$$

Therefore, in the case of Fig. 2, the delay introduced by the filter is $D = 690$ samples, or 14 ms.

III. IFIR Multi-way Crossover design

In the proposed approach IFIR filters are applied to the realization of a multi-way crossover filter. Fig. 3 shows the scheme of the proposed crossover. Considering N ways, the cut-off frequencies of the N bands of the crossover are f_{c_1}, f_{c_2}, ..., $f_{c_{N-1}}$, where f_{c_1} is the cut-off frequency of the first low-pass filter and $f_{c_{N-1}}$ is the cut-off frequency of the last high-pass filter. Starting from $N-1$ basis low-pass filters $H_1(z)$, $H_2(z)$, ..., $H_{N-1}(z)$ with cut-off frequencies of f_{c_1}, f_{c_2}, ..., $f_{c_{N-1}}$, respectively, the combination of these low-pass filters and their high-pass complementary filters allows to obtain the N outputs of the crossover network. The basis low-pass filters $H_i(z)$ are IFIR filters obtained, as shown in Fig. 4, as follows:

$$H_i(z) = F_i(z^{L_i})F_i(z), \tag{9}$$

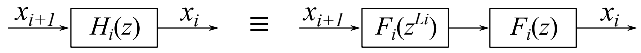

Fig. 4. Design of the i-th basis low-pass filter using IFIR method, with $i = 1, ..., N-1$.

where $F_i(z)$ is the i-th model filter and is designed following the specifications discussed in Sec. II and Eq.s (6)-(4), with $i = 1, 2, ..., N-1$. In the case of $L_i = 1$ the Eq. (9) is not applied and the filter $H_i(z)$ is designed as a single FIR filter, so it is equal to the filter $F_i(z)$, e.g., $H_i(z) = F_i(z)$. The respective high-pass filter $H_i^{\mathrm{H}}(z)$ with cut-off frequency f_{c_i} is obtained as the complementary filter of $H_i(z)$ as follows:

$$H_i^{\mathrm{H}}(z) = z^{-D_i} - H_i(z), \tag{10}$$

where D_i is the delay calculated following Eq. (8). The use of complementary filters allows to reduce the computational complexity and to guarantee a flat magnitude response of the combined outputs, verifying the requirement (a). Taking into account Fig. 3, the kth output of the crossover network $Y_k(z)$ is computed as follows:

$$Y_k(z) = [X_k(z)z^{-D_{k-1}} - X_{k-1}(z)]z^{-\Delta_k}, \tag{11}$$

where X_{k-1} is obtained as follows:

$$X_{k-1} = X(z)\prod_{i=k-1}^{N-1} H_i(z), \tag{12}$$

with $k = 2, ..., N$ and considering $X_N(z) = X(z)$. The synchronization delay Δ_k is applied starting from the third band and is defined as follows:

$$\Delta_k = \sum_{i=1}^{k-2} D_i, \tag{13}$$

with $k = 3, ..., N$ and D_i is the delay introduced by the i-th basis filter and it is calculated following Eq. (8). The output of the first band $Y_1(z)$ is simply equal to $X_1(z)$ that is obtained by Eq. (12). Finally, the total delay of the crossover network τ is computed as follows:

$$\tau = \sum_{i=1}^{N-1} D_i. \tag{14}$$

The computational complexity is given by the number of operations per sample. The number of multiplications of the proposed crossover network is computed as follows:

$$\text{n}^\circ \text{ Mul} = \sum_{i=1}^{N-1} c_i M_i + c_i, \tag{15}$$

and the number of additions is calculated as follows:

$$\text{n}^\circ \text{ Sum} = \sum_{i=1}^{N-1} c_i M_i + 1, \tag{16}$$

where M_i is the order of the i-th model filter $F_i(z)$, N is the number of ways of the crossover and c_i is a parameter that depends on the value of the i-th interpolation factor L_i as follows:

$$c_i = \begin{cases} 2, & \text{if } L_i > 1, \\ 1, & \text{if } L_i = 1. \end{cases} \tag{17}$$

IV. Experimental Results

The proposed crossover has been tested considering a 4-way configuration with the following cut-off frequencies: $f_{c_1} = 120$ Hz, $f_{c_2} = 1000$ Hz and $f_{c_3} = 8000$ Hz and a sampling frequency of $F_s = 48$ kHz. In this case, three basis low-pass filters $H_1(z)$, $H_2(z)$ and $H_3(z)$ have been designed using the IFIR technique, as explained in Sec. II, obtaining the following interpolation factors: $L_1 = 14$, $L_2 = 4$ and $L_3 = 1$, and the following filters orders: $M_1 = 92$, $M_2 = 38$ and $M_3 = 20$. The evaluation has been carried out examining the four requirements listed in Sec. I, comparing the proposed crossover with the Linkwitz-Riley state-of-the-art approach, with the time filtering of equivalent FIR filters and with the FFT implementation. The Linkwitz-Riley crossover network [13] is obtained considering 4th order filters. The FIR crossover is obtained implementing the same scheme of Fig. 3, but the basis filters $H_i(z)$ are designed as normal FIR filters with the Kaiser window with a shape parameter of $\beta = 10$ and the following orders: $M_1 = 1282$, $M_2 = 154$, $M_3 = 20$. The FFT method is obtained by calculating the frequency response of each band of the FIR crossover and applying the Overlap and Save algorithm considering a FFT length of 1024 samples. Tab. I shows the results obtained by the experimental tests. In the table, the checkmark means the verification of the considered requirement, while the distortion index (DI) quantifies the level of distortion and it is calculated as follows:

$$DI = \frac{\max |T(e^{j\omega})|_{\text{dB}} + \min |T(e^{j\omega})|_{\text{dB}}}{2}, \tag{18}$$

where $T(z)$ is the sum of all the bands frequency responses of the crossover. The DI should take values close to 0 dB to have a flat response. The Linkwitz-Riley crossover guarantees only the requirements (b) and (c). In particular, regarding the magnitude flatness, Fig. 5(i) shows the magnitude frequency response of the combined outputs comparing Linkwitz-Riley with the proposed system, confirming the results obtained for the distortion index. In fact, Linkwitz-Riley presents a distortion of 0.5 dB, while the proposed crossover shows a completely flat response. Fig. 5(ii) shows the comparison in terms of magnitude frequency response of the four bands. In the proposed approach, the stop-bands at the low frequencies have a smaller attenuation than the Linkwitz-Riley crossover, while the high frequencies are more attenuated. However, a good suppression of the low frequencies that reach the last driver (generally a tweeter) and could damage the loudspeaker is obtained. For this reason, the requirement (b) on the cut-off rate is verified by both the techniques. Fig. 6 shows the polar diagrams corresponding to the considered cut-off frequencies of the 4-way crossover. The figure is obtained taking into account the equations reported in [26] and compares the proposed IFIR crossover with the Linkwitz-Riley method. Both of them show an acceptable polar response, verifying the requirement (c). Fig. 7 shows the combined phase response of the total 4-way crossover, comparing Linkwitz-Riley method with the proposed one. As expected, the proposed crossover presents a linear phase and this means a symmetric time response, satisfying the requirement (d). Regarding the latency and computational cost, the Linkwitz-Riley method presents the lowest computational cost and the lowest latency, as expected. All the other FIR methods are based on the proposed system changing the implementation, so they verify all the four requirements but they differ in computational cost and latency. The FIR method is the most expensive in terms of number of operations reaching a total of 2918 operations per sample, while the FFT implementation shows the highest latency (21 ms). The proposed method has a latency of 16 ms similar to the FIR method and requires a total of 568 operations per sample (285 multiplications and 283 additions), that is smaller than both the FIR method and the FFT implementation.

TABLE I
Comparison between crossovers, evaluating the requirements, the distortion index, the latency and the computational cost

Crossover	Magnitude flatness (a)	Cut-off rate (b)	Polar response (c)	Phase response (d)	DI (desired 0 dB)	Latency	Number of Sum	Number of Mul	Total Operations
Linkwitz-Riley		✓	✓		0.5 dB	4 ms	32	36	68
FIR	✓	✓	✓	✓	0 dB	15 ms	1459	1459	2918
FFT	✓	✓	✓	✓	0 dB	21 ms	360	244	604
Proposed	✓	✓	✓	✓	0 dB	16 ms	283	285	568

V. Conclusions

In this paper, the IFIR filters are applied for the implementation of a digital multi-way crossover network. The experimental results have demonstrated the great performances of proposed approach in terms of flatness of the magnitude response of the combined outputs, adequate steep cut-off rates of the filters, acceptable polar response and linear phase response, verifying all the requirements needed for a high quality loudspeaker crossover network. Moreover, an analysis of the computational complexity has been carried out, demonstrating a low computational cost in comparison with other linear-phase implementations.

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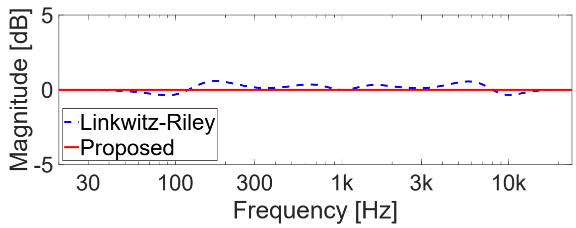
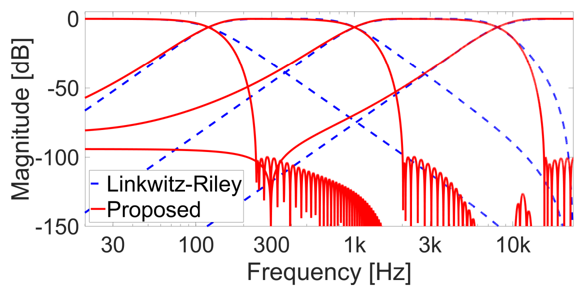
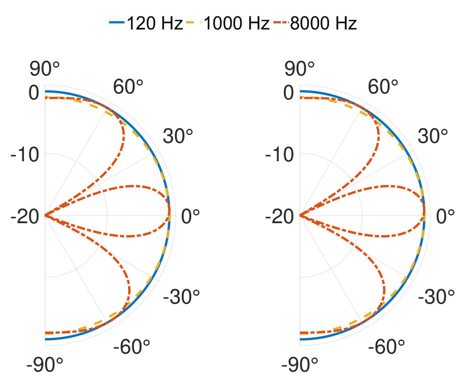


(i)



(ii)

Fig. 5. Comparison between 4th order Linkwitz-Riley crossover with the proposed IFIR crossover considering the following 4 bands: <120Hz, 120Hz-1kHz, 1kHz-8kHz, >8kHz. Fig. (i) is the total magnitude frequency response of the combined outputs of the crossover, while Fig. (ii) shows the magnitude frequency response of each band.



(i) (ii)

Fig. 6. Polar plot of the considered 4-way crossover network using (i) the Linkwitz-Riley method and (ii) the proposed IFIR method for the filters design, considering a distance between loudspeakers of 5 cm and distance from the origin of 1 m.

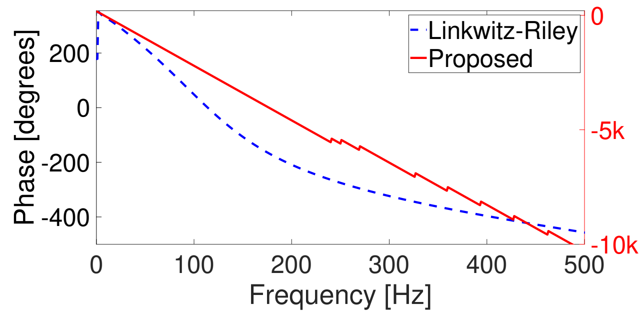


Fig. 7. Combined phase response of the 4-way crossover network, comparing 4th order Linkwitz-Riley crossover with the proposed IFIR crossover.

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