Waveform Design for a Joint MIMO Radar and Communication Systems

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***Abstract*—We consider the waveform design for a joint multiple-input multiple-output (MIMO) radar and communication systems. The aim of the waveform design is to approximate a desired radar beampattern and achieve certain communication rate under radar power constraints. Since the resulting fractional problem is nonconvex, we first convert the problem to an integral problem, and then solve it based on alternating direction method of multipliers (ADMM) algorithm. Finally, numerical results are provider to verify the performance of the proposed method.**

***Index Terms*—MIMO Radar, MIMO Communication, Beampattern, ADMM, Communication rate**

I. Introduction

With the advent of the 5G Era, high-frequency wireless communication design has gained popularity. The growing scarcity of spectrum resources in this context. To solve this problem it is necessary to explore the possibility of coexistence of communication systems with other electronic devices in the same frequency band, and radar equipment is considered. For example, the 5G millimeter wave communication band is very close to the operating band of the vehicle-mounted millimeter wave radar [1]. For the problem of radar communication spectrum sharing, there are two main research ideas. One is Radar-Communication-Coexistence (RCC), and the other is Dual-Functional Radar-Communication system (DFRC). The former considers that the radar and communication systems are independent of each other, but share the same spectrum, and the interference between the two systems is reduced by designing the relevant parameters. The latter considers that the radar and communication systems are on the same hardware platform, and the spectrum coexistence between communication and radar is achieved by designing an integrated signal processing scheme [2]. In this paper, the former is mainly studied.

For the RCC technique, the [3] establishes a spectrum sharing model for pulsed radar and communication systems, precoding on each channel as an optimization variable, introducing the concept of compound rate in the communication system, and considering the signal-to-noise ratio (SINR) of the radar system to establish a joint optimization model. The [4] studies the coexistence problem of MIMO radar and MIMO communication system under clutter interference by jointly optimizing the radar transmit waveform, the receive filter, and the communication space-time transmission covariance matrix to establish an optimization model with maximizing the radar signal-to-noise ratio as the objective function.

In this paper, we consider the spectral coexistence problem of a MIMO radar and a MIMO communication system [5]–[7], taking the radar's transmit waveform as the optimization variable and the radar's primary flap power minimum as the objective function, and matching the radar's transmit beampattern with the desired beampattern by constraining the primary flap power sum and the secondary flap power sum respectively while ensuring a certain lower limit on the communication rate. For the constructed optimization problem, firstly, the communication rate constraint is approximated by using the first-order Taylor expansion; then the objective function is approximated as a convex function by using the log-sum-exp formula and mathematical transformation [8]; finally, the convergent solution about the waveform vector is obtained by using the ADMM method to eliminate the equation constraint.

II. Signal Model

Consider the coexistence of a MIMO communication system and a co-located MIMO radar, both of which are in the same frequency band. The MIMO radar transmits narrowband signals with M uniform line arrays, while the communication system has Nt transmit arrays and Nr receive arrays and the transmitted signals are also narrowband. It is assumed that the symbol rates of both systems are the same and synchronized. In order to reduce the interference of the radar system to the communication system, the transmit direction map of the MIMO radar can be designed so that the transmit power is mainly concentrated in the non-communication receiving array area. Assume that the narrowband emission signal at moment l of the M array elements is $\mathbf{s}_l = [s_1(l), ..., s_i(l), ...s_M(l)]^T$, where $s_i(l)$ is the transmitted signal of the i-th array element at time l, $i = 1, 2, \ldots, M; l = 1, 2, \ldots, L$, and L represents code length. The synthesis signal seen at the direction θ is given by

$$\mathbf{y}^{\mathrm{H}}(\theta) = \mathbf{a}^H(\theta)\mathbf{S} \tag{1}$$

where $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_L] \in \mathbb{C}^{M\times L}$ is the space-time transmit waveform matrix, $\mathbf{a}(\theta) = \frac{1}{\sqrt{M}}\left[1, e^{\frac{j2\pi d \sin\theta}{\lambda}}, \ldots, e^{\frac{j2\pi(M-1)d\sin\theta}{\lambda}}\right]^T$ is guidance vector of transmitting array. d represents the array element spacing, λ indicates the wavelength of the signal. The signal power received at direction θ can be expressed as

$$\begin{aligned} p(\theta) = E[\mathbf{y}^H(\theta)\mathbf{y}(\theta)] &= \left(\mathbf{S}^H\mathbf{a}(\theta)\right)^H \left(\mathbf{S}^H\mathbf{a}(\theta)\right) \\ &= \mathbf{a}^H(\theta)\mathbf{S}\mathbf{S}^H\mathbf{a}(\theta) \end{aligned} \tag{2}$$

Define s=vec($\mathbf{S}$), the power received at direction can also be written as

$$p(\theta) = \mathbf{s}^H\left(\mathbf{I}_L \otimes \mathbf{R}(\theta)\right)\mathbf{s} \tag{3}$$

where $\mathbf{I_L}$ is L-dimensional identity matrix, $\mathbf{R}(\theta) = \mathbf{a}(\theta)\mathbf{a}^H(\theta)$.

For a communication system, its transmit signal matrix $\mathbf{X} = [\mathbf{x}(1), \ldots, \mathbf{x}(L)]$ satisfies Gaussian space-time random distribution. Thus, if $\mathbf{x} = vec(\mathbf{X})$, $\mathbf{x} \sim \mathcal{CN}(0, \mathbf{R}_{\mathrm{x}})$. For a fixed MIMO communication system, there is a fixed channel matrix $\mathbf{H}$ between the transmitting and receiving arrays of the system, and the signal from the transmitting end is received at the receiving end of the communication satisfying

$$\mathbf{x}_{\mathrm{r}} = (\mathbf{I}_L \otimes \mathbf{H})\,\mathbf{x} = \tilde{\mathbf{H}}\mathbf{x} \tag{4}$$

Considering the interference of the radar to the communication system, assuming that P channels exist between the radar and the communication receiver, the pth path has steering angle of departure , θ_p, and angle of arrival, φ_p, for $p = 1, ..., P$. The radar signal received by the communication system can be expressed as

$$\begin{aligned} \mathbf{s}_{\mathrm{r}}(l) &= \sum_{p=1}^{P} e^{j2\pi f_p(l-1)}\beta(p)\,\mathbf{v_r}^*(\varphi_p)\,\mathbf{a}^H(\theta_p)\,\mathbf{s}_l \\ &= \sum_{p=1}^{P} e^{j2\pi f_p(l-1)}\mathbf{G}_p(\theta_p, \varphi_p)\mathbf{s}_l \end{aligned} \tag{5}$$

where f_p represents the normalized Doppler frequency of the p-th channel, $\beta(p)$ is the attenuation coefficient on the pth transmission path, $\mathbf{v}_r(\varphi)$ is steering vector of communication receiver, which has the following general form

$$\mathbf{v}_r(\varphi) = \frac{1}{\sqrt{N_r}}\left[1, e^{j2\pi d_r\sin\varphi/\lambda} \ldots e^{j2\pi(N_r-1)d_r\sin\varphi/\lambda}\right]^T \tag{6}$$

where d_r is antenna spacing at the communication receiver.

Then the signal received by the communication system from the radar can be expressed as

$$\mathbf{s}_{\mathrm{r}} = \sum_{p=1}^{P}\mathbf{D}_f \otimes \mathbf{G}_p(\theta_p, \varphi_p)\mathbf{s} = \sum_{p=1}^{P}\tilde{\mathbf{G}}_p(f_p, \theta_p, \varphi_p)\mathbf{s}, \tag{7}$$

where $\mathbf{D}_f = diag(1, \ldots, e^{j2\pi f_p(l-1)}, \ldots, e^{j2\pi f_p(L-1)})$. There is also interference from Gaussian white noise in the communication system, the noise signal vector $\mathbf{n}$ satisfies $\mathbf{n} \sim \mathcal{CN}\left(0, \sigma_{\mathrm{C}}^2\mathbf{I}_{N_rL}\right)$, where σ_{C}^2 is variance of the noise signa. Thus the received signal at the receiver of the communication system can be expressed as

$$\mathbf{y}_C = \mathbf{x}_r + \mathbf{s}_{\mathrm{r}} + \mathbf{n} \tag{8}$$

III. Problem Description

In this section, with the help of the communication rate measure of the [9], i.e., the system capacity per channel use and per DOF under Gaussian interference, we define

$$\mathrm{C}(\mathbf{R}_{\mathrm{x}},\mathbf{s}) = \frac{1}{N_tN_rL}\log_2\det(\mathbf{I}_{NrL} + \mathbf{R}_{Cin}^{-1}\tilde{\mathbf{H}}\mathbf{R}_x\tilde{\mathbf{H}}^H) \tag{9}$$

where

$$\begin{aligned} &\mathbf{R}_{Cin} = E[(\mathbf{s}_r + \mathbf{n})(\mathbf{s}_{\mathrm{r}}+\mathbf{n})^{\mathrm{H}}] \\ &= \sum_{p=1}^{P}\tilde{\mathbf{G}}_p(f_p, \theta_p, \varphi_p)\mathbf{s}\mathbf{s}^H\tilde{\mathbf{G}}_p^H(f_p, \theta_p, \varphi_p) + +\sigma_C^2\mathbf{I}_{NrL} \end{aligned} \tag{10}$$

Consider the communication rate constraint under radar interference, and then combine the radar's emission beampattern to establish the following optimization model

$$\begin{aligned} \max \quad & \min\left\{\boldsymbol{s}^{\mathrm{H}}\tilde{\boldsymbol{R}}(\theta_{\mathrm{m}})\,\boldsymbol{s}\right\}, \theta_{\mathrm{m}} \in \Theta_m \\ s.t. \quad & \mathrm{C}(\boldsymbol{R}_{\mathrm{x}}, \boldsymbol{s}) \geq \mathrm{C_t} \\ & \sum_{\theta_{\mathrm{m}}\in\Theta_m}\boldsymbol{s}^{\mathrm{H}}\tilde{\boldsymbol{R}}(\theta_{\mathrm{m}})\,\boldsymbol{s} \leq P_m \\ & \sum_{\theta_s\in\Theta_s}\boldsymbol{s}^{\mathrm{H}}\tilde{\boldsymbol{R}}(\theta_s)\,\boldsymbol{s} = P_s \\ & \|\boldsymbol{s}\|_2^2 = P_0 \end{aligned} \tag{11}$$

In (10), the main lobe region where the desired target is located and side lobe region are defined as Θ_m and Θ_s respectively. The P_s and P_m, respectively, represent the sum of side lobe power and sum of main lobe power.

Since problem (10) is still a non-differentiable non-convex function, we can use the method in [10]. If the function $f(x) = \log(e^{x_1}+...+e^{x_n})$, the function satisfies the following theorem

$$\max(x_1, \ldots, x_n) \leq f(x) \leq \max(x_1, \ldots, x_n) + \log n \tag{12}$$

Also the objective function can be equated as

$$\min\max\{-\mathbf{s}^{\mathrm{H}}\tilde{\mathbf{R}}(\theta_{\mathrm{m}})\mathbf{s}\}, \theta_{\mathrm{m}} \in \Theta_m \tag{13}$$

Then the objective function is approximated as

$$\max\{-\mathbf{s}^{\mathrm{H}}\tilde{\mathbf{R}}(\theta_{\mathrm{m}})\mathbf{s}\} \approx \alpha \log \sum_{\theta_{\mathrm{m}}\in\Theta_m} \exp(\frac{-\mathbf{s}^{\mathrm{H}}\tilde{\mathbf{R}}(\theta_{\mathrm{m}})\mathbf{s}}{\alpha}) \quad (14)$$

To ensure that the covariance matrix is positive definite, the objective function is then deformed as

$$\begin{aligned} &\alpha \log \sum_{\theta m\in\Theta m} \exp(-\frac{\mathbf{s}^H\tilde{\mathbf{R}}(\theta_m)\mathbf{s}}{\alpha}) \\ &= \alpha(\frac{\xi P_0}{\alpha} + \log \sum_{\theta m\in\Theta m} \exp(-\frac{\mathbf{s}^H(\tilde{\mathbf{R}}(\theta_m)+\xi\mathbf{I}_{ML})\mathbf{s}}{\alpha})) \\ &= \xi P_0 + \alpha \log \sum_{\theta m\in\Theta m} \exp(-\frac{\mathbf{s}^H\hat{\mathbf{F}}(\theta_m)\mathbf{s}}{\alpha}) \end{aligned} \quad (15)$$

where $\hat{\mathbf{F}}(\theta_m) = (\tilde{\mathbf{R}}(\theta_m) + \xi\mathbf{I}_{ML}) \succ 0$, ξ is a very small constant greater than 0. The communication rate constraint still needs to be deformed for the subsequent algorithm solution, first let $\mathbf{V} = \mathbf{s}\mathbf{s}^H$, then a first-order Taylor expansion is applied to $\mathrm{C}(\mathbf{R}_{\mathrm{x}},\mathbf{V})$

$$\mathrm{C}(\mathbf{R}_{\mathrm{x}},\mathbf{V}) \approx \mathrm{C}(\mathbf{R}_{\mathrm{x}},\mathbf{V}^k) - (\mathrm{Tr}(\mathbf{D}(\mathbf{V}-\mathbf{V}^{\mathrm{k}}))) \quad (16)$$

where $\mathbf{V}^k$ is the value of the kth iteration,

$$\begin{aligned} \mathbf{D} &= -\Big(\frac{(\partial\mathbf{C}(\mathbf{R_x},\mathbf{V}))}{\partial\mathbf{V}}\Big)^{\mathbf{T}}_{\mathbf{V}=\mathbf{V^k}} \\ &= \sum_p^P \tilde{\mathbf{G}}_p^H(f_p,\theta_p,\varphi_p)[(\sum_p^P \tilde{\mathbf{G}}_p(f_p,\theta_p,\varphi_p)\mathbf{V}^k\tilde{\mathbf{G}}_p^H(f_p,\theta_p,\varphi_p) \\ &+ \sigma_c^2\mathbf{I}_{N_rL})^{-1} - (\sum_p^P \tilde{\mathbf{G}}_p(f_p,\theta_p,\varphi_p)\mathbf{V}^k\tilde{\mathbf{G}}_p^H(f_p,\theta_p,\varphi_p) \\ &+ \sigma_c^2\mathbf{I}_{N_rL} + \tilde{\mathbf{H}}\mathbf{R}_x\tilde{\mathbf{H}}^H)^{-1}]\tilde{\mathbf{G}}_p(f_p,\theta_p,\varphi_p) \end{aligned} \quad (17)$$

Since $Tr(\mathbf{DV}) = \mathbf{s^H D s}$, the original communication constraint can be deformed as

$$\mathbf{s}^H\mathbf{D}\mathbf{s} \le \tilde{C} = C(\mathbf{R}_x, \mathbf{V}^k) + Tr(\mathbf{D}\mathbf{V}^k) - N_tN_rLC_t \quad (18)$$

so problem (11) can be redescribed as

$$\begin{aligned} \min \quad & \log \sum_{\theta m\in\Theta m} \exp(-\frac{\mathbf{s}^H\hat{\mathbf{F}}(\theta_m)\mathbf{s}}{\alpha}), \theta_{\mathrm{m}} \in \Theta_m \\ s.t. \quad & \mathbf{s}^H\mathbf{D}\mathbf{s} \le \tilde{C} \\ & \sum_{\theta_{\mathrm{m}}\in\Theta_m} \mathbf{s}^{\mathrm{H}}\tilde{\mathbf{R}}(\theta_{\mathrm{m}})\mathbf{s} \le P_m \\ & \sum_{\theta_s\in\Theta s} \mathbf{s}^{\mathrm{H}}\tilde{\mathbf{R}}(\theta_s)\mathbf{s} = P_s \\ & \|\mathbf{s}\|_2^2 = P_0 \end{aligned} \quad (19)$$

IV. ADMM ALGORITHM TO SOLVE

For the optimization problem (19), this paper intends to use the ADMM method to solve this optimization model. To prevent the subsequent fourth-order polynomial on $\mathbf{s}$, introduce the auxiliary variable $\mathbf{h}$, and let $\mathbf{s} = \mathbf{h}$, while transforming the inequality constraint into an equation constraint, then (19) is converted into

$$\begin{aligned} \min \quad & \log \sum_{\theta m\in\Theta m} \exp(-\frac{\mathbf{s}^H\hat{\mathbf{F}}(\theta_m)\mathbf{h}}{\alpha}), \theta_{\mathrm{m}} \in \Theta_m \\ s.t. \quad & \mathbf{s}=\mathbf{h} \\ & \mathbf{s}^H\mathbf{D}\mathbf{h} - \tilde{C} + \lambda_1 = 0, \lambda_1 \ge 0 \\ & \sum_{\theta_{\mathrm{m}}\in\Theta_m} \mathbf{s}^{\mathrm{H}}\tilde{\mathbf{R}}(\theta_{\mathrm{m}})\mathbf{h} - P_m + \lambda_2 = 0, \lambda_2 \ge 0 \\ & \sum_{\theta_s\in\Theta s} \mathbf{s}^{\mathrm{H}}\tilde{\mathbf{R}}(\theta_s)\mathbf{h} = P_s \\ & \mathbf{s}^{\mathrm{H}}\mathbf{h} = \mathrm{P}_0 \end{aligned} \quad (20)$$

Thus, the augmented Lagrangian function of (20) is

$$\begin{aligned} & L(\mathbf{s},\mathbf{h},\mathbf{u},\tau_1,\tau_2,\tau_3,v) \\ &= \log \sum_{\theta m\in\Theta m} \exp(-\frac{\mathbf{s}^H\hat{\mathbf{F}}(\theta_m)\mathbf{h}}{\alpha}) \\ &+ \frac{\rho_1}{2}\|\mathbf{s}-\mathbf{h}+\mathbf{u}\|_2^2 + \frac{\rho_2}{2}\left|\mathbf{s}^H\mathbf{D}\mathbf{h} - \tilde{C} + \lambda_1 + \tau_1\right|^2 \\ &+ \frac{\rho_3}{2}\left|\sum_{\theta m\in\Theta m} \mathbf{s}^H\tilde{\mathbf{R}}(\theta_m)\mathbf{h} - P_m + \lambda_2 + \tau_2\right|^2 \\ &+ \frac{\rho_4}{2}\left|\sum_{\theta s\in\Theta s} \mathbf{s}^H\tilde{\mathbf{R}}(\theta_s)\mathbf{h} - P_s + \tau_3\right|^2 \\ &+ \frac{\rho_5}{2}\left|\mathbf{s}^H\mathbf{h} - P_0 + v\right|^2 \end{aligned} \quad (21)$$

where $\rho_1, \rho_2, \rho_3, \rho_4$ and ρ_5 represent the penalty factors for each constraint, respectively; and $\mathbf{u}, \tau_1, \tau_2, \tau_3$ and v represent the Lagrange multipliers, respectively.

In this paper, we use the ADMM method to (21), then at the (k+1)-th iteration, the updates of each variable are

$$\mathbf{s}^{k+1} = \arg\min_{\mathrm{s}} \left\{\mathrm{L}\left(\mathbf{s},\mathbf{h}^k,\mathbf{u}^k,\tau_1^k,\tau_2^k,\tau_3^k,\lambda_1^k,\lambda_2^k,v^k\right)\right\} \quad (22)$$

$$\mathbf{h}^{k+1} = \arg\min_{\mathrm{h}} \left\{\mathrm{L}\left(\mathbf{h},\mathbf{s}^{k+1},\mathbf{u}^k,\tau_1^k,\tau_2^k,\tau_3^k,\lambda_1^k,\lambda_2^k,v^k\right)\right\} \quad (23)$$

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \mathbf{s}^{k+1} - \mathbf{h}^{k+1} \quad (24)$$

$$\lambda_1^{k+1} = \max\left\{0, \left\{-(\mathbf{s^{k+1}})^H\mathbf{D}\mathbf{h}^{k+1} + \tilde{C} - \tau_1^k\right\}\right\} \quad (25)$$

$$\tau_1^{k+1} = \tau_1^k + \left(\mathbf{s}^{k+1}\right)^H\mathbf{D}\mathbf{h}^{k+1} - \tilde{C} + \lambda_1^{k+1} \quad (26)$$

$$\lambda_2^{k+1} = \max\left\{0, \left\{\begin{array}{l} \sum\limits_{\theta_m\in\Theta_m} -(\mathbf{s}^{(k+1)^{\mathrm{H}}}\tilde{\mathrm{R}}(\theta_m)\mathbf{h}^{(k+1)}) \\ +P_m - \tau_2^k \end{array}\right\}\right\} \quad (27)$$

$$\tau_2^{k+1} = \tau_2^k + \sum_{\theta_m\in\Theta_m} (\mathbf{s}^{(n+1)^{\mathrm{H}}}\tilde{\mathrm{R}}(\theta_m)\mathbf{h}^{(n+1)}) - P_m + \lambda_2^{k+1} \quad (28)$$

$$\tau_3^{k+1} = \tau_3^k + \sum_{\theta_s\in\Theta_s} (\mathbf{s}^{(n+1)^{\mathrm{H}}}\tilde{\mathrm{R}}(\theta_s)\mathbf{h}^{(n+1)}) - P_s \quad (29)$$

$$v^{k+1} = v^k + \left(\mathbf{s}^{k+1}\right)^H \mathbf{h}^{k+1} - P_0 \quad (30)$$

For (22) and (23), which still contain the logarithm function and exponential function, such an optimization problem is any badly solved, and the text combines with the literature [11] to treat the compound function separately, and only the approximation methods for the objective function related to s are listed here in the case of fixed $\mathbf{h}$. The logarithmic approximation of the first term in (21) is

$$\begin{aligned} &\log \sum_{\theta_m \in \Theta_m} \exp(-\mathbf{s}^H \hat{\mathbf{F}}_{\mathrm{m}}(\theta_m)\mathbf{h}) \\ &\leq \sum_{\theta_m \in \Theta_m} (\tfrac{1}{2a_{\mathrm{m}}^k} \mathbf{s}^H \hat{\mathbf{F}}_m^H(\theta_m)\mathbf{h}\mathbf{h}^H \hat{\mathbf{F}}_m(\theta_m)\mathbf{s}) \\ &- \sum_{\theta_m \in \Theta_m} (\tfrac{b_{\theta_m}^k}{a_m^k} \mathbf{s}^H \hat{\mathbf{F}}_m(\theta_m)\mathbf{h}) + \text{constant} \end{aligned} \tag{31}$$

where

$$a_m^k = \sum_{\theta_m \in \Theta_m} \exp(-(\mathbf{s}^k)^H \hat{\mathbf{F}}_m(\theta_m)\mathbf{h}^k) \tag{32}$$

$$\mathrm{b}_{\theta_m}^k = (\mathbf{s}^k)^H \hat{\mathbf{F}}_m(\theta_m)\mathbf{h}^k + \exp(-(\mathbf{s}^k)^H \hat{\mathbf{F}}_m(\theta_m)\mathbf{h}^k) \tag{33}$$

Algorithm 1 The proposed ADMM algorithm for (20).

Input:
$\mathbf{H}, \tilde{\mathbf{G}}_p(f_p, \theta_p, \varphi_p), \mathbf{s_0}, P_s, P_m, P_0, \sigma_C, C_t, \alpha, \xi, \varepsilon, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5$;
Initialization:
Set: $\mathbf{s^k} = \mathbf{s_0}, \mathbf{u}^k = 0, \mathbf{h}^k = 0, \tau_1^k, \tau_2^k, \tau_3^k, \lambda_1^k, \lambda_2^k, v^k, \mathbf{R}_x = \frac{E_t}{\sqrt{L}}\mathbf{I}_{N_t L}, k = 0$;
Repeat
STEP 1:
Compute D **and** $\tilde{C}$ **according to** (17) **and** (18)
STEP 2:
Approximation of (21) **according to** (31)
STEP 3:
Update h^{k+1} **using** (23)
STEP 4:
Update s^{k+1} **using** (23)
STEP 5:
Update Lagrange multipliers and related parameters using (24) **to** (30)
STEP 6:
SET $k = k + 1$
Until:
$\left|\min\{(\mathbf{s}^{k+1})^{\mathrm{H}}\tilde{\mathbf{R}}(\theta_{\mathrm{m}})\mathbf{s}^{k+1}\} - \min\{(\mathbf{s}^{k})^{\mathrm{H}}\tilde{\mathbf{R}}(\theta_{\mathrm{m}})\mathbf{s}^{k}\}\right| \leq \varepsilon$

V. Simulation Results

In MIMO radar system, we consider a uniform linear array of M=10 with half wavelength array element spacing, and each transmit pulse has L=32 samples. The carrier frequency f_0 = 3GHz, the expected normalized gain of main lobe amplitude is 1 and the gain of side lobe region is 0. The communication system consists of $N_t = 10$ transmit elements and $N_r = 10$ receive elements spaced half wavelength apart from each other.

In radar pattern,we define the main lobe region $\Theta_{\mathrm{m}} = [-20°, 20°]$, the number of discretization points is 41, sidelobe region $\Theta_{\mathrm{s}} = [-90° - 30°] \cup [30°, 90°]$, the number of discretization points is 120.

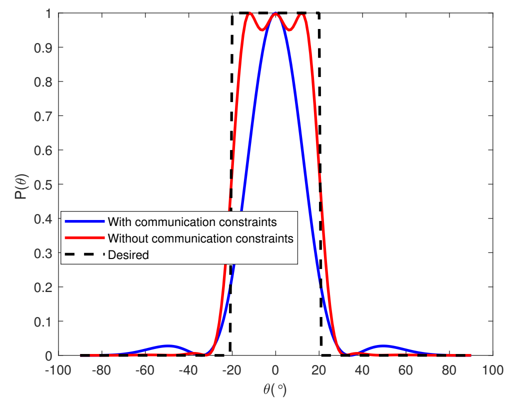

Fig. 1. Pattern matching results

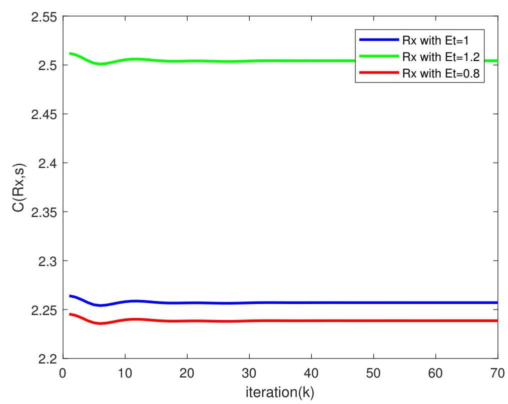

Fig. 2. Communication rate with different $\mathbf{R}_x$

For the interferences between radar and communication system, we set P=21, for the channel matrix $\mathbf{G}$, we select angle parameters $\theta_p = \varphi_p$ are uniformly located at $[-30°, 10°]$.

We assume that $C_t = 1$, $\xi = 0.001$, $\varepsilon = 0.01$, $\sigma_C^2 = 0.001$, respectively. The entries of $\mathbf{H}$ are independently generated following the distribution of $\mathcal{CN}(0,1)$. Fig.1 compares the effect of matching the radar emission beampattern with and without the communication rate constraint. The results show that the radar beampattern matching effect is adversely affected when a certain communication rate is guaranteed. Large fluctuations in the main lobe of the radar and poor suppression of the side lobe under communication constraints.

Fig.2 shows the iterative curve of the communication rate C($\mathbf{R}_{\mathrm{x}}$,s). It can be seen that the algorithm in this paper can better meet the requirements of the communication rate constraint, and while matching the radar emission beampattern.

VI. Conclusion

In this paper, we consider the spectral coexistence model of a MIMO radar and a MIMO communication system. Radar transmit beampattern matching problem under communication rate constraint solved by ADMM algorithm. The simulation results show that the rate lower limit of the communication system can be improved by optimizing the transmit waveform of the radar under a deterministic communication system.

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